(a) (b) (c)

Fig. 2.36 Question 2.3

Question 2.3

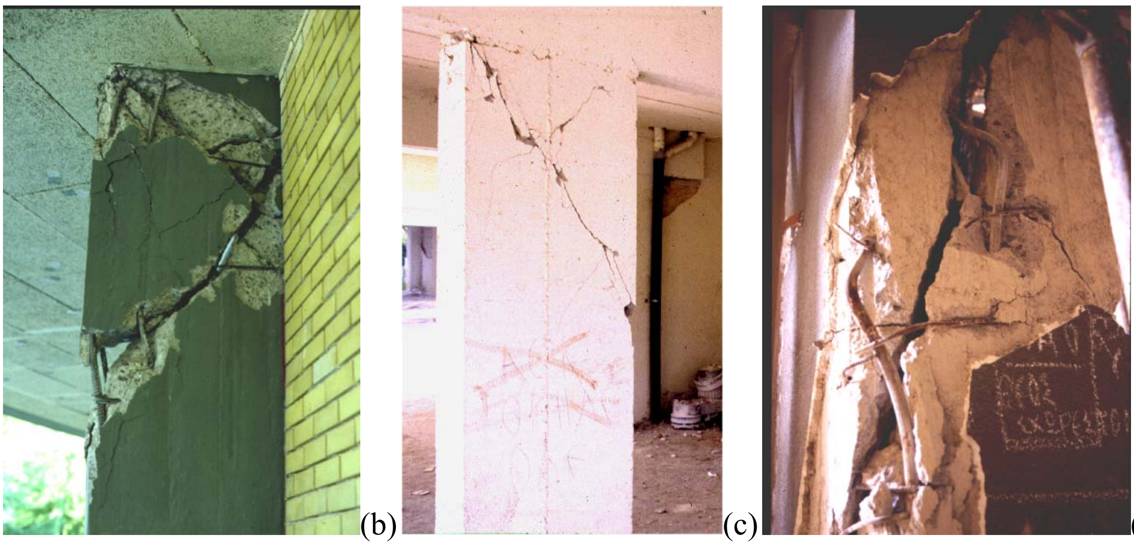
What is the mode of failure or damage of the beams in Fig. 2.36? Would you characterise this case as damage or as failure?

Answer to Question 2.3

All three: Flexural damage (not failure).

Question 2.4

What is the mode of failure or damage of the columns in Fig. 2.37? Would you characterise this case as damage or as failure?



(a) (b) (c) (d)

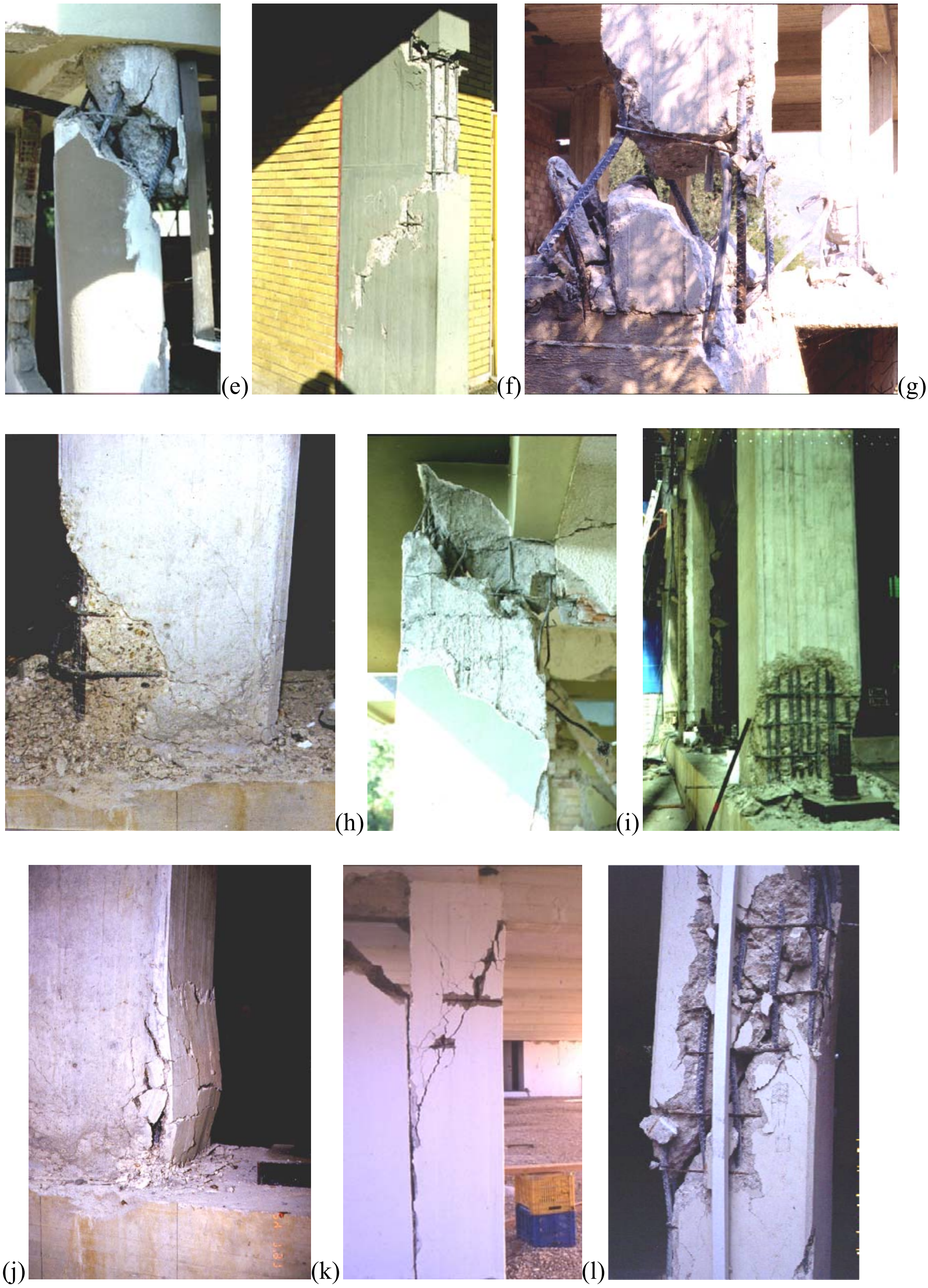


Fig. 2.37 Question 2.4

Answer to Question 2.4

(f), (g), (i), (j): Flexure; all others: shear;

(b), (k): damage; all others: failure (possible exception: (j), (i)).

Question 2.5

What is the mode of failure or damage of the concrete walls in Fig. 2.38? Would you characterise this case as damage or as failure?

(a) (b) (c) (d) (e) (f)

Answer to Question 2.5

All: Shear.

All, except (f): Failure; (f): damage.

Chapter 4

Example 1: A system, whose centres of mass and lateral stiffness coincide in plan, has three uncoupled DOFs: the translations in the two orthogonal horizontal directions, X and Y, and twisting about the vertical axis, Z. It can be shown that the torsional rigidity conditions of EC8 (i.e., that the torsional radii exceed the radius of gyration of the mass) imply that the period of the twisting mode, T_θ, is shorter than those of the translational ones in X, T_X, and Y, T_Y.

Answer:

$T_X = 2\pi\sqrt{(M/K_X)}$, $T_Y = 2\pi\sqrt{(M/K_Y)}$, $T_\theta = 2\pi\sqrt{(I_\theta/K_\theta)}$, where: K_X, K_Y, K_θ: lateral stiffness in X, Y, torsional stiffness about a vertical axis through the centre of mass and stiffness, M, I_θ: mass and rotary moment of inertia about vertical axis through centre of mass and stiffness.

$$T_X > T_\theta \rightarrow M/K_X > I_\theta/K_\theta \rightarrow K_\theta/K_X > I_\theta/M \rightarrow r_Y = \sqrt{(K_\theta/K_X)} > l_s = \sqrt{(I_\theta/M)}$$

$$T_Y > T_\theta \rightarrow M/K_Y > I_\theta/K_\theta \rightarrow K_\theta/K_Y > I_\theta/M \rightarrow r_X = \sqrt{(K_\theta/K_Y)} > l_s = \sqrt{(I_\theta/M)}$$

Example 2: A building has storey masses uniformly distributed ever the floor area, and a structural system consisting of several regularly spaced and similar plane frames in each one of the two orthogonal horizontal directions, X and Y, except for the two exterior frames in each direction, which have half the lateral stiffness of an individual interior frame of the same direction. Such a building cannot fulfill the torsional rigidity conditions in EC8 ($r_X \geq l_s$, $r_Y \geq l_s$), except as equalities and, indeed, only in the special case where the total lateral stiffness is the same in the two directions X and Y.

Answer:

Let's denote by k_X, k_Y, m the lateral stiffness in X, Y, and the mass per unit floor area; they all have a constant value over the plan. Moreover, because of the uniformity of k_X, k_Y, m over the plan, the centres of mass and of lateral stiffness coincide. Let's introduce $a = k_Y/k_X$. The total lateral stiffness in X, Y, and the torsional stiffness about a vertical axis through the centre of stiffness are:

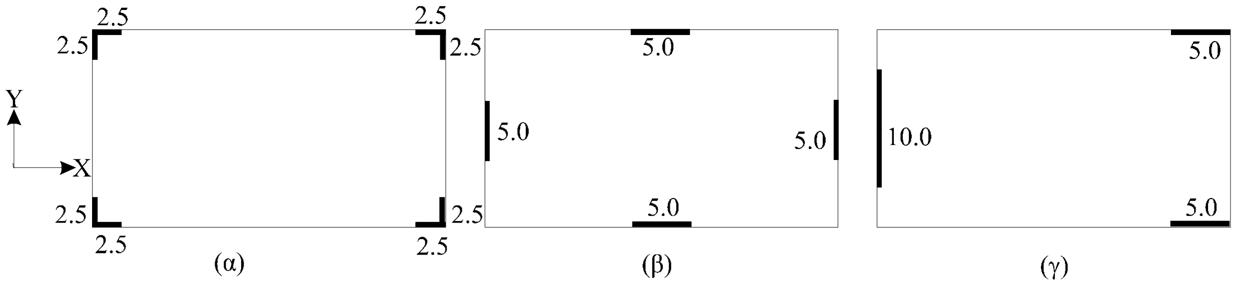
$K_X = \int_A k_X dA = k_X A$, $K_Y = \int_A k_Y dA = a k_X A$, $K_\theta = \int_A (y^2 k_X + x^2 k_Y) dA = k_X \int_A (y^2 + a x^2) dA = k_X(I_X + a I_Y)$, where A, I_X, I_Y are the area and the moments of inertia with respect to centroidal axes X and Y of the floor plan.

The torsional radii are: $r_Y = \sqrt{(K_\theta/K_X)} = \sqrt{[(I_X + aI_Y)/A]}$, $r_X = \sqrt{(K_\theta/K_Y)} = \sqrt{[(I_X + aI_Y)/(aA)]}$.

The radius of gyration of the mass is: $l_s = \sqrt{(I_\theta/M)} = \sqrt{[\int_A(y^2m+x^2m)dA]/[\int_A m dA]} = \sqrt{[(I_X + I_Y)/A]}$.

$r_X \geq l_s \rightarrow (I_X + aI_Y)/(aA) \geq (I_X + I_Y)/A \rightarrow, 1 \geq a; r_Y \geq l_s \rightarrow (I_X + aI_Y)/A \geq (I_X + I_Y)/A \rightarrow, a \geq 1.$

Therefore: $a = 1$, $r_X = l_s$, $r_Y = l_s$.

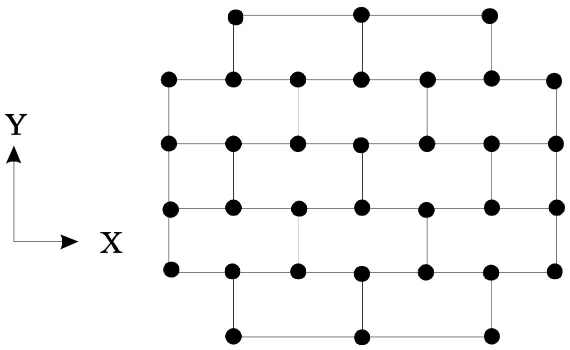


Example 3: A building, 20×35 m in plan, has columns on a 5×5 m grid and shear walls (with 250 mm thickness) in three alternative arrangements, (a), (b), (c), which are compared taking into account the restraint of floor shrinkage, the lateral stiffness and the torsional one with respect to the vertical axis, the vertical reinforcement required for the same total flexural capacity at the base, the static eccentricity, the system's redundancy, etc.

Answer:

The volume of concrete is the same in all three options. At first sight, option (a) seems to make better use of it, because all four walls have biaxial strength and stiffness and to be well placed to maximise the overall torsional stiffness with respect to the vertical axis. However, the walls of the two other options provide larger total lateral stiffness to both horizontal directions, as well as torsional one with respect to the vertical. For the same vertical reinforcement ratio, they also give larger flexural resistance than those in option (a), thanks to their geometry and, secondarily, their larger axial load (due to their larger tributary floor area). Moreover, in option (a) the walls restrain shrinkage of the floors and may lead to cracking. It is also difficult to provide an effective foundation to a wall at a corner in plan, as in option (a). Compared to (b), option (c) provides larger total lateral stiffness and flexural resistance in horizontal direction Y, as well as torsional stiffness with respect to the vertical axis. It has very large eccentricity of the centre of mass with respect to those of stiffness and resistance (which are almost at the centre of the 10 m long wall); this large eccentricity is less of a problem than it seems at first sight, because it is partly resisted by the contribution to torsion about the vertical axis of the two walls in X. The main handicap of option (c) is its lack of redundancy in direction Y and the lack of a load path other than through the 10 m long wall. For these reasons, the ideally balanced option (b) seems better. However, its two walls per direction still provide poor redundancy.

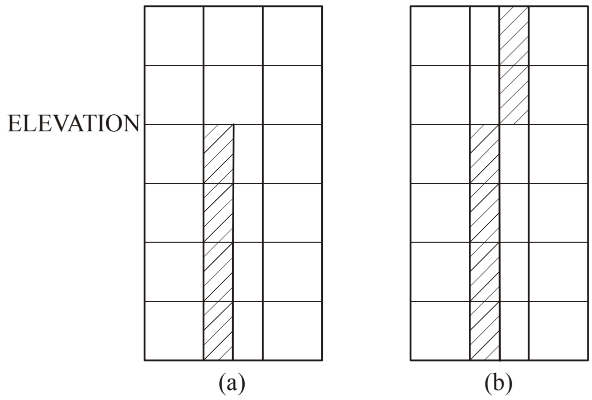
Example 4: Comments on the layout of the framing plan concerning earthquake resistance in the two horizontal directions X or Y (dots are columns, lines depict beams)



Answer:

The building is characterised by perfect symmetry and uniformity in plan. At each corner, the area between the outline of the floor and the convex polygonal line enveloping the floor is less than 2 % of the floor area, well below the 5% limit set in EC8 for regularity in plan. In direction X all the frames are continuous from one side to the opposite. However, in Y, all interior frames are one-bay; there is no continuous frame from one side to the other, except for the two 3-bay exterior ones. So, the building suffers in that direction from lower redundancy and multiplicity of load paths, fewer plastic hinges in beams and less cost-effective use of the concrete in the frames.

Example 5: In the structural systems sketched in elevation as (a) and (b), cross-hatched regions denote walls and vertical lines are columns. Comments and comparison of the two systems from the point of view of regularity in elevation and suitability for earthquake resistance.

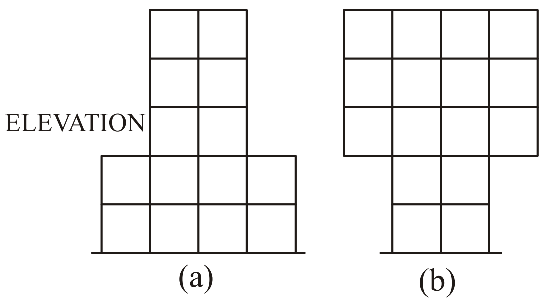


Answer:

Regularity in elevation: System (a) is irregular in elevation, because the wall, which is its main source of lateral force resistance, does not continue to the top. If the criterion for irregularity in elevation is storey lateral stiffness and resistance, system (b) may nominally be less irregular than (a), because these properties are nominally not so much affected by the offset in the wall at floor 4, as by the termination of the wall there in case (a).

Suitability for earthquake resistance: System (b) has a very severe discontinuity in the load path at floor 4, which will lead to more adverse and uncertain response than the termination of the wall at that floor in system (a). System (a) can, in principle, be designed and detailed for the concentration of inelastic deformation demands at the bottom of the 5th-storey columns and capacity-designed against a soft-storey mechanism at that storey. System (b) cannot be reliably designed for predictable seismic response; it is absolutely unsuitable for earthquake resistance.

Example 6: Comments and comparison of the two systems (a) and (b) concerning earthquake resistance.

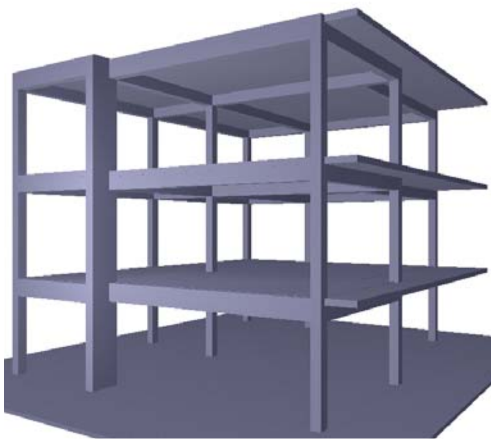


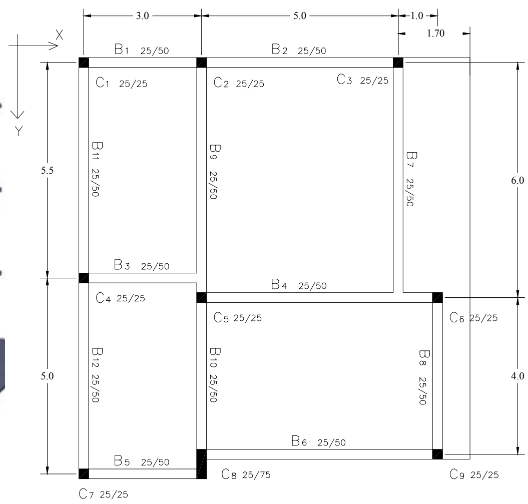
Answer:

Both systems are irregular in elevation, owing to the drastic change of the horizontal dimension at floor 2. However, system (b) is much more adverse for earthquake resistance for many reasons: 1) The outer columns do not continue to the ground; at the 2nd floor their action effects need to be transferred to the central columns, which continue to the ground, via the horizontal elements and the floor diaphragm at that level. 2) Above floor 2, only the central part of the frame is engaged in inelastic action for earthquake resistance; the outer ones follow its displacements, staying in the elastic regime. 3) The central part of the frame, which provides almost all of the earthquake resistance, has less redundancy and a smaller number of possible load paths. 4) The resultant of lateral forces is applied higher up, while the width of the base (distance between the outer columns) is much smaller; this combination increases very much the seismic axial forces at the base of the outer columns and the footings underneath, making very difficult the verification of these columns at the ULS in flexure with axial load, as well of their footings for the corresponding seismic action effects.

Example 7: Suitability for earthquake resistance of the 3-storey building (cross-sectional dimensions in cm), eccentricity of Centre of Mass (as centroid of floor plan) to the Centre of Stiffness (from the basis of the moments of inertia of the columns) and comparison with torsional radii.

Answer:





Judging on the basis of cross-sectional size alone, the columns, unless much heavier reinforced than the beams, are weaker than them at all interior or exterior joints, except that of C8 and B10. So, the building is prone to soft-storey collapse. Beam B3 is indirectly supported on B9 and B7 on B4; so, B3 does not form a proper moment resisting frame with C4, nor B7 with C3. Beams B5, B6 are offset; so, their connection to column C8 is doubly eccentric, and the behaviour of that beam-column joint for bending around global axis X is uncertain; the same can be said for the 2-bay frame these beams form with C7, C8, C9. There are only three frames which are continuous from one side in plan to the opposite without offsets: that of B1, B2 along direction X, those of B9, B10 and B11, B12 in Y. There is two-way eccentricity of the Centre of Mass with respect to the Centre of Stiffness, estimated below in a co-ordinate system X-Y with origin at the exterior corner of column C1:

Floor area = 9.825×10.25+3.25×0.5 = 102.33125 m^2

Co-ordinates, X_{CM}, Y_{CM}, of Centre of Mass, as centroid of the floor plan:

- $X_{CM} = (9.825^2\times 10.25+3.25^2\times 0.5)/(2\times 102.33125) = 4.86$ m,
- $Y_{CM} = (9.825\times 10.25^2+3.25\times 0.5\times 10.5)/(2\times 102.33125) = 5.127$ m.

Moments of inertia of the floor plan with respect to its centroid:

- $I_X = (9.825^3\times 10.25+3.25^3\times 0.5)/3 - 102.3312\times 4.86^2 = 829.11\ m^4$,
- $I_Y = (3.25\times 10.75^3+6.575\times 10.25^3)/3 - 102.3312\times 5.127^2 = 1016.12\ m^4$.

Radius of gyration of the floor plan area with respect to its centroid:

$l_s = \sqrt{[(829.11+1016.12)/102.33125]} = 4.246$ m

Co-ordinates, X_{CS}, Y_{CS}, of Centre of Stiffness, as centroid of the moments of inertia of the columns (the moment of inertia of a 0.25 m square column is symbolised by I):

- For bending in a plane parallel to X: $\sum I_X = 3I$ (for C8) + $8I$ (for C1-C7, C9) = $11I$.
- For bending in a plane parallel to Y: $\sum I_Y = 8I$ (for C8) + $8I$ (for C1-C7, C9) = $16I$.
- $X_{CS} = (0.125\times 3I +3.125\times 10I +8.125\times I +9.125\times 2I)/(16I)= 3.625$ m,
- $Y_{CS} = (0.125\times 3I +5.625\times I+6.125\times 2I +10.125\times I +10.625\times I +10.375\times 3I)/(11I) = 6.375$ m.

Torsional stiffness: $(0.125-3.625)^2\times 3I +(3.125-3.625)^2\times 10I +(8.125-3.625)^2\times I +(9.125-3.625)^2\times 2I + (0.125-6.375)^2\times 3I +(5.625-6.375)^2\times I +(6.125-6.375)^2\times 2I +(10.125-6.375)^2\times I +(10.625-6.375)^2\times I +(10.375-6.375)^2\times 3I = 318(m^2)I$

Torsional radii with respect to the Centre of Stiffness and comparison with radius of gyration of floor plan:

- $r_X = \sqrt{[318I/16I]} = 4.458\ m > l_s= 4.246$ m,
- $r_Y = \sqrt{[318I/11I]} = 5.377\ m > l_s= 4.246$ m.

The building is torsionally stiff, albeit marginally.

Eccentricities, e_X, e_Y, of Centre of Mass with respect to Centre of Stiffness:

- $e_X = X_{CM} - X_{CS} = 4.86 - 3.625 = 1.135$ m, $|e_X| < 0.3r_X = 1.337$ m,
- $e_Y = Y_{CM} - Y_{CS} = 5.127 - 6.375 = -1.248$ m, $|e_Y| < 0.3r_Y = 1.613$ m.

The eccentricities are not large enough to consider the building as irregular in plan.

Example 8: A large building in a moderate seismicity region has 3 to 5 storeys over different parts of its plan and continuous concrete walls over most of the perimeter, with irregularly placed openings of various sizes. Choice of the best option for its seismic design: low strength and high ductility or the opposite?

Answer:

A large number of concrete walls, be it with openings, can provide a low-to-mid-rise building with sufficient strength to resist the design seismic action in a moderate seismicity region elastically (i.e., with q = 1.5) even with little reinforcement. Moreover, EC8's design and detailing rules for ductile walls of DC M or H, were not meant for long walls with irregular openings. Such walls would be better be designed for nearly elastic response. Last, but not least, linear analysis with a q-factor significantly larger than 1.5 cannot predict with any confidence the inelastic response of a system of geometrically complex walls to the design earthquake. Therefore, the prudent and, in all likelihood, most cost-effective choice for its seismic design is for high strength and low ductility.

Example 9: A cooling tower, with circular horizontal section and concrete shell thickness of 120 mm, is designed for wind with an average design value (including the partial safety factor) of p = 2 kN/m^2 of projected vertical surface area. The thin tower shell, with its double curvature, is fairly stiff: its dynamic response is like that of a rigid body on flexible supports (a series of diagonal concrete columns), with uniform response acceleration up the tower and fundamental period in the constant pseudo-acceleration spectral range. Estimation of the design ground acceleration at the site (including the importance factor), Sa_g, above which seismic design for DC L (i.e., with q = 1.5) governs over design for wind actions.

Answer:

If H denotes the total height of the tower's shell and R_m the mean value of its diameter up the height, the design value of the lateral wind force is $2R_mHp$ and that of the seismic base shear (lateral seismic load resultant) is $0.85\times(2\pi(R_m Ht)\varepsilon S_{a,d})$, where ε = 25 kN/m^3 is the unit weight of RC and $S_{a,d} = 2.5Sa_g/q$ the design spectral acceleration (Sa_g in g's). For $2R_m Hp > 0.85\times2\pi(R_m Ht)\varepsilon S_{a,d} = 0.85\times2\pi(R_m H\varepsilon t)(2.5Sa_g/q)$, we need: $0.4qp/(0.85\pi\varepsilon t)$, i.e., $0.4\times1.5\times2/(0.85\times\pi\times25\times0.12)$ = 0.15g > Sa_g, for wind to govern over seismic design with q = 1.5. For this light structure, with large vertically projected area, the Sa_g value exceeds by 50% EC8's recommended limit of 0.1g for design with DC L.

Example 10: A concrete building has aspect ratio ("slenderness") in elevation (: ratio of height from the foundation, H, to width of the base, B, parallel to the seismic action) of 5. The design ground acceleration at the site is Sa_g = 0.3g, the corner period of the spectrum is T_C = 0.6 sec and the fundamental period T = 0.8 sec. The building is designed with EC8's default values of q for planwise irregular, heightwise regular multi-storey, multi-bay frame- or frame-equivalent dual systems: 4.5×1.15 = 5.175 for DC H, 3×1.15=3.45 for DC M, 1.5 for DC L. The DC appropriate for the design of the building is determined, if the design requirement is to have the resultant of the seismic lateral force (acting at 2/3 of the building's height from the foundation, H) and of the total weight of the building passing through: (a) the edge of the foundation in plan (nominal overturning, failure of the ground under the toe of the foundation); (b) one-third of the base width, B, from the centre in plan (: safety factor of 1.5 against overturning); (c) one-sixth of B from the centre in plan (: uplift starts, for linear distribution of soil pressures).

Answer:

The total design lateral force, V, equal to the building weight, W, times $0.85\times(2.5Sa_g/q)(T_C/T)$ (Sa_g in g's), acting at 2/3 of the building's height from the foundation, H, produces an overturning moment at the base:

$M_o = 0.85\times(2.5Sa_g/q)(T_C/T)(2H/3)W$

For the resultant of V and W to pass through the edge of the base in plan (nominal overturning):

$M_o = 0.85\times(2.5Sa_g/q)(T_C/T)(2H/3)W \leq WB/2$: $q \geq 3.19$.

For the resultant of V and W to pass from a point at one-third of the base width, B, from the centre:

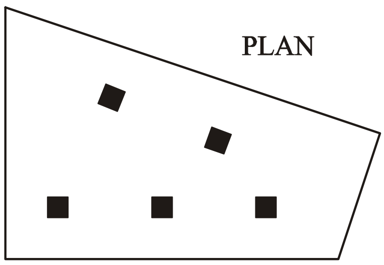
$M_o = 0.85\times(2.5Sa_g/q)(T_C/T)(2H/3)W \leq WB/3$: $q \geq 4.78$.

For the resultant of V and W to pass from a point $B/6$ from the centre:

$M_o = 0.85\times(2.5Sa_g/q)(T_C/T)(2H/3)W \leq WB/6$: $q \geq 9.56$.

Witness how the choice of Ductility Class affects the design of the foundation. Even for a tall building in a high seismicity area, it is easy to prevent nominal overturning or failure of the ground under the toe of the foundation, if design is for DC M or H. If the goal is to retain a safety factor of 1.5 against nominal overturning (a common conventional goal in foundation design), design can only be for DC H. However, it is far from feasible to prevent uplift of the foundation under these conditions.

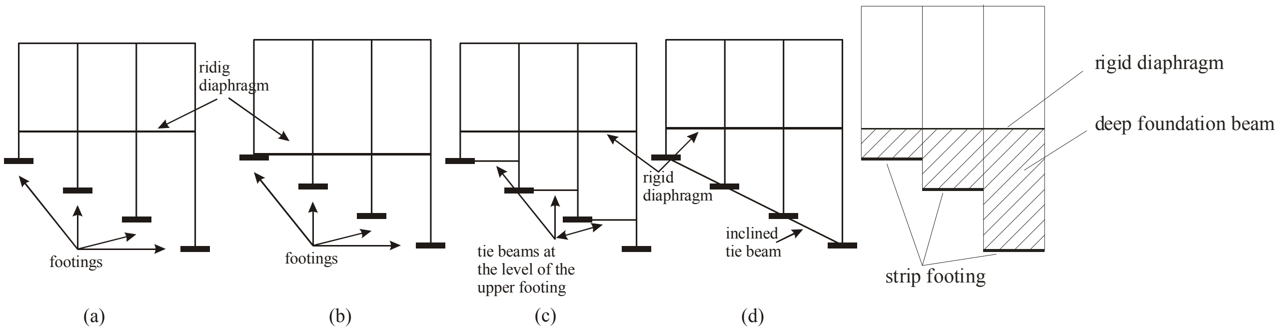
Example 11: A multi-storey building, with quadrilateral plan as shown in the figure, has interior columns in an irregular pattern in plan that serves architectural and functional considerations. Partition walls and interior beams supporting the slab have different layout in different storeys. However, there is no constraint to the type, location and size of lateral force resisting components and sub-systems on the perimeter. Proposals are made and justified for the choice of the lateral-load resisting system and its foundation.



Answer:

The irregular pattern of interior columns in plan and the varying layout of interior beams at different storeys, prohibit the use of continuous in plan and elevation, clear frames inside the building. So, the seismic action should be fully resisted by strong frames around the perimeter, preferably combined with a wall at about length of each side. Interior beams should serve the support of slabs, as well as the pattern and the constraints due to architectural/functional considerations, with the minimum possible cross-section, to minimise the share of the seismic base shears resisted by the interior frames, at the expense of the contribution of the exterior lateral-load resisting system; flat slabs, directly supported on the columns without beams, may be used at the interior. Only the lateral-load resisting system on the perimeter may then be taken as one of "primary seismic elements" per EC8; the interior system may be considered to comprise only "secondary seismic elements" per EC8, taken into account only against gravity loads. A (nearly-basement-high) box foundation system is most appropriate, comprising a deep foundation beam on the perimeter for the lateral-load-resisting elements, footings for the interior columns, a top slab and a grid of tie-beams or a concrete slab at the bottom, connecting the footings with the base of the perimeter foundation beam, as convenient.

Example 12: Pros and cons for earthquake resistance of the alternative foundation schemes (a) to (d) in a building on a steep slope; proposed alternative scheme, with justification.



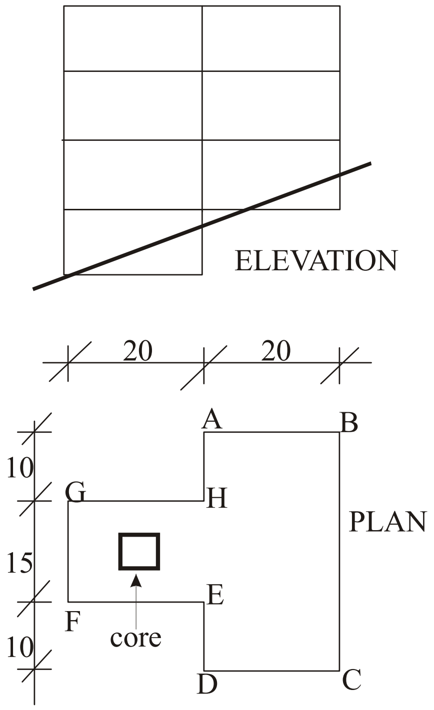
Answer:

In options (a) to (c), there is no assurance that the horizontal seismic displacement time-history will be the same for all footings: they may differ, owing to the incident seismic waves or the seismic response of the superstructure. In that respect, the uncertainty concerning the seismic response of the building or its parts is higher than for option (d). If all footings have the same horizontal seismic displacements, then, owing to the rigid diaphragm above them, each footing, and the length of vertical element immediately above it, develops elastic shears (about) proportional to the stiffness of that element. In cases (a) and (d), that stiffness is inversely proportional to the cube of the clear length of that element; in (b), the entire shear goes to the shallowest footing, on the right. In case (c), the total shear is more uniformly shared by the four columns, because connection to a transverse beam at about mid-height increases their lateral stiffness. The left-most column above the footing in cases (a), (c), (d), the second one from the left in (b) and the lower part of all columns in (b) are squat, hence vulnerable in shear.

None of this options is appropriate for earthquake resistance. A suitable one is depicted on the left; it ensures common displacement of all footings, avoids squat columns above the footings, avoids excessive excavation, in order to bring all footings to the same horizontal level, yet, it considerably increases the volume of excavation compared to (a) to (d); it suits a building with a basement at the part of the building to the right.

Example 13: A 3-to-4-storey building is built on a slope. Wing ABCD (in plan) has 3 storeys and a frame structural system. Wing EFGH has a concrete core at the centre for an elevator shaft and staircase. Proposed foundation system for the two wings of the building and structural system for the superstructure and justification.

Answer:



As there is no basement under wing ABCD, a general excavation to achieve the same foundation level, or to rigidly connect the foundation of the two wings, for them to have the same horizontal displacements, is not cost-effective. Moreover, the T-shape of the building in plan and the eccentric position of the elevator-cum-staircase shaft, make the building irregular and introduce considerable uncertainty concerning its seismic response. Besides, if part ABCD does not have a basement anyway, it is not sensible to construct one just to provide a footing for the central core and to a box foundation to the whole building.

The best option is to separate ABCD and EFGH into two statically independent, planwise regular wings, founded at different levels. Stiff lateral-load-resisting elements are needed on the perimeter of EFGH, to increase the torsional stiffness of that part and balance the effect of the central core for the shaft.

Fig. 4.21

Question 4.1

The building shown in Fig. 4.21 consists of several structurally independent units, separated by wide joints. All elements shown in gray are of structural concrete. Do the criteria of Eurocode 8 for regularity in plan and elevation seem overall to be met? Which structural features of the building seem favourable for its earthquake resistance and which ones adverse? Does the building give an overall impression of being deficient in terms of seismic resistance?

Answer to Question 2.1:

Each independent unit seems to have a wall structural system, or possibly a dual one. The layout of the frames and walls in these systems seems to be strongly irregular in plan, with asymmetric or eccentric arrangement of the walls. The units seem to have a good number of walls, but these walls are very eccentric in plan within the unit. The walls, though, are mostly arranged on the perimeter of each unit, providing significant torsional stiffness. Some of these walls increase in size from the ground floor to the others; they also have openings. The heightwise pattern of the walls creates a strong irregularity in eleveation, which seems to be the most disconcerting feature for the earthquake resistance of the building. Some frames also have irregular geometry and even deep beams, clearly stronger than the columns.

Despite its stong irregularities, the building seems to have good earthquake resistance, thanks to the large size of its vertical members.

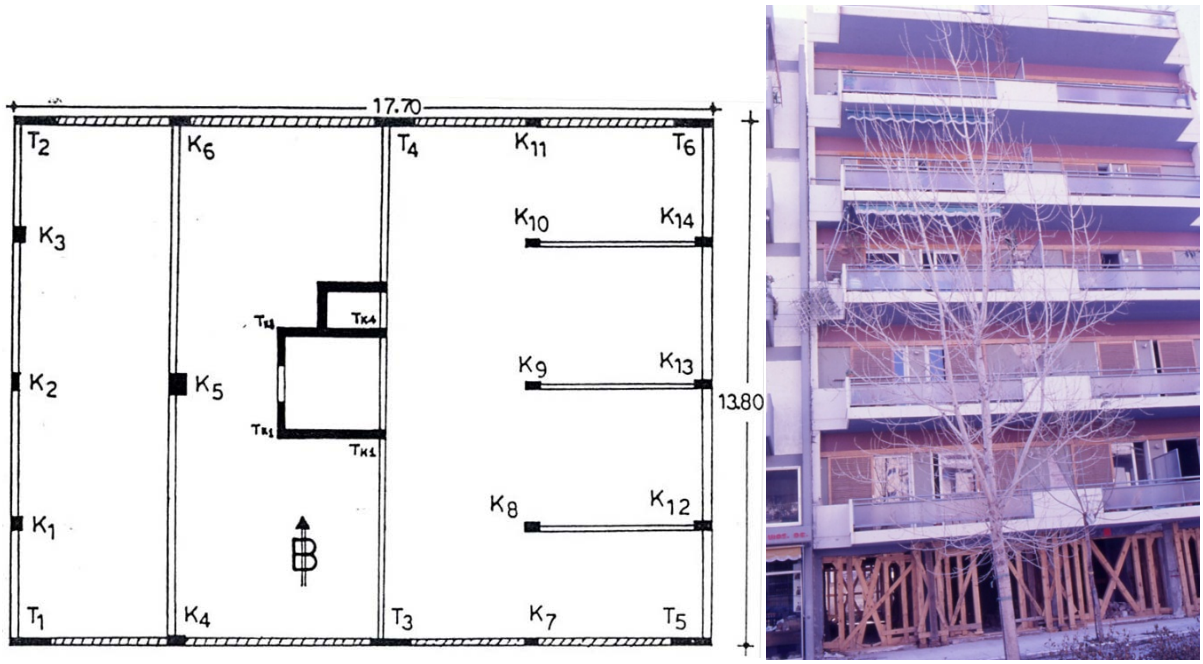


Fig. 4.22

Question 4.2

The 6-storey building in Fig. 4.22 has an open ground floor, except for the 200mm-thick solid masonry infills (shown cross-hatched) along the property lines between walls T2, T6 and T1, T5. There are similar infills in the 5 storeys above, supplemented with 200 mm-thick infills with many openings on the street sides between walls T1, T2 and T5, T6 (shown in elevation), and 100mm-thick masonry partitions at the interior, solid or with openings. Columns (denoted by K) and walls (denoted by T) are shown in solid dark. The complex core of walls at the centre houses an elevator and stairs. Ground storey beams are shown with the width of their web. Which features of the structural system and of the layout of the infills may adversely affect the earthquake resistance of the building? What may have contributed to the full failure/disintegration of all intermediate columns K1 to K3 and K12 to K14 of the façades at the ground floor in a past earthquake? What may have kept the beams supported on these columns from collapsing upon losing their intermediate supports and before propping?

Answer to Question 4.2:

Thanks to the 200 mm-thick solid masonry infills along the property lines between T2, T6 and T1, T5 in the ground storey, the building has the ground storey open only in the short direction in plan (N-S). Besides, the six walls T1 to T6, together with the predominant orientation of the components of the complex wall system at the centre in plan (the core housing the elevator and the staicase) provide significant lateral stiffness and resistance in the long direction in plan. These six walls, alongide the solid infilling of the bays on the property lines between T1 and T5 on one hand and T2 and T6 on the other, provide significant torsional stiffness and resistance, counterbalancing the concentration of stiffness in the central core and preventing it from causing a torsionally flexible system. That said, where are the adverse features for earthquake resistance? A clear one is the lack of lateral stiffness and strength in the short (N-S) direction in plan: Apart from the central core, which presents its weak axis in that direction, there are three N-S frames: a) the two-bay frame between columns K4, K5, K6 - which is handicapped by its long bays; and b) the two 4-bay frames along the façades - of which the one with K12, K13, K14 and T5, T6 involves the weak direction of these vertical elements. Notable is the lack of beams and of frame action along the column line K7 to K10. Note also that, with the central core working as a vertical cantilever, the beams connecting it to the weak direction of walls T3 and T4 do not produce an effective frame. Last, but not least, the large central core is effective as a vertical cantilever, only if it is fixed against rotation at the base - which can be achieved only by a box foundation system; if it rotates there, the burden falls on the other lateral force resisting elements, of which, those along the short sides in plan, are not stiff and strong enough.

The deficiency in the N-S direction led to the complete failure in an earthquake of all six columns K1 to K3 and K12 to K14 at the ground floor. Having lost all their intermediate support, the continuous beams between T1 and T2 on one hand and T5 and T6 on the other, sagged, mobilising the infilled façade above to work as a deep beam, extending from the 1st storey floor to the top floor. These floors can work as tension and compression chords ("flanges"), respectively, thanks to the infills of the façade, which - despite their openings - constitute the web between the "flanges". If these infills were not there, the entire façade would had collapsed: its 4-span continuous beams would had sagged alike at all floors, the columns connecting them being useless as supporting elements, with their continuity to the ground broken.

Question 4.3

A 4-storey hotel building (Fig. 4.23) has an open ground floor for the restaurant. Storeys 2 to 4 have one row of rooms along each long side in plan, separated by a corridor. The two short sides of the perimeter are fully infilled in all storeys, except for certain openings at the ends of the corridor at storeys 2 to 4 and along the right-hand side of the ground floor. There is a staircase near the upper left-hand corner, with straight flights between landings at floor levels and in-between floors. Interior and exterior walls are of 0.1 m- or 0.2 m-thick brick masonry, respectively. Columns, denoted by C.., are shown with their rectangular or L- shaped section; beams, denoted by B.., are shown with the width of their web; cross-section dimensions are written next to the member no. in meters (e.g., 0.2/0.7 next to a beam means a web width of 0.2 m and a section depth of 0.7 cm). Comment on the features of the structural design and of the layout of infills which are important for earthquake resistance and seismic performance. How do they relate to the almost full collapse of this building in an earthquake (the extreme left-hand bay with the staircase survived, as well as one long-side façade and the frame along the right-hand side in plan).

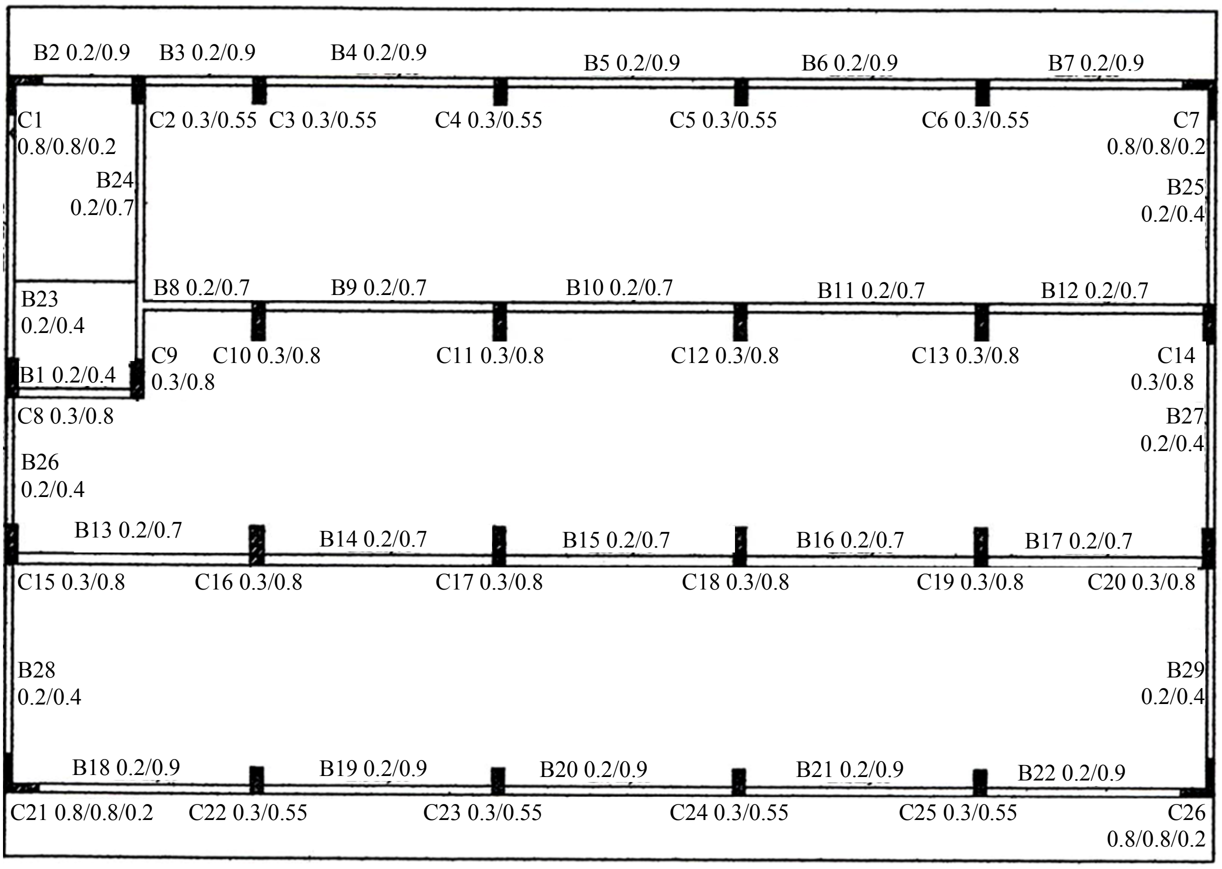


Fig. 4.23 Question 4.3

Answer for Question 4.3:

Lateral force resistance in the short direction in plan is provided only by the two 4-bay outer frames, between columns C1 and C21 on one hand and C7, C26 on the other. In fact, these two frames are the only elements in the building with features favourable for earthquake resistance: good-size columns (close to walls), clearly stronger than the beams; that's why these frames were the only ones to avoid collapse. The four frames in the long direction are continuous, but the size and orientation of their columns makes them much weaker than the beams. The ground storey was effectively free of infills in both horizontal directions and failed first. The regularity in plan and near-symmetry of the building were not enough to prevent collapse. Note also that floor slabs were all one-way; with very little reinforcement (if any) in the secondary direction (the long one of the building). So, once the frames other than the two outer and strong ones in the short direction started to succumb, the floor diaphragms did not have reinforcement for the in-plane bending needed to transfer forces from the distressed interior to the two outer frames and to receive support from them. So, as shown in the picture on the right, the interior in plan collapsed (except for the façade shown on the left), while the lateral sides stayed intact.

Question 4.4

In a 6-storey building (Fig. 4.24) the ground floor is open, except for the 200 mm-thick solid masonry infills (shown hatched) along the property line on the left-hand-side K1-T1-K10 and the three bays around the staircase/elevator shaft area K8-K12-T5-T4. There are similar infills at the 5 overlying storeys, but are supplemented with 200 mm-thick solid infills along the property line at the top side K15-K16-K17, 200 mm-thick infills with many openings along the rest of the perimeter and 100 mm-thick masonry partitions at the interior, solid or with openings. Columns (denoted by K) and walls (denoted by T) are shown solid dark, while beams with the projection of their web. Where do the centres of mass and stiffness seem to be located at the ground floor level (considering also the effect of the infills)? Does the building seem regular in plan according to the criteria of Eurocode 8? Comment on those features of the structural system and of the layout of infills in plan and elevation that adversely affect the seismic resistance and performance. Which side of the building seems more likely to have kick-started its collapse in a past earthquake?

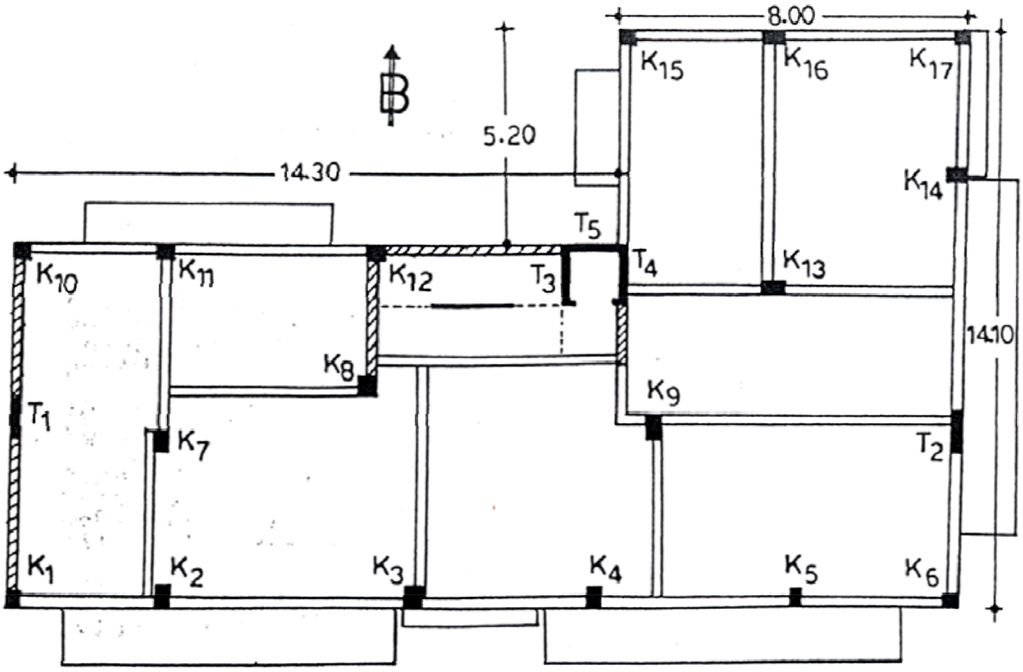


Fig. 4.24 Question 4.4

Answer to Question 4.4:

The centre of mass is about mid-way between T3 and K9. Because of: a) the larger density of the left-hand-side in plan in lateral force resisting elements, b) the solid infilling of the bays between K1 and T1 and T1 and K10, c) the C-shaped wall composed of T3, T4, T5 and the staircase between that wall and K8, K12 and d) the infills surrounding the staircase area, the centre of stiffness is between K8 and T3. The eccentricity produces twisting, which increases the displacements of the sides furthest from the stiffness centre, i.e, the two side meeting at K6.

The building is clearly irregular in plan per Eurocode 8, not only owing to the likely eccentricity according to what has been said so far, but also because of the shape in plan, which deviated significantly from a compact rectangular one. The sparsity of infills at the ground floor relative to the overlying storeys causes a strong irregularity in elevation.

Besides the irregularity in elevation and in plan (the latter having to do with the eccentricity discussed above), the building has other features adverse for earthquake resistance; the main ones are: a) the lack of continuous frames, b) the abudance of indirect supports of beams on other beams instead of columns (see beams strarting from K3, K8, K9, K13, etc), c) the offset of the beams with respect to K2, K7, K3, K9, etc, d) the failure to use columns K4, K5, K14 in two-way frame action, etc.

Twisting due to the eccentricity discussed in the first paragraph increased the most the displacements of the "flexible" side furthest from the centre of stiffness, i.e., the one between K6 and K17. Indeed, the columns on that side failed first (this is the side shown in the photo, and K17 is the column at the extreme right).

Question 4.5

A 7-storey building has the ground storey open. Floors 6 and 7 are set back along the façade; floor 6 along the left-hand side too. The framing plan of storeys 2 to 5 is shown in Fig. 4.25(a) and the foundation plan in Fig. 4.25(b). Five columns of the façade in storeys 2 to 5 (at the bottom side in Fig. 4.25(a)) are supported at the tip of cantilevering beams without continuing to the foundation. Three concrete walls – shown hatched in Figs. 4.25(a) and 4.25 (b) – are added at the only feasible locations on the perimeter for the purposes of seismic strengthening; the new footings for the added walls are shown with a dashed outline in Fig. 4.25(b). The axonometric view in Fig. 4.25(c) shows the as-built configuration; that in 4.25(d) has the 3 added walls.

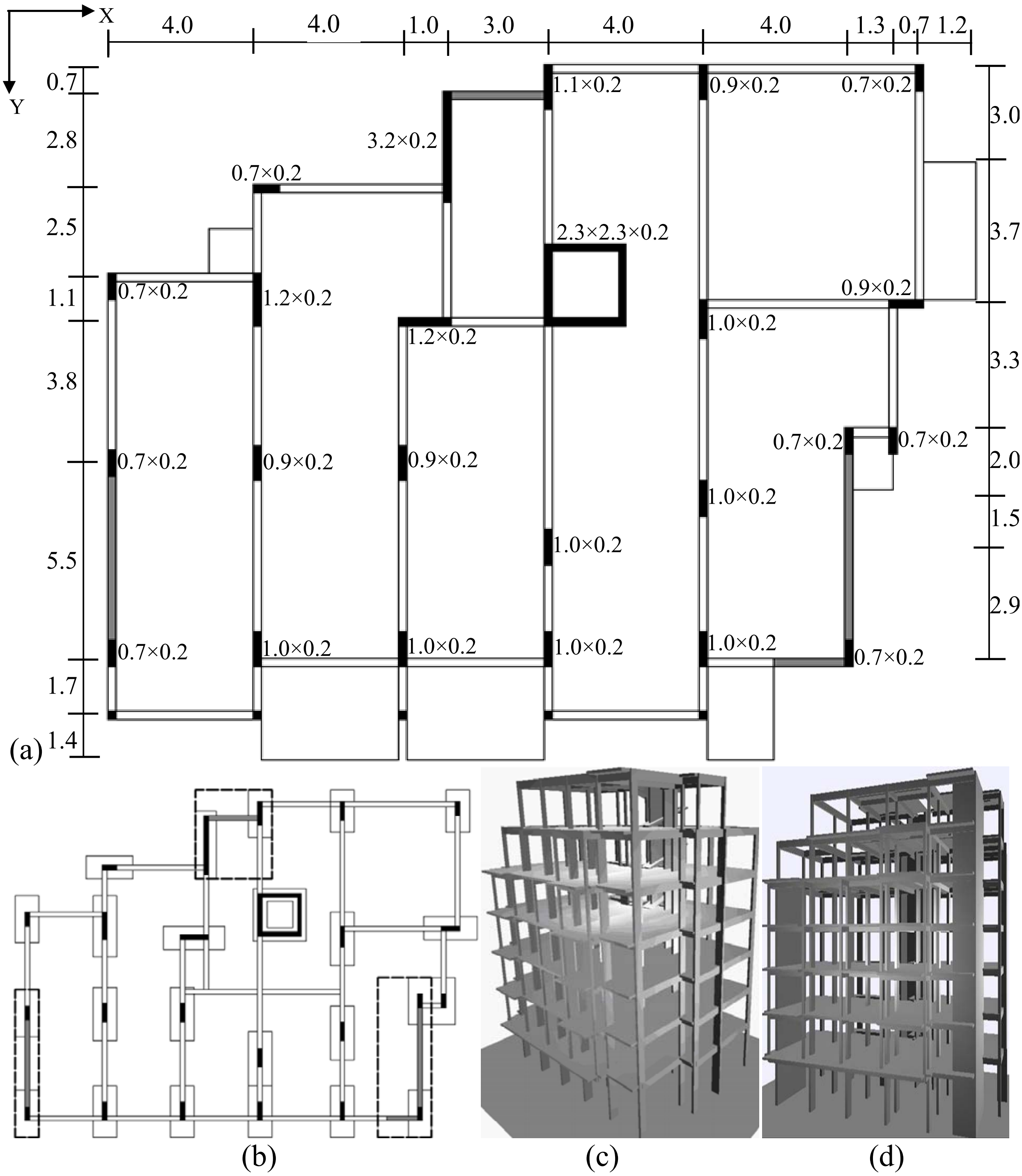


Fig. 4.25 Question 4.5

Calculate the co-ordinates of the centres of mass and stiffness at the ground storey (from the outline of the plan and the moments of inertia of vertical elements, respectively), the eccentricities, the torsional radii and the radius of gyration of the as-built and the retrofitted building, and characterise both of them as regular or not in plan and elevation, according to all Eurocode 8 criteria. Comment on the effectiveness of the retrofitting concerning regularity and on features of the structural layout of both the as-built and the retrofitted building which are important for its earthquake resistance, stressing the ones you consider adverse.

Answer to Question 4.5:

The basis for the calculation of the centre of mass is the ceiling of the ground floor, which has the same plan as storeys 2 to 5, shown in Fig. 4.25(a). The co-ordinates of the centroid of the ground floor plan area, with respect to the upper-left corner of a rectangle circumscribing the ground floor are:

x_{CM}= 11.39 m and

y_{CM}= 9.9 m.

The surface area of the ground floor and its polar moment of inertia with respect to its centroid are 353.1 m^2 and 22377 m^4, respectively. Therefore, the radius of gyration is

$l_s = \sqrt{(22377/353.1)} = 7.96$ m.

Note that the length of the longest diagonal of the ground floor plan area (i.e., that of the circumscribing rectangle) divided by √12 is 8.22 m, just 3.3% larger.

The co-ordinates of the centre of stiffness are calculated from the moments of inertia of the vertical elements continuing to the ground (with the five columns of the façade, which do not continue, excluded), listed in the table below (where I_x denotes the moment of inertia for bending in a vertical plane parallel to the X-axis; I_y is for bending parallel to Y). With respect to the same origin as the centre of mass, these copordinates are:

x_{CK}= 11.675 m and

y_{CK} = 6.10 m,

Their eccentricities with respect to the centre of mass are:

e_x = 11.675-11.39 = 0.285 m and

e_y = 6.10-9.9 = -3.80 m.

Co-ordinates and moments of inertia of vertical elements - As built structure

x-coordinate (m)	*y*-coordinate (m)	I_x (m^4)	I_y (m^4)	Wall in X: 1; column: 0	Wall in Y: 1; column: 0
0	6.35	0.000467	0.005717	0	0
0	10.9	0.000467	0.005717	0	0
0	16.15	0.000467	0.005717	0	0
4.25	3.5	0.005717	0.000467	0	0
4	6.55	0.0008	0.0288	0	1
4	10.9	0.0006	0.01215	0	1
4	16	0.000667	0.016667	0	1
8.5	7.1	0.0288	0.0008	1	0
8	10.9	0.0006	0.01215	0	1
8	16	0.000667	0.016667	0	1
9	2.3	0.002133	0.546133	0	1

12	0.45	0.000733	0.022183	0	1
12	13.1	0.000667	0.016667	0	1
12	16	0.000667	0.016667	0	1
16	0.35	0.0006	0.01215	0	1
16	7.1	0.000667	0.016667	0	1
16	12	0.000667	0.016667	0	1
16	16	0.000667	0.016667	0	1
20	10.25	0.000467	0.005717	0	0
20	16.15	0.000467	0.005717	0	0
21.3	10.25	0.000467	0.005717	0	0
21.65	6.7	0.01215	0.0006	1	0
22	0.35	0.000467	0.005717	0	0
13.05	6.05	1.246	1.246	1	1

The torsional radii are:

$r_x = 2.82$ m $< l_s = 7.96$ m and

$r_y = 3.52$ m $< l_s = 7.96$ m,

and do not meet the Eurocode 8 criteria for regularity in plan and torsional flexibity.

The eccentricity in y also fails to meet the Eurocode 8 criteria for regularity in plan:

$e_x = 0.285$ m $< 0.3r_x = 0.845$ m, and

$e_y = 3.80$ m $> 0.3r_y = 1.055$ m.

If we consider only the vertical members which qualify as walls (on the basis of the 4:1 aspect ratio of the section), and indeed only in their strong direction, the centre of stiffness, the eccentricities and the conclusions above to not change much:

$x_{CK}= 11.67$ m and

$y_{CK} = 6.08$ m,

$r_x = 2.375$ m $< l_s = 7.96$ m and

$r_y = 2.955$ m $< l_s = 7.96$ m.

$e_x = 11.67\text{-}11.39 = 0.28$ m $< 0.3r_x = 0.712$ m, and

$e_y = 6.08\text{-}9.9 = 3.82$ m $> 0.3r_y = 0.885$ m.

As a matter of fact, the lateral stiffness is controlled by the wall with the hollow section, which is eccentric along Y and far from the perimeter; hence the failure to meet all the criteria above.

Adding the three walls for retrofitting changes the situation as follows:

The new co-ordinates of the centre of stiffness are:

x_{CK}= 13.225 m and

y_{CK} = 4.69 m,

at eccentricities with respect to the centre of mass of :

e_x = 13.225-11.39 = 1.835 m and

e_y = 4.69-9.9 = -5.21 m.

The torsional radii increase dramatically and meet the Eurocode 8 criteria against torsional flexibity:

r_x = 8.525 m > l_s = 7.96 m and

r_y = 16.48 m > l_s = 7.96 m.

The eccentricities increase considerably but, thanks to the increased torsional radii, they are now closer to meeting the Eurocode 8 criteria for regularity in plan:

e_x = 1.835 m < 0.3r_x = 2.56 m, and

e_y = 5.21 m > 0.3r_y = 4.945 m.

If only the vertical members which qualify as walls are considered, the picture changes very little:

x_{CK}= 13.22 m and

y_{CK} = 4.67 m,

r_x = 8.52 m > l_s = 7.96 m and

r_y = 16.5 m > l_s = 7.96 m.

e_x = 13.22-11.39 = 1.83 m < 0.3r_x = 2.555 m, and

e_y = 4.67-9.9 = 5.23 m > 0.3r_y = 4.95 m.

Details on the retroffited structure are given in the table below:

Co-ordinates and moments of inertia of vertical elements - Retrofiitted structure

x-coordinate (m)	*y*-coordinate (m)	I_x (m^4)	I_y (m^4)	Wall in X: 1; column: 0	Wall in Y: 1; column: 0
0	6.35	0.000467	0.005717	0	0
0	13.525	0.003967	3.510748	0	1
4.25	3.5	0.005717	0.000467	0	0
4	6.55	0.0008	0.0288	0	1
4	10.9	0.0006	0.01215	0	1
4	16	0.000667	0.016667	0	1
8.5	7.1	0.0288	0.0008	1	0
8	10.9	0.0006	0.01215	0	1
8	16	0.000667	0.016667	0	1
12	13.1	0.000667	0.016667	0	1
12	16	0.000667	0.016667	0	1

16	0.35	0.0006	0.01215	0	1
16	7.1	0.000667	0.016667	0	1
16	12	0.000667	0.016667	0	1
16	16	0.000667	0.016667	0	1
21.3	10.25	0.000467	0.005717	0	0
21.65	6.7	0.01215	0.0006	1	0
22	0.35	0.000467	0.005717	0	0
13.05	6.05	1.246	1.246	1	1
19.74	13.94	0.509175	7.936375	1	1
10.056	1.44	2.024226	1.447155	1	1

Neither the as-built nor the retroffited structure meet both eccentricity conditions; the as-built one is also torsionally flexible. The slenderness and compactness of the floors in plan are acceptable, as far as regularity in plan is concerned: the re-entrant corners and recesseses in plan are not large enough to violate the 5% regularity criterion of Eurocode 8. However, Fig. 4.25(d) shows that at the location of the setbacks in the two upper floors the floor diaphragm is not complete: it has large openings, violating another criterion for regularity in plan.

The building is clearly irregular in elevation, owing to the open ground floor, the large asymmetric setbacks at the two upper floors (in excess of 10% of the parallel dimension in plan), the five columns of the facade which do not continue to the foundation, etc. These defficiencies are not corrected by adding the walls; moreover, as shown in Fig. 4.25(d), one of these walls stops two storeys below the top level of the buidling. In addition to the clear drawbacks which came out of the discussion above concerning regularity, there is a serious shortfall of frame action, lateral strength and stiffness in the X direction: frames have one or two-bays only, columns which are aligned in the X-direction are not connected with beams to form frames, most columns present their weak direction to an earthquake along X, etc. This shortcoming is not addressed by the retrofiiting, as the added walls are almost exclusively in the Y direction. In fact, the as-built structure has quite a few complete and continuous frames, with good-size vertical elements in the Y-direction; it is the X-direction that is wanting.

Question 4.6

For the depicted 2-storey building (Fig. 4.26), locate the centres of mass and stiffness at the ground storey from the outline of the plan and the moments of inertia of vertical elements (estimating their cross-sectional size from the other dimensions in plan, including a beam width of 0.3 m). Determine the eccentricities, the torsional radii and the radius of gyration.

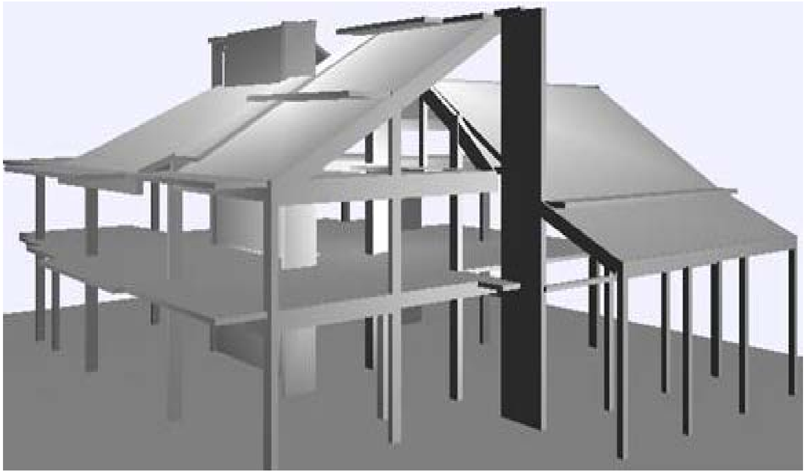
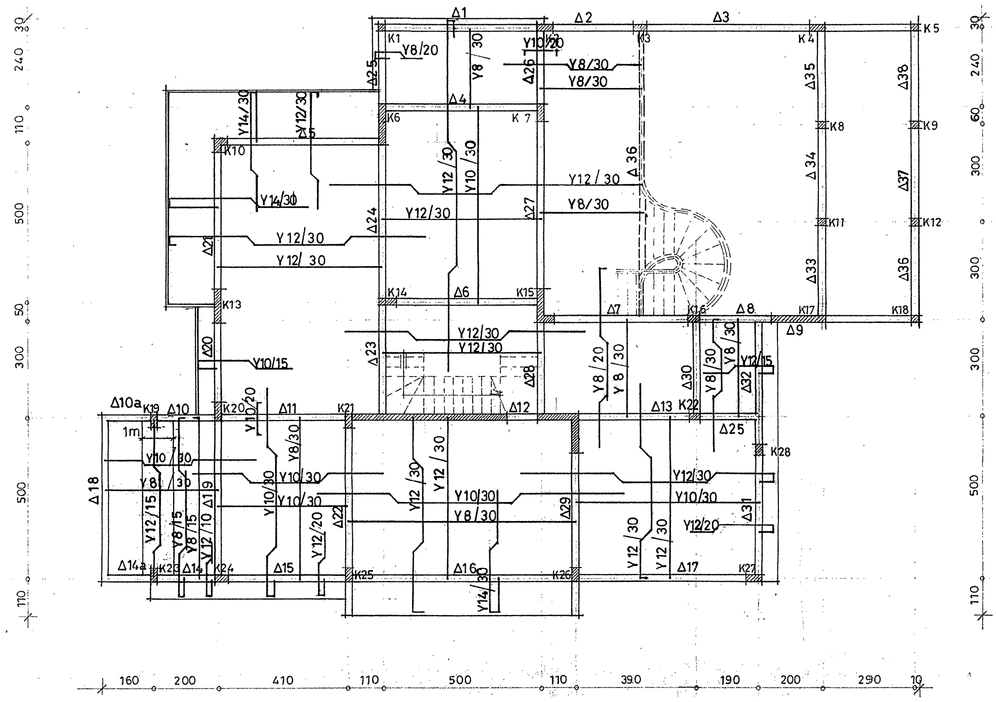
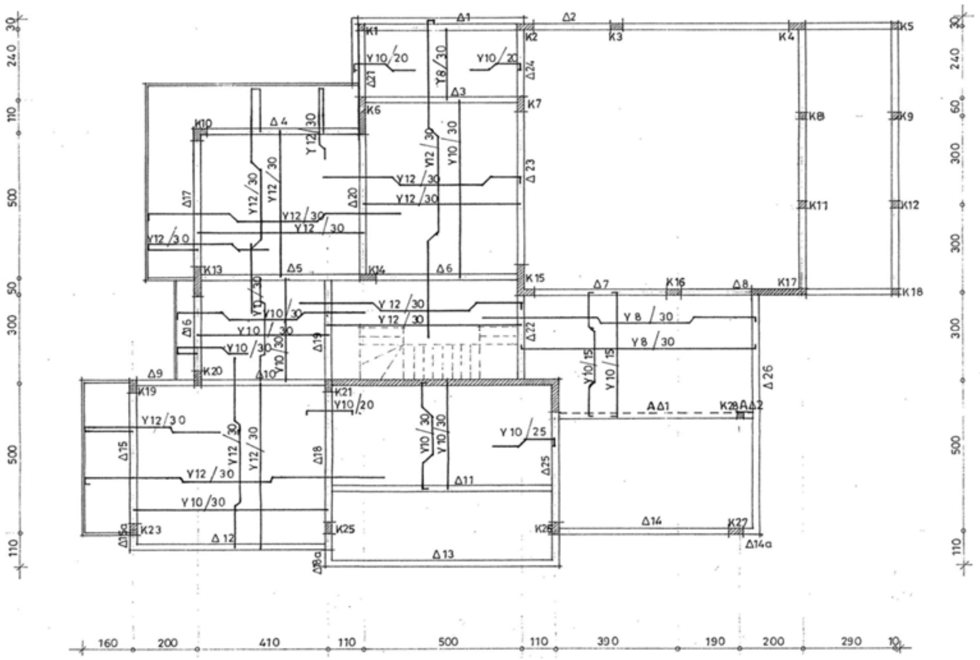


Fig. 4.26 Question 4.6



Ground storey



Second storey

Characterise the building as regular or not in plan and elevation according to all Eurocode 8 criteria. Comment on the features of the structural layout which are important for the earthquake resistance of the building, pointing out the one you consider unfavourable.

Answer to Question 4.6:

For an origin at the lower left-hand-corner in plan, the centre of mass has co-ordinates:

x_{CM}= 13.18 m and

y_{CM}= 8.4 m,

i.e., just to the left of column K15.

The surface area of the ground floor and its polar moment of inertia with respect to its centroid are 358.2 m^2 and 15856 m^4, respectively. Therefore, the radius of gyration is

$l_s = \sqrt{(15856/358.2)} = 6.65$ m.

which is the same as the length of the shorter of the two diagonals in plan divided by $\sqrt{12}$.

The co-ordinates of the centre of stiffness are calculated from the moments of inertia of the vertical elements of the ground floor with respect to the same origin as the centre of mass:

x_{CK}= 10.91 m and

y_{CK} = 4.835 m,

at eccentricities with respect to the centre of mass of:

e_x = 13.18-10.91 = 2.27 m and

e_y = 8.4-4.835 = 3.565 m.

The centre of stiffness almost coincides with the centroid of the large wall next to the staircase.

The torsional radii violate the Eurocode 8 criteria for regularity in plan and torsional flexibity:

$r_x = 5.31$ m < $l_s = 6.65$ m and

$r_y = 0.785$ m < $l_s = 6.65$ m.

The eccentricities also fails the Eurocode 8 check for regularity in plan:

$e_x = 2.27$ m > $0.3r_x = 1.595$ m, and

$e_y = 3.565$ m > $0.3r_y = 0.24$ m.

Co-ordinates and moments of inertia of vertical elements

x-coordinate (m)	*y*-coordinate (m)	I_x (m^4)	I_y (m^4)
1.6	0	0.001125	0.003125
3.6	0	0.003125	0.001125
7.7	0	0.001125	0.003125
14.9	0	0.001125	0.003125
20.7	0	0.0054	0.00135
1.6	5	0.001125	0.003125

3.6	5	0.00135	0.0054
11.8	4.8	14.7225	0.152493
18.8	5	0.0016	0.0009
20.7	4	0.0009	0.0016
3.6	8.5	0.002025	0.018225
8.8	8.5	0.003125	0.001125
13.8	8.2	0.009156	0.034156
18.8	8	0.0009	0.0016
21.9	8	0.084375	0.003375
25.5	8	0.000675	0.000675
25.5	11	0.000675	0.000675
22.6	11	0.000675	0.000675
22.6	14	0.000675	0.000675
25.6	14	0.000675	0.000675
3.6	13.5	0.005208	0.005208
8.8	14	0.00315	0.0686
13.7	14.5	0.001575	0.008575
13.8	17.3	0.0054	0.00135
16.8	17.3	0.003125	0.001125
8.8	17.3	0.000675	0.000675
25.6	17.3	0.000675	0.000675
22.6	17.3	0.003125	0.001125

The building is apparently very irregular in plan and elevation.

As far as the Eurocode 8 criteria for irregularity in elevation are concerned, the changes in plan from the first storey to the second exceed the set-back limits beyond which Eurocode 8 characterises the building as irregular in elevation. Moreover, quite a few vertical elements do not continue to the top (although, strictly speaking, they do so because the building stops vertically at that level).

Concerning irregularity in plan, apart from the large eccentricities and the torsional flexibility that came out from the numerical checks above, there are large recesses and deviations from a compact, close to rectangular, plan.

Overall, the building has numerous features which are adverse for earthquake resistance. By far the most important one is the very strong and stiff wall near the centre in plan: it is this wall that makes the system strongly eccentric and torsionally flexible and puts at risk the perimeter columns, which may fail due to twisting about that wall. Being squat, the wall itself is prone to brittle shear failure. If the wall were no there, the building might had been sufficient, despite all its other deficiencies, as it has only two storeys.

Question 4.7

For the building of Question 3.3:

1. Calculate the torsional radii and the radius of gyration for:
   - the space truss roof itself, on bearings; or
   - the perimeter frame, with the roof mass considered fixed to the cap beam,

   and check the condition for torsional flexibility, Eq. (4.7). What is the conclusion of this check concerning regularity in plan? how does it compare with the conclusion from the natural periods calculated in Question 3.3?
2. Would you characterise the building as regular in plan and/or elevation?
3. Propose an appropriate Ductility Class and behaviour factor value for the design.

Answer to Question 4.7:

1. For the roof:

$l_{s,roof}=\sqrt{}(28^2+52^2)/\sqrt{}12=17.5\cdot m$

$K_{X,roof} = K_{Y,roof} = 12\times700=8400$ kN/m

$K_{\theta,roof} =2\times(5\times12.3^2+2\times12.15^2+3\times24.3^2)\times700=3{,}952{,}000$ kNm/rad

$r_{X,roof} = r_{Y,roof} = \sqrt{}(3{,}952{,}000/8{,}400)=22.23 \text{ m} > l_{s,roof}$

For the frame, taken as monolithically connected to the roof:

Frame and roof have comparable masses:

$M_{roof}=28\times52\times1/9.81 = 148.4$ tons

$M_{frame} = \{200+25\times[0.64\times2\times(24+48)+7.5/3\times(16\times0.2+4\times0.16)]\}/9.81=280$ tons.

The individual moments of these masses with respect to the centre in plan are:

$I_{roof}= M_{roof}(28^2+52^2)/12=43{,}135 \text{ ton}\cdot\text{m}^2$

$I_{frame}= M_{frame}(24+48)^2/12=120{,}840 \text{ ton}\cdot\text{m}^2$

Then, $l_{s,mon}=\sqrt{}[(120840+43135)/(280+148.4)]=19.56\cdot m$

Torsional radii, calculated from the column moments of inertia, per Eqs. (4.4):

Intermediate columns:

strong direction: $(EI)_{c,s} = 0.5\times30{,}000{,}000\times0.4\times0.5^3/12= 62{,}500 \text{ kNm}^2$

weak direction: $(EI)_{c,w} = 0.5\times30{,}000{,}000\times0.5\times0.4^3/12= 40{,}000 \text{ kNm}^2$

Corner columns:

$(EI)_{c,s}$ = 0.5×30,000,000×0.4^4/12= 32,000 kNm^2

$\sum(EI)_X$= 2×7×62500+2×40000+4×32000= 1,083,000 kNm^2

$\sum(EI)_Y$= 2×7×40000+2×62500+4×32000= 813,000 kNm^2

$\sum[y^2(EI)_X+x^2\sum(EI)_Y]$= 2×(7×62500+2×32000)×12^2+

2×(62500+2×32000)×24^2+4×40000×(18^2+12^2+6^2) = 370,800,000 kNm^2

$r_{X,mon}$ = √(370,800,000/813,000)=21.35 m > $l_{s,mon}$

$r_{Y,mon}$ = √(370,800,000/1,083,000)=18.5 m < $l_{s,mon}$

However, the columns have completely different fixity conditions at the top in the in-plane or out-of-plane direction of the frame; so, the torsional radii are calculated more accurately, from the frame stiffness:

$k_Y=(EI)_b/(EI)_{c,s}(H/L)$= (1088/62.5)(8/12)=11.6

$K_{X,frame}$ = 2×3×(7+1)×62500×(12×23.21+1)/[(3×23.21+1)×8^3] +2×3×40000/8^3 = 23190+2×234.5 = 23659 kN/m

$K_{Y,frame}$=2×3×(1+1)×62500×(12×11.6+1)/[(3×11.6+1)×8^3] +14×3×40000/8^3=5737+14×234.5 = 9020 kN/m

$K_{\theta,frame}$ = 23190×12^2+5737×24^2+4×234.5×(18^2+12^2+6^2) = 7,116,600 kNm/rad

$r_{X,mon}$ = √(7,116,600/9,020)=28.1 m > $l_{s,mon}$

$r_{Y,mon}$ = √(7,116,600/23,659)=17.35 m < $l_{s,mon}$

The more representative values are higher than the approximate ones in direction X, lower in direction Y, but the conclusion has not changed: Eq. (4.7) is violated by the frame considered monolithic with the roof, along direction Y. The roof itself on bearings satisfies this condition.

The checks of Eq. (4.7) concur with the relative magnitude of periods in Question 3.3:

- for the roof itself, the torsional period is shorter than the translational ones;
- for the frame considered monolithic with the roof, the torsional period is shorter than the translational along Y, but longer than along X (which has to do with r_Y).

However, in the real structure, with the roof on bearings, the fundamental torsional period is shorter than both translational ones. This conclusion cannot be drawn with back-of-the-envelope calculations of the type required for Eq. (4.7). Such calculation might had been meaningful, if the entire mass were lumped at the

DOF of the roof; that DOF would then be the only one per direction, and a composite stiffness could be computed for it from those of the frame and the bearings:

$K_{X,eq} = 1/(1/K_{X,roof}+1/K_{X,frame}) = 1/(1/8400+1/23659)=6200$ kN/m

$K_{Y,eq} = 1/(1/K_{Y,roof}+1/K_{Y,frame}) = 1/(1/8400+1/9020)=4350$ kN/m

$K_{\theta,eq}=1/(1/K_{\theta,roof}+1/K_{\theta,frame}) = 1/(1/3,952,000+1/7,116,600)=2,540,000$ kNm/rad

$r_{X,eq}=\sqrt{(2,540,000/4,350)}=24.15$ m $> l_{s,eq}=\sqrt{[(120840+43135)/(280+148.4)]} =19.56\cdot$m

$r_{Y,eq} = \sqrt{(2,540,000/6,200)}=20.2$ m $> l_{s,eq}=19.56\cdot$m

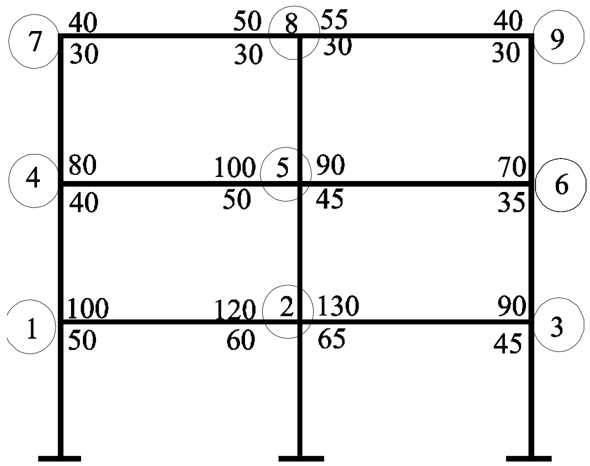
With this consideration, Eq. (4.7) is satisfied. Although this check does not refer to the real structure, the conclusion agrees with the hierarchy of its modes.

2) Owing to the abrupt discontinuity in stiffness at the interface between the roof and the frame, the building is not regular in elevation. No matter the outcome of the check of Eq. (4.7), it cannot be characterised as regular in plan either, because there is no rigid diaphragm (condition 3 in Sect. 4.3.2).

3) The roof on bearings, which accounts for a good part of the total mass, will have elastic response. Thanks to the large sections of its beams and columns, the concrete frame building is judged to have significant lateral load resistance; so, it may well be designed for DC L (Low) with a q factor value of 1.5, even if it is in a moderate seismicity region. If it is in a high seismicity zone, DC M (Medium) is more appropriate, still with q = 1.5, as the building is an inverted pendulum system.

Chapter 5

Example 1: The design values of moment resistance of the beams, $M_{Rb,d,}$, in a 3-storey frame are displayed in the figure below (in kNm) next to the corresponding tension side of the beam (top or bottom). Calculation of the minimum design values of moment resistances of the columns, $M_{Rc,d,}$ to meet Eq. (5.21), assuming that the columns have symmetric section and reinforcement and that, if the cross-section and the reinforcement above and below a joint are the same, the higher axial load at the column section below the joint increases the moment resistance by 10% compared to the section above.



Answer:

Below the joint: $M_{Rc,d1} \geq 1.10{\times}1.3(\sum M_{Rb,d})/2.1$

Above the joint: $M_{Rc,d2} \geq 1.3(\sum M_{Rb,d})/2.1 = 1.3(\sum M_{Rb,d})\text{-}M_{Rc,d1}$

Node 1: Below: $M_{Rc,d1} \geq (1.1/2.1){\times}1.3\max(100, 50) =$ 68 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1){\times}1.3\max(100, 50) =$ 62 kNm

Node 2: Below: $M_{Rc,d1} \geq (1.1/2.1){\times}1.3\max(120{+}65, 130{+}60) =$ 129.4 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1){\times}1.3\max(120{+}65, 130{+}60) =$ 117.6 kNm

Node 3: Below: $M_{Rc,d1} \geq (1.1/2.1){\times}1.3\max(90, 45) =$ 61.3 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1){\times}1.3\max(90, 45) =$ 55.7 kNm

Node 4: Below: $M_{Rc,d1} \geq (1.1/2.1){\times}1.3\max(80, 40) =$ 54.5 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1){\times}1.3\max(80, 40) =$ 49.5 kNm

Node 5: Below: $M_{Rc,d1} \geq (1.1/2.1){\times}1.3\max(100{+}45, 90{+}50) =$ 98.7 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1)\times1.3\max(100+45, 90+50) =$ 89.8 kNm

Node 6: Below: $M_{Rc,d1} \geq (1.1/2.1)\times1.3\max(70, 35) =$ 47.7 kNm

Above: $M_{Rc,d2} \geq (1.0/2.1)\times1.3\max(70, 35) =$ 43.3 kNm

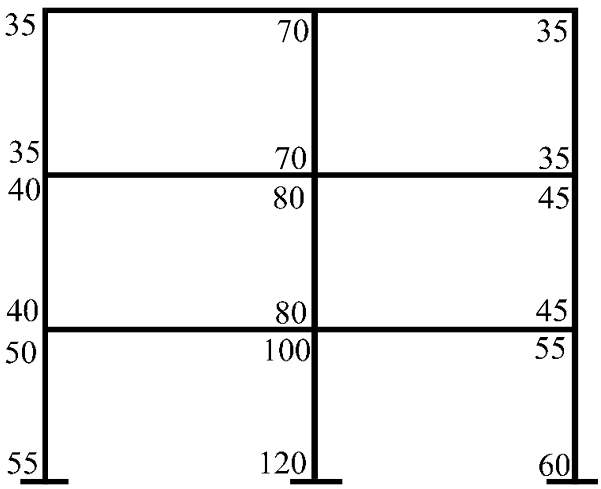
At the nodes of the roof, capacity design per Eq. (5.21) is not required; as a matter of fact, it is meaningless. However, it is extended here to these nodes, to show that it sometimes leads to absurdly large column moment capacities:

Node 7: Below: $M_{Rc,d1} \geq 1.3\max(40, 30) =$ 92 kNm

Node 8: Below: $M_{Rc,d1} \geq 1.3\max(50+30, 55+30) =$ 110.5 kNm

Node 9: Below: $M_{Rc,d1} \geq 1.3\max(40, 30) =$ 52 kNm

Example 2: For the same design values of moment resistances of the beams, $M_{Rb,d,}$, as in the previous example, the design values of moment resistances of the columns, $M_{Rc,d,}$ are depicted in the figure below. Estimation of the likely location of plastic hinges form, if the response to the seismic action is from the left to the right or from the right to the left.



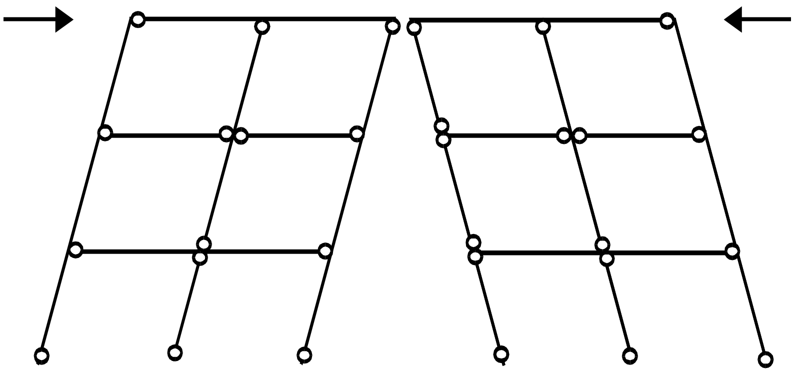
Answer:

For seismic response from left to right, sagging bending at the left end of each beam, hogging at the right one; the reverse for seismic response from right to left.

Node $\sum M_{Rc}$ Response from left to right Response from right to left

(kNm) $\sum M_{Rb}$ (kNm) Plastic hinges in: $\sum M_{Rb}$ (kNm) Plastic hinges in:

1	90	> 50	Beam	<100	Columns
2	180	<185	Columns	<190	Columns
3	100	> 90	Beam	> 45	Beam
4	75	> 40	Beam	< 80	Columns
5	150	>145	Beams	>140	Beams
6	80	> 70	Beam	> 35	Beam
7	35	> 30	Beam	< 40	Column
8	70	< 80	Column	< 85	Column
9	35	< 40	Column	> 30	Beam



A storey-sway mechanism ("soft-storey") is not apparent. The closest a storey comes to such a mechanism is when the seismic response is from right to left, with plastic hinges forming at top and bottom of two columns of the ground storey and the bottom of the third one. Conclusions for the top storey do not change, no matter whether the hinge forms at the top of the column or at the beam end next to it.

When the plastic mechanism is mixed, as at all storeys in both present cases, a storey-sway mechanism is more likely to happen if at the storey top nodes the value of the storey index, $\sum(\sum M_{Rb})/\sum(\sum M_{Rc})$, exceeds 1.0 (with the outer sums in the numerator and the denominator of the index extending over all top nodes of the storey).

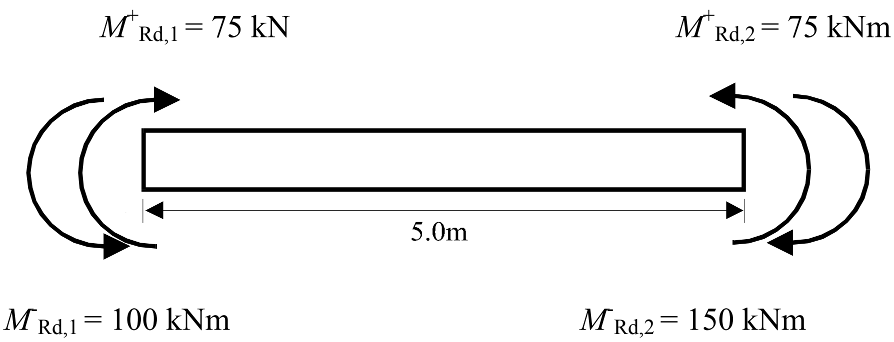
Storey	$\sum(\sum M_{Rc})$ (kNm)	Response from left to right		Response from right to left	
		$\sum(\sum M_{Rb})$ (kNm)	$\sum(\sum M_{Rb})/\sum(\sum M_{Rc})$	$\sum(\sum M_{Rb})$ (kNm)	$\sum(\sum M_{Rb})/\sum(\sum M_{Rc})$
1	90+180+100=370	50+185+90=325	0.88<1: beam-sway	100+190+45=335	0.91<1: beam-sway
2	75+150+80=305	40+145+70=255	0.84<1: beam-sway	80+140+35 = 255	0.84<1: beam-sway
3	35+ 70 +35=140	30+80+40 = 150	1.07>1: neutral	40+85+30 = 155	1.11>1: neutral

Note the value of the index at the ground storey for response from right to left: despite the hinging at two column tops out of three, in that case the index is still less than 1.0, thanks to the large margin of the total column moment resistance at the top of the third column with respect to the beam connected to it. The greater than 1.0 values of the index at the top storey have no practical impact, because it does not matter whether the hinge forms at column tops or at the roof beam.

Example 3: A DC H beam has the following design values of moment resistance:

- sagging M^{+}_{Rd} = 75 kNm, constant all along the span;
- hogging $M^{-}_{Rd,1}$ = 100 kNm at the left end (index: 1), $M^{-}_{Rd,2}$ = 150 kNm at the right one (index: 2).

The beam spans L_n = 5.0 m between the faces of its supporting columns, which are stronger in flexure than the beams around the two beam ends ($\sum M_{Rb} < \sum M_{Rc}$). The design shear forces at the two ends of the beam are computed by capacity design for two values of the quasi-permanent transverse load: $g+\psi q$ = 14 kN/m, and $g+\psi q$ = 20 kN/m, considering the possibility that the plastic hinge in positive (sagging) bending may form at some distance from the end section.



Answer:

(a) For $g+\psi q$ = 14 kN/m: $V_{g+\psi q,o,1}$=$V_{g+\psi q,o,2}$ = 14x5.0/2 = 35 kN

If $\sum M_{Rb} < \sum M_{Rc}$, Eq. (5.32a) gives for γ_{Rd} = 1.2 (DC H):

max$V_{d,1}$ = 1.2×(150+75)/5 + 35 = 89 kN, max$V_{d,2}$ = 1.2×(100+75)/5 + 35 = 77 kN

(b) For $g+\psi q$ = 20 kN/m: $V_{g+\psi q,o,1}$ = $V_{g+\psi q,o,2}$ = 20×5.0/2 = 50 kN

At first sight, the design shears increase by 50-35 = 15 kN, due to the increase in $V_{g+\psi q,o}$. This holds for

$\max V_{d,1}$, but not for $\max V_{d,2}$, for the following reasons:

At the instance $\max V_{d,1}$ occurs at end 1, the concurrent shear force at end 2 is: $\min V_{d,2} = -1.2\times(150+175)/5 + 50 = 4$ kN > 0; i.e., the shear does not change sign between the two ends, implying that there is no local maximum of the sagging bending moment along the beam in this case of the seismic design situation. By contrast, when $\max V_{d,2} = 1.2\times(100+75)/5 + 50 = 92$ kN develops at end 2, the value of $V_{d,1}$ at end 1 is $\min V_{d,1} = 1.2\times(100+75)/5 - 50 = -8$kN; i.e., the shear changes sign along the span, going through zero at a point where the bending moment attains its maximum value. As the slope (derivative) of the V-diagram is equal to the transverse load, an estimate of the distance of the maximum moment point to end 1 is $x = |\min V_{d,1}|/(g+\psi q) = 8/20 = 0.4$ m. That maximum moment is equal to the moment at end 1, taken for the present purposes equal to $\gamma_{Rd}M^{+}_{Rd} = 1.2\times75 = 90$ kNm, plus the area under the V-diagram between the maximum moment point and end 1. This area is equal to $|\min V_{d,1}|x/2 = 8\times0.4/2 = 1.6$ kNm, giving a maximum moment of 90 + 1.6 = 91.6 kNm. As expected, this value exceeds the overstrength sagging moment resistance, which is equal to $\gamma_{Rd}M^{+}_{Rd} = 90$ kNm all along the span. Therefore, the values calculated in the present paragraph, including the capacity design shear of $\max V_{d,2} = 1.2\times(100+75)/5 + 50 = 92$ kN cannot materialise, without violating the overstrength sagging moment resistance somewhere along the span; so, they are invalid.

The true value $\max V_{d,2}$ is equal to $20l_x/2 + 1.2\times(100 + 75)/l_x$, where $l_x = L_n - x$ is the distance from the maximum moment (and zero shear) point to end 2. The value of x estimated in the paragraph above as giving a maximum moment of 91.6 kNm (greater than the overstrength sagging moment resistance of 90 kNm), gives first trial values of $l_x \approx 5.0 - 0.4 = 4.6$ m and $\max V_{d,2} \approx 20\times4.6/2 + 1.2\times(100+75)/4.6 = 91.65$ kN. If the maximum moment at a distance $x = 0.4$m from end 1 is equal to $\gamma_{Rd}M^{+}_{Rd} = 90$ kNm, the moment at end 1 is equal to that maximum moment minus the area under the V-diagram between the maximum moment point and end 1, i.e., to 90 - 0.4×8/2 = 88.4 kNm. This new moment value at end 1 corresponds to a shear force value at that end equal to $\min V_{d,1} = (1.2\times100 + 88.4)/5 - 50 = -8.32$ kN and to a new estimate for the distance of that end to the maximum moment (and zero shear) point of: $x \approx 8.32/20 = 0.416$ m. The new x-value gives a new moment estimate at end 1, equal to: 90 - 0.416×8.32/2 = 88.27 kNm, which in turn yields an estimate of $\max V_{d,2} = (1.2\times100+88.27)/5 + 50 = 91.65$ kN, coinciding with the value $20l_x/2 + 1.2\times(100 + 75)/l_x = 10\times(5 - 0.416) + 1.2\times175/(5 - 0.416) = 91.65$ kN. This is taken as the final value of

$\max V_{d,2}$. Note that it could had been computed from the value of $x = 0.4$ m estimated in the paragraph above, without iterations. More important, the difference with the outcome of Eq. (5.32), namely $\max V_{d,2} = 1.2\times(100+75)/5 + 50 = 92$ kN, is minor and to the side of safety.

Question 5.1

One end of beam B7 in Example 4.7 (Fig. 4.18) is indirectly supported on beam B4. How would you take that into account in the calculation of the capacity design shear force at the other end of B3 (the one connected to column C3)?

Answer to Question 5.1

The indirectly supported end will not develop a seismic moment, let alone a plastic hinge. The capacity design shear at the other end may well be calculated setting the beam moment resistance at the indirectly supported end equal to zero.