CHAPTER 5: DETAILED SEISMIC DESIGN OF CONCRETE BUILDINGS

5.1 Introduction

5.1.1 Sequence of operations in the detailed design for earthquake resistance

The subject of the present Chapter, detailed design, is the third stage in the overall design process. The second stage is the analysis for the design actions; Chapter 3 focused specifically on the analysis for the seismic action. The analysis stage is preceded by conceptual design, the subject of Chapter 4. A sub-stage of conceptual design is "sizing of members", i.e., the selection of their cross-sectional dimensions, which in turn determine the member elastic stiffness, a necessary input to the analysis of the structural system for any action – including the seismic one. Therefore, "sizing" should take place before any analysis and, as such, is part of conceptual design. However, it is addressed in this Chapter, because it relates closely to detailed design rules and requirements for them, which are dealt with in other sections of Chapter 5.

Capacity design introduces strong interdependence across various phases of detailed design of a building per Eurocode 8, in the same member as well as between different ones, especially in frames:

- If the columns are capacity-designed around joints to be stronger in flexure than the beams (see Section 4.5.2 and Eq. (5.31) in Section 5.4.2), the longitudinal reinforcement of the beams should be known beforehand; to this end, the beams are the first members to be dimensioned; in fact, beam ends and the base section of walls are normally the only places whose detailed design is based exclusively on analysis results – in this case on their bending moments.
- Dimensioning of a column or a beam in shear depends on the longitudinal reinforcement of the column/beam itself and of that of the members framing into it at either end; so, it is carried out after the amount and layout of the beam and column reinforcement have been determined (see Eqs. (5.42),

(5.44)) in Sect. 5.5.1).

- Dimensioning of any storey of a DC H wall in shear depends on the vertical reinforcement at the base of the bottom storey (see first bullet point above and Eqs. (5.54) in Sect. 5.6.2.1); it should be undertaken after the amount and layout of the latter is determined.
- The design of isolated footings and of their tie-beams and the verification of the soil underneath depend on the longitudinal reinforcement of the vertical elements they support (see Section 6.3.2); so, it should take place afterwards.

The operations in detailed design should follow the sequence highlighted above, so that all the information needed at every step is available beforehand. Integrated computer programs for detailed design per Eurocode 8 should be structured to perform these operations in the above sequence; even when the columns do not need to be capacity-designed in order to be stronger than the beams per Eq.(5.31) in Sect. 5.4.1, it is convenient to follow the same general sequence, but using, in that case, for the dimensioning of the column in flexure its bending moments from the analysis, $M_{Ed,c}$, instead of the beam moment resistances, $M_{Rd,b}$.

The sequence suggested above is followed in the detailed description of the dimensioning steps across this Chapter and in Section 6.3 of Chapter 6, as well as in the full example of Chapter 7.

5.1.2 Material partial factors in Ultimate Limit State (ULS) dimensioning of members

Eurocode 8 adopts the Eurocode 2 approach for ULS design, where the general Ultimate Limit State (ULS) verification, Eq. (1.1), uses a design value of force or moment resistance, R_d, calculated from the design values of material strengths; the latter are obtained by dividing the nominal or characteristic values by the corresponding material partial factors:

- $f_{cd} = f_{ck}/\gamma_c$, for concrete;
- $f_{yd} = f_{yk}/\gamma_s$, for steel.

Being safety elements, the partial factors γ_c and γ_s are NDPs (Nationally Determined Parameters). Eurocode 8 does not recommend values for them, but mentions the options of:

1. using the values $\gamma_c = 1.5$, $\gamma_s = 1.15$, recommended in Eurocode 2 for the ULS design against non-seismic actions, or
2. setting $\gamma_c = 1.0$, $\gamma_s = 1.0$, which are the recommended values for design against accidental actions.

Option 1 is very convenient for the designer, as he/she may then dimension the elements to provide a design value of force resistance, R_d, at least equal to the largest design value of the action effect due to the non-seismic or the seismic combinations of actions. With option 2, elements have to be dimensioned once for the action effect due to the non-seismic combinations and then for that due to the seismic ones, each time using different values of γ_c and γ_s for R_d.

5.2 Sizing of frame members

5.2.1 Introduction

If member sizes are not judiciously selected from the outset, the designer will encounter problems in the detailed design phase after the analysis, e.g.:

- failure of undersized members to meet the ULS verification in shear or (more rarely) in flexure, no matter the amount of their reinforcement;
- extreme congestion of reinforcement in undersized members;
- poor utilization of materials in oversized members (which may thus have the minimum longitudinal reinforcement only), with undesirable distribution of overstrengths over the structure, leading to a concentration of inelasticity in members which are not oversized, etc.

Such problems, especially of the first two types, require revising the member sizes and repeating the analysis. Besides, even if these two types of problem do not arise, a poor choice of member sizes may lead to substandard overall seismic performance and low cost-effectiveness of the building.

The following paragraphs give guidance on how to size beams and columns, in order to help meet the rules of Eurocodes 8 and 2 during the detailed design phase. Except for the procedure in Sect. 5.2.3.4, which, strictly speaking, requires knowledge of beam longitudinal reinforcement, the rest may be used during conceptual design before any analysis.

5.2.2 Sizing of beams

To facilitate continuity of the beam top and bottom bars across a column, the cross-section of beams should be the same in all bays of a plane frame.

Beam depth is often controlled by gravity loads, or (in frame buildings without shear walls) by drift control under the damage limitation earthquake (see Section 1.3.2); it is normally chosen around one-tenth of the average bay length in the frame.

The importance of a sizeable web width is sometimes overlooked; instead, the designer often tries to accommodate the beam within the thickness of a nonstructural wall under the beam. The web should be sufficiently wide:

1. to avoid undue congestion of longitudinal bars (preferably placed in one layer),
2. to provide at least the minimum concrete cover of stirrups at the sides of the beam per Part 1-1 of Eurocode 2, and
3. to provide at least the minimum mean axial distance of longitudinal bars to the concrete surface per Part 1-2 of Eurocode 2.

Note that, depending on the environmental exposure class and the specified fire rating, requirements under 2 and 3 respectively may be quite restrictive. Concerning point 1: at the supports on columns, most of the top beam bars should be placed within the beam stirrups, but some may be outside these stirrups in a top slab; if h_f is the thickness of that slab, top bars may be placed within an effective flange width in tension (cf. Fig. 2.22(b)) extending per Eurocode 8 on each side of the beam to the face of the column parallel to it, and even beyond, by:

– $4h_f$, if the column is interior in the direction of the beam and a similarly deep beam frames into the column in the transverse direction;

– $2h_f$, if the column is exterior in the direction of the beam and supports a similar transverse beam, or is interior but without a transverse beam.

Note also that if the column cross-sectional depth in the direction of the beam is small, the onerous restriction of the beam bar size per Eurocode 8 (highlighted in Section 5.2.3.3) may result in a large number of small diameter bars, aggravating bar congestion at the supports on columns and requiring even wider beam webs. Examples at beam ends supported by relatively thin perimeter walls or columns may be found in Chapter 7 (see Figs. 7.34 to 7.39).

The ideal connection of a beam with a column is concentric, with the column being wider than the web of the beam on each side by at least 50 mm, to allow the beam longitudinal bars to pass through the confined core of the column section between its outermost bars. If a fully concentric connection is not feasible, the eccentricity between the beam and the column centroidal axes is limited by Eurocode 8 according to Eq. (4.10). To meet this condition, perimeter beams having the exterior side flush with that of the exterior columns should have a web wider than one-half of the largest cross-sectional dimension of the column at right angles to the beam axis. An example of this may be seen at the corner columns of the building in Chapter 7 (see Fig. 7.2).

5.2.3 Sizing the columns

5.2.3.1 Introduction

Storey seismic shears and column axial forces decrease from the base to the roof; so, one may be tempted to reduce the column section in the upper storeys. However, field experience and tests provide strong evidence that such a reduction is detrimental for the seismic performance of columns at intermediate or upper storeys, especially in medium- to high-rise buildings. Besides, when the column section changes from one storey to the next, it is difficult to detail the transition of column bars through the joint. Moreover, for the same reinforcement ratio (often the minimum of 1% per Eurocode 8) and cross section, the column moment resistance decreases in the upper storeys owing to the reduction in column axial compression. Recall in this respect from Section 4.4.4.1 (and witness in Figs. 7.9, 7.12, 7.15, 7.22) that the column seismic moments in dual (frame-wall) systems are often smaller in the lower storeys, while those due to gravity loads are invariably larger at the top storey (see Figs. 7.27, 7.28, 7.30); so, if the column is smaller in the upper storeys, it may require more vertical reinforcement there. Therefore, except for serious architectural reasons, the size of a column should be kept constant in all storeys, as determined from the most critical one.

The most cost-effective option, which also serves the requirement to have a clear structural system, is to have as uniform a size of columns in the building as feasible: experience from past earthquakes show that larger columns in the system are more likely to fail than the smaller ones, even when they have higher vertical steel ratio.

Eurocode 8 sets a minimum length of 200 or 250 mm, for a side of a DC M or H column, respectively. In addition, if the sensitivity coefficient for second-order effects, $\theta = P\delta/Vh$ (see Section 3.1.12) exceeds 0.1, the column sides should be at least equal to 5% of the distance of the inflection point to the column end further away, for bending within a plane parallel to the side. It should be pointed out, though, that these minimum values may govern only the narrow sides of sections composed of more than one rectangular parts (T-, L-, C-, etc). For all other column sections, the Eurocode 8 or 2 rules highlighted in Sections 5.2.3.3 to 5.2.3.4 are normally far more restrictive.

5.2.3.2 Upper limit on normalised axial load in columns

To ensure a minimum flexural ductility of the column, Eurocode 8 sets upper limits on its axial load ratio:

- For DC M: $\nu_d \leq 0.65$ (5.1a)
- For DC H: $\nu_d \leq 0.55$ (5.1b)

where $\nu_d = N_d/(A_c f_{cd})$, with N_d denoting the column axial load in the seismic design situation and A_c the column cross-sectional area. In order to choose A_c from the outset, so that Eqs. (5.1) and other restrictions listed below are met, a value of N_d should be estimated before the analysis:

- If beams parallel to the horizontal seismic action component considered frame into the column from both sides (as they normally do in interior columns), a back-of-the-envelope calculation may give N_d as the total column tributary plan area in all floors (: column tributary plan area in a typical floor times the number of overlying storeys) times the estimated quasi-permanent gravity load per unit floor area in the seismic design situation (typically from 7 to 9 kN/m^2).
- If beams frame into the column along the horizontal direction of the seismic action only from one side (e.g., as in a column which is exterior in that direction of the earthquake), this value of N_d due to quasi-permanent gravity loads should be increased at each storey by the maximum possible beam shear, taken as the sum of the hogging moment resistance at the beam end framing into the column, plus the sagging one at the opposite end, divided by the clear beam span; if the minimum value of N_d in the seismic design situation is sought (e.g., for use in Eqs. (5.2) below), the beam shear is computed from the sum of the sagging moment resistance at the end framing into the column plus the hogging one at the opposite end and subtracted from the value of N_d due to quasi-permanent gravity loads. This calculation cannot be practically done before dimensioning the beams; so:

– if the column in question is exterior, the total seismic axial force in the row of exterior columns of rectangular in plan buildings may be taken equal to the total seismic overturning moment at storey mid-height, divided by the plan dimension parallel to the horizontal direction of the seismic action: if the total building height is H_{tot} and all storeys have about the same mass, the total seismic overturning moment at mid-height of the ground storey – with storey height H_{st} – may be taken as the base shear times $(2/3)H_{tot}$ - $H_{st}/6$; exterior columns share this force in proportion to their cross-sectional area;

– if the column is interior in plan, its seismic axial force may be neglected (but this is a approximation questionable for columns not connected to beams on both sides)

5.2.3.3 Column size for bond of beam bars in beam-column joints

Eurocode 8 sets a very restrictive, albeit fully warranted, lower limit to the column depth, h_c, parallel to a beam framing into the column, to accommodate the very high bond stresses along the length of a beam bar inside an interior beam-column joint, or the anchorage of beam bars terminating in a joint, exterior or not (cf. Fig. 2.22(a)). If the bar diameter is d_{bL}, then the limit is:

- in an interior beam-column joint:

$$\frac{d_{bL}}{h_c} \leq \frac{7.5 f_{ctm}}{\gamma_{Rd} f_{yd}} \cdot \frac{1+0.8\nu_d}{1+k\dfrac{\rho_2}{\rho_{1,max}}} \tag{5.2a}$$

- in a beam-column joint which is exterior in the direction of the beam:

$$\frac{d_{bL}}{h_c} \leq \frac{7.5 f_{ctm}}{\gamma_{Rd} f_{yd}} \cdot \left(1+0.8\nu_d\right) \tag{5.2b}$$

where:

- for DC M, $\gamma_{Rd} = 1.0$, $k = 0.5$;
- for DC H, $\gamma_{Rd} = 1.2$, $k = 0.75$;

- the value of $\nu_d = N_{Ed}/f_{cd}A_c$ is the minimum in all combinations of the design seismic action with the quasi-permanent gravity loads from the analysis or the rough estimation outlined in the previous sub-section ($\nu_d = 0$ for net axial tension, as may occur in exterior columns of medium- or high-rise buildings);
- $\rho_{1,max}$ is the maximum value of beam top reinforcement ratio and ρ_2 the bottom steel ratio (taken as $\rho_2 = 0.5\rho_{1,max}$, if the beam reinforcement is not known yet and only the combination of its maximum allowed diameter and the column depth is being sought).

Most critical for Eqs. (5.2) appears to be the top storey, but as the dependence on ν_d is rather weak, those actually controlling are normally the storeys which require the largest amount of beam reinforcement at the support, to be accommodated with the minimum possible number of (larger) bars. For common axial load ratio values (e.g., ν_d ~0.2) and steel grades (500 MPa) and for a low concrete grade (f_{ck} = 20 MPa), a column depth h_c of about $40d_{bL}$ is required for DC H! The required size is relaxed to about $30d_{bL}$ for medium-high axial loads and higher concrete grades. If DC M is chosen, the required column size is reduced by about 25%.

For a sample application of this Section to the beams and columns of the 7-storey example building, see Section 7.6.2.1.

5.2.3.4 Sizing of columns to meet the slenderness limits in Eurocode 2

The EC2 rules concerning second-order effects in the analysis and verifications for gravity loads pose strong demands on the size of columns. Buildings designed for earthquake resistance do not necessarily meet by default the complex local and global lateral stiffness rules of Eurocode 2 against second-order effects. According to them, if such effects are important, the ULS resistance of members should be verified for internal forces from an analysis satisfying equilibrium in the deformed state and accounting for all effects that increase local and global deformations: cracking, material nonlinearities, creep, biaxial bending, soil flexibility, soil-structure interaction, postulated deviations of vertical members from the vertical, etc. This applies to the combination of actions taken into account at the ULS, i.e., the "persistent and transient" design situation in EN1990, where permanent and variable actions enter multiplied with partial load factors and deviations of vertical members from the vertical are considered. The seismic design situation is excluded, as it is covered by the specific provisions of Eurocode 8 for second-order effects (see Section 3.1.12). Both Eurocodes 2 and 8 allow ignoring these effects, if they are less than 10% of the first-order ones and give simplified criteria to check whether they are. Given that the type of analysis per Eurocode 2 which accounts for second-order effects is onerous, the designer should avoid it by meeting these criteria. Being deemed-to-satisfy rules, these simplified criteria are safe-sided and give larger member sizes than required by a rigorous analysis with second-order effects.

Eurocode 2 gives a simplified criterion for the slenderness ratio of isolated columns, λ:

$$\lambda \equiv \frac{l_0}{i_c} \le \lambda_{\min} = 20\frac{ABC}{\sqrt{n}} \tag{5.3}$$

where l_0 is the column effective length, i_c the radius of gyration of the uncracked column section, and $n=N_{Ed}/A_c f_{cd}$, with N_{Ed} the column axial force in the "persistent and transient" design situation in EN1990 (i.e., with permanent and imposed actions multiplied with their partial load factors). If necessary, N_{Ed} may be estimated before the analysis as the product of the total column tributary plan area in all floors (: column tributary plan in a typical floor times the number of overlying storeys) times the sum of factored permanent load per unit floor area (estimated between 10 and 12 kN/m^2 in typical buildings) and the factored specified imposed load per unit floor area. Eurocode 2 gives default values for A, B and C:

- A=0.7 for A, corresponding to an "effective" creep coefficient of 2.1;
- B=1.1 for $B=\sqrt{(1+2\rho_{tot} f_{yd}/f_{cd})}$; the default value corresponds to a column steel ratio, ρ_{tot}, slightly over the minimum of 0.4% recommended in Eurocode 2; B = 1.2 suits better the 1% minimum steel ratio

in Eurocode 8;

- C=0.7 for C=1-M_{01}/M_{02}, where M_{01}, M_{02} are the first-order end moments of the column, with $|M_{02}| \geq |M_{01}|$; C=0.7 is recommended if the building is not braced by walls per Eq. (5.11) below, or if the column's first-order moments are mainly due to lateral loads or postulated deviations from the vertical.

The effective length of the column is derived from its clear height, H_{cl}, as follows:

- For buildings not braced by walls with total moment of inertia meeting Eq. (5.11):

$$l_0 = H_{cl} \cdot \max\left\{\sqrt{1+10\frac{k_1 k_2}{k_1+k_2}};\left(1+\frac{k_1}{1+k_1}\right)\left(1+\frac{k_2}{1+k_2}\right)\right\} \tag{5.4a}$$

- For buildings braced by walls per Eq. (5.11):

$$l_0 = 0.5 H_{cl} \cdot \sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)\cdot\left(1+\frac{k_2}{0.45+k_2}\right)} \tag{5.4b}$$

In Eqs. (5.4) k_i is the column rotational stiffness at end node i (= 1, 2) relative to the total restraining stiffness (moment, M_i, applied to the node divided by the resulting rotation, θ_i) of the members framing in node i in the plane of column bending considered:

$$k_i = \frac{\theta_i}{M_i}\sum\frac{EI_{c,eff}}{H_{cl}} = \frac{\sum\frac{EI_{c,eff}}{H_{cl}}}{4\sum\frac{EI_{c,eff}}{H_{cl}}+4\sum\frac{EI_{b,eff}}{L_{cl}}} \tag{5.5}$$

The sums of column or beam stiffness values in Eq. (5.5) are taken around node i. L_{cl} is the clear length of a beam framing into node i within the plane of column bending considered and $EI_{b,eff}$ is this beams's cracked flexural rigidity, taking into account creep, i.e., computed with the design concrete modulus, $E_{cd} = E_{cm}/1.2$, divided further by $(1+\varphi_{eff})$, where φ_{eff} is the final creep coefficient times the fraction of the total bending moment in the combination of actions which is due to quasi-permanent loads. If d is the beam effective depth, b the width of a flange which is in compression over its whole thickness, t, and b_w

the thickness of the web, $EI_{b,eff}$ may be taken as:

$$EI_{b,eff}=E_s bd^3\left\{\begin{array}{l}\dfrac{1}{a}\left[\dfrac{\xi^2}{2}\left(\dfrac{1+\delta}{2}-\dfrac{\xi}{3}\right)\dfrac{b_w}{b}+\left(1-\dfrac{b_w}{b}\right)\left(\xi-\dfrac{t}{2d}\right)\left(1-\dfrac{t}{2d}\right)\dfrac{t}{2d}\right]+\\ \dfrac{(1-\delta)}{2}\left[(1-\xi)\rho_1+(\xi-\delta)\rho_2+\dfrac{\rho_V}{6}(1-\delta)\right]\end{array}\right\} \tag{5.6}$$

where α = $(1+\varphi_{eff})E_s/E_{cd}$ is the ratio of effective elastic moduli (steel-to-concrete) and the neutral axis depth (normalised to d) is computed as:

$$\xi=\sqrt{\alpha^2A^2+2\alpha B}-\alpha A \tag{5.7}$$

where:

$$A=\frac{b}{b_w}(\rho_1+\rho_2+\rho_v)+\frac{1}{\alpha}\frac{t}{d}\left(\frac{b}{b_w}-1\right),\ B=\frac{b}{b_w}(\rho_1+\rho_2\delta+0.5\rho_v(1+\delta))+\frac{1}{2\alpha}\left(\frac{t}{d}\right)^2\left(\frac{b}{b_w}-1\right) \tag{5.8}$$

In Eqs. (5.6)-(5.8) ρ_1, ρ_2 are the ratios of tension and compression reinforcement, ρ_V the ratio of longitudinal reinforcement at the sides of the web between the tension and compression steel (all ratios normalised to bd) and $\delta = d_1/d$ the centroidal distance of compression bars from the extreme compression fibres, normalised to d.

If there is no distinct compression flange, Eqs. (5.6)-(5.8) are applied with $b = b_w$. If there is one, but Eqs. (5.7), (5.8) give a neutral axis depth, ξd, less than the compression flange thickness, t, then Eqs. (5.6)-(5.8) are (re-)applied in a simplified form, with b_w taken equal to b.

If two beams parallel to the plane of column bending frame into opposite faces of node i, they should be considered in turn with the top flange of one beam taken in tension and that of the other in compression. Note that, although strictly speaking the effective width of the slab should be included in b when the compression flange includes the slab, it makes little difference in the outcome of Eq. (5.6) if the web width is taken as b, provided that this value is also used when ρ_1, ρ_2, ρ_V are normalised to bd.

Concerning the cracked flexural rigidity of a column, $EI_{c,eff}$, Eurocode 2 gives an approximation:

$$EI_{c,eff} = E_s I_s + E_{cd} \sqrt{\frac{f_{ck}(MPa)}{20}} \frac{K_2 I_c}{1+\varphi_{eff}} \tag{5.9}$$

where E_s and I_s are the elastic modulus and the moment of inertia of the section's reinforcement (which, if unknown, may obtained at this stage from the minimum steel ratio of 1% in Eurocode 8) with respect to the centroid of the section, I_c is the moment of inertia of the uncracked gross concrete section and K_2 is taken as:

$$K_2 = \frac{n\lambda}{170} = \frac{1}{170} \frac{N_{Ed}}{A_c f_{cd}} \frac{l_0}{i_c} \le 0.20 \tag{5.10}$$

A minimum value k_i = 0.1 is recommended in Eurocode 2 at a column end where the column is fixed against rotation (here at the base of the ground storey). Note that the column one storey above has its lower end less restrained and hence may be more critical, despite its lower axial load. So, the minimum column size meeting Eq. (5.3) throughout all storeys should be sought in the two lowest storeys.

As both the unknown effective length of the column, l_0, and the size of its section (through i_c and A_c) enter in Eqs. (5.3)-(5.5) and (5.9), (5.10) in an implicit nonlinear way, they have to be found by iterations, after dimensioning the top beam reinforcement at the supports to determine $EI_{b,eff}$.

Eurocode 2 allows to consider the building as braced in a given horizontal direction and to apply Eq. (5.4b) in lieu of (5.4a), if it has walls with a total moment of inertia of the uncracked gross section in that horizontal direction, $\sum I_w$, which meets the following condition at the top of the foundation or of a rigid basement:

$$F_{V,Ed} \le 0.31 \frac{n_{st}}{n_{st}+1.6} \frac{E_{cd} \sum I_w}{H_{tot}^2} \tag{5.11}$$

where $F_{V,Ed}$ is the total vertical load acting on all n_{st} overlying storeys of the building throughout the plan area and H_{tot} the height of the walls above the top of the foundation or of a rigid basement.

Eq. (5.11) presumes that the bracing walls are cracked. If it can be shown that they stay uncracked while performing their bracing function in the ULS combination of actions considered (i.e., for the "persistent and transient" design situation, with factored permanent and imposed loads and geometric imperfections), then the right-hand-side of Eq. (5.11) is multiplied by 2, reducing by 50% the minimum required value of $\sum I_w$. The bracing walls should be dimensioned at the ULS to resist the full lateral force on the building due to the deviation from the vertical postulated in Eurocode 2.

If the building is laterally braced by walls meeting the criterion of the above paragraph in a horizontal direction, then Eq. (5.3) can be met with a column depth of reasonable magnitude in that direction (as in the example building of Chapter 7); otherwise they may come out quite large (Fardis et al, 2012). Walls which are collectively sufficient to laterally brace the building per the above paragraph, normally take a large enough fraction of the elastic base shear for the lateral force resisting system to qualify as dual (be it frame-equivalent). So, it is the columns of frame systems that are more severely penalised by the Eurocode 2 rules on second-order effects. On the positive side, the resulting large columns of laterally unbraced buildings, as well as the large walls necessary in braced ones, impart significant lateral force resistance and stiffness, thanks to which a building may perform well in an earthquake it has not been designed for (Fardis et al, 2012).

As pointed out at the closing of Sect. 5.2.3.1, in order to apply Eqs. (5.6)-(5.8) the beam longitudinal reinforcement should be known. This is possible only when the procedure in the present sub-section is applied in the context of detailed design after the beams are fully dimensioned and the designer wants to check if second-order effects may indeed be neglected. In that phase the value of N_{Ed} to be used in Eqs. (5.3), (5.10) is the one from the analysis in EN1990's "persistent and transient" design situation. To size the columns at the conceptual design phase, the procedure may be applied with the beam longitudinal reinforcement estimated in the two lowest storeys for the purposes of Eqs. (5.6)-(5.8): e.g. with $\rho_v = 0$, and the top and bottom reinforcement taken from the corresponding maximum and minimum steel ratios, respectively, per Eurocode 8.

For a sample application of this Section to one of the columns of the 7-storey example building, see Section 7.6.2.2.

5.3 Detailed design of beams in flexure

5.3.1 Dimensioning of the beam longitudinal reinforcement for the ULS in flexure

The top and bottom bars at the two ends of each beam are dimensioned for the ULS in flexure with no axial force for the envelope of bending moments resulting from the analysis under:

(a) the combination of factored gravity loads ("persistent and transient design situation" per EN1990), and

(b) the combination of quasi-permanent gravity loads, G+ψ_2Q, with plus and minus the design seismic action.

The beam seismic moments in (b) are the final outcome of the combination of the moments due to the horizontal components of the design seismic action, E_X, E_Y, per Eqs. (3.99) or (3.100) in Section 3.1.7 and include the effect of the accidental eccentricities of these components (see Section 3.1.8).

The cross-sectional area of the top reinforcement, A_{s1}, of each end region is dimensioned as the tension reinforcement required for an acting moment, M_{Ed}, equal to the maximum hogging moment at the column face; normally in this dimensioning, combination (b), with the beam seismic moments taken as hogging, controls over (a). The cross-sectional area of the bottom reinforcement, A_{s2}, is dimensioned as the tension reinforcement for an acting moment, M_{Ed}, equal to that at the column face, or at a nearby section where the sagging moment attains its maximum; in this case M_{Ed} is obtained from combination (b), but with the beam seismic moment taken as sagging. Besides, the main bottom bars of the beam are dimensioned from a section around mid-span, normally where the sagging moment from combination

(a) attains its maximum value within the span.

The cross-sectional area, A_s, of the tension reinforcement may be conveniently dimensioned from the extreme value of the pertinent acting moment M_{Ed} (i.e., the extreme sagging moment for the bottom reinforcement or the extreme hogging one for the top bars), by taking the internal lever arm of the beam (between its tension and compression chords), z, as equal to the distance between the tension and compression bars, d-d_2, where d is the effective depth of the section and d_2 the distance of the centroid of the compression bars from the extreme compression fibres:

$$A_s = \left|M_{Ed}\right| / \left(f_{yd}\left(d - d_2\right)\right) \tag{5.12}$$

where f_{yd} is the design yield stress of steel. Note that the absolute value of M_{Ed} is used; its sign determines the side of the section (top or bottom) where the tension fibres are and the tension reinforcement area, A_s, is placed.

Alternatively to Eq. (5.12), the reinforcement may be dimensioned with more strict adherence to the assumptions in Eurocode 2 for the ULS in flexure without axial force for concrete grade, f_{ck}, up to 50 MPa. This altenative employs the dimensionless acting moment:

$$\mu_d = \left|M_{Ed}\right| / \left(b_{eff} d^2 f_{cd}\right) \tag{5.13}$$

where b_{eff} is the effective width of the compression flange and f_{cd} the design strength of concrete. From the value of μ_d, the mechanical ratio of tension reinforcement, defined as:

$$\omega = A_s/(b_{eff}d)\cdot(f_{yd}/f_{cd}) \tag{5.14a}$$

is computed as:

$$\omega = 0.973\left(1 - \sqrt{1 - \frac{2\mu_d}{0.973}}\right), \text{ or} \tag{5.15a}$$

$$\omega = 1 - \sqrt{1 - 2\mu_d} \tag{5.15b}$$

Eq. (5.15a) is obtained if the standard parabolic-rectangular σ-ε law of concrete is adopted for ULS design; Eq. (5.15b) results, instead, from the rectangular stress block in the extreme 80% of the compression zone. Neither of these expressions accounts for the presence of longitudinal bars in the compression zone. It is necessary to account for it, if the dimensionless acting moment, μ_d, is so large that the – normalized to d – neutral axis depth, ξ, reaches a value beyond which the tension reinforcement is not even in the yielding state when the extreme compression fibres exhaust the ultimate strain of concrete in ULS design for bending, ε_{cu2} (= 0.35% for $f_{ck} \leq 50$ MPa), i.e., when ξ reaches the value:

$$\xi_{\lim} = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + f_{yd}/E_s} \tag{5.16}$$

If the standard parabolic-rectangular σ-ε law of concrete is adopted for the ULS design, the limit value of μ_d corresponding to the value of ξ from Eq.(5.16) is:

$$\mu_{d,\lim} = 0.81\xi(1 - 0.416\xi_{\lim}) \tag{5.17a}$$

whereas, if the rectangular stress block in the extreme 80% of the compression zone is adopted:

$$\mu_{d,\lim} = 0.8\xi(1 - 0.4\xi_{\lim}) \tag{5.17b}$$

The part of μ_d which exceeds the value of $\mu_{d,\lim}$ from Eqs.(5.17) is assigned to a resisting moment produced by compression reinforcement with cross-sectional area A_s' and a mechanical ratio defined as:

$$\omega' = A_s'/(b_{eff}d)\cdot(f_{yd}/f_{cd}) \tag{5.14b}$$

and computed from:

$$\omega' = \frac{\mu_d - \mu_{d,\lim}}{1 - d_2/d} \tag{5.18}$$

Then the tension reinforcement is obtained as:

$$\omega = \omega_{\lim} + \omega' \tag{5.19}$$

where $\omega_{\lim}$ is the value of ω given by Eqs. (5.15) for $\mu_d = \mu_{d,\lim}$. The second term is the part of the tension reinforcement needed to act as a couple with the compression reinforcement from Eq. (5.18).

The required cross-sectional areas of the top and bottom reinforcement, A_{s1}, A_{s2}, are determined as above, using as acting moment, M_{Ed}, the extreme hogging moment for A_{s1} and the extreme sagging one for A_{s2}. The final value of A_{s2} should not be taken less than the compression reinforcement area, A_s', obtained from the extreme hogging moment via Eqs. (5.14b), (5.13) and (5.15) to (5.18) (if Eqs. (5.13) to (5.19) are used in lieu of (5.12)).

Eurocode 8 allows to count in A_{s1} the cross-sectional area, $\Delta A_{s,slab}$, of all slab bars which are:

– parallel to the beam,
– within the effective flange width in tension per Eurocode 8, which extends on each side of the web beyond the face of the column parallel to the beam by the widths given in Section 5.2.2, and
– are well anchored within the joint or beyond.

However, the design of the beams in flexure is normally a separate procedure from the design of the slabs; therefore, $\Delta A_{s,slab}$ is not available at this phase of detailed design. So, most often the designer fails to profit from this allowance, to reduce the amount of real beam top reinforcement by $\Delta A_{s,slab}$, presuming this convenient omission to be safe-sided. Indeed it is so for the ULS in flexure of the beam, but is unsafe wherever the beam moment resistance is used as a demand in "capacity-design" calculations (see Sections 5.3.4 and 5.4.1).

All the above apply to beams in pure bending, without axial load. As a matter of fact, the values of beam axial forces which may come out of the analysis depend heavily on the modelling of the floor diaphragms and/or the way the external lateral loads are applied to the floors. So, normally they are fictitious and would better be neglected in dimensioning the beams. At any rate, the way to consider a (real) axial force in dimensioning a beam for the ULS in flexure is presented in Section 6.3.8, on the occasion of a postulated axial force for the design of tie-beams between footings.

5.3.2 Detailing of beam longitudinal reinforcement

In translating A_{s1} and A_{s2} into a combination of bar diameters and numbers, the designer should respect the detailing rules of Eurocode 8 summarised in Table 5.1. The rule at the fourth row concerning the maximum ratio of tension reinforcement, ρ_{max}, is the only one of these rules which is not prescriptive; at the same time it is the most restrictive: as the value of A_{s1} to be accommodated within a given beam width, b, cannot be reduced below what is necessary to resist the acting moment, M_{Ed}, the best way to meet the rule for ρ_{max} is by increasing the ratio of compression reinforcement, ρ', at the end section.

The bar diameters chosen should also respect the maximum allowed by Eqs. (5.2) for a given section depth, h_c, of the column where these bars are anchored (at exterior columns) or pass through (at interior ones).

Table 5.1 EC8 detailing of the longitudinal bars in primary beams (in secondary ones as in DCL)

	DC H	**DC M**	**DC L**
"critical region" length at member end	$1.5h$	H	
$\rho_{min} = A_{s,min}/bd$ at the tension side	$0.5f_{ctm}/f_{yk}$ (1)		$0.26f_{ctm}/f_{yk}$ (1), 0.13% (2)
$\rho_{max} = A_{s,max}/bd$ in critical regions (2)	$\rho'+0.0018f_{cd}/(\mu_\phi\varepsilon_{yd}f_{yd})$ (3)		0.04
$A_{s,min}$, top and bottom bars	$2\Phi14$ ($308mm^2$)	-	
$A_{s,min}$, top bars in the span	$0.25A_{s,top\text{-}supports}$	-	
$A_{s,min}$, bottom bars in critical regions	$0.5A_{s,top}$ (4)		-
$A_{s,min}$, bottom bars at supports	$0.25A_{s,bottom\text{-}span}$ (2)		
anchorage length for diameter d_{bL} (5)	$l_{bd} = a_{tr}[1-0.15(c_d/d_{bL}-1)](d_{bL}/4)f_{yd}/(2.25f_{ctd}a_{poor})$ (6),(7),(8),(9)		

(1) f_{ctm} (MPa) = $0.3(f_{ck}(\text{MPa}))^{2/3}$: 28-day, mean tensile strength of concrete; f_{yk} (MPa): nominal yield stress of longitudinal steel.
(2) NDP (Nationally Determined Parameter) per EC2; the value recommended in EC2 is given here.
(3) ρ': steel ratio at the opposite side of the section; μ_ϕ: curvature ductility factor corresponding via Eqs. (5.64) to the basic value of the behaviour factor, q_o, applicable to the design; $\varepsilon_{yd} = f_{yd}/E_s$.
(4) This $A_{s,min}$ is additional to the compression steel from the ULS verification of the end section in flexure under the extreme hogging moment from the analysis for the seismic design situation.
(5) Anchorage length in tension is reduced by 30% if the bar end extends by $\geq 5d_{bL}$ beyond a bend $\geq 90^o$.
(6) c_d: concrete cover of anchored bar, or one-half the clear spacing to the nearest parallel anchored bar, whichever is smaller.
(7) $a_{tr} = 1-k(n_wA_{sw}-A_{s,t,min})/A_s \geq 0.7$, with A_{sw}: cross-sectional area of tie-leg within the cover of the anchored bar; n_w: number of such tie legs over the length l_{bd}; $k = 0.1$ if the bar is at a corner of a hoop or tie, $k = 0.05$ otherwise; $A_s = \pi d_{bL}^2/4$ and $A_{s,t,min}$ is specified in EC2 as equal to $0.25A_s$.
(8) $f_{ctd}=f_{ctk,0.05}/\gamma_c = 0.7f_{ctm}/\gamma_c = 0.21f_{ck}^{2/3}/\gamma_c$: design value of 5%-fractile tensile strength of concrete.
(9) $a_{poor} = 1.0$ if the bar is in the bottom 0.25 m of the beam depth, or (in beams deeper than 0.6 m) ≥ 0.3 m from the beam top; otherwise, $a_{poor} = 0.7$.

The bars chosen on the basis of the two end sections and the one around mid-span where the span bottom bars are determined, are terminated according to the positive and negative moment envelope; they extend beyond the point where they are not needed according to the envelope by the "tension shift" length $z = 0.9d\text{-}d_2$ in Eurocode 2.

Eurocode 2 considers the bar stress to drop off linearly along the anchorage length, l_{bd} (given at the last row of Table 5.1), from f_{yd} to zero. So, if the inclination of the moment envelope (i.e., the shear force, V) exceeds the bar yield force, $f_{yd}\pi d_{bL}^2/4$, times the ratio of the internal lever arm, z, to l_{bd} (i.e., if $V > (2.25\pi f_{ctd}a_{poor})d_{bL}z/\{a_{tr}[1\text{-}0.15(c_d/d_{bL}\text{-}1)]\}$ with the notation of Table 5.1), then the bar has to be further extended by l_{bd} beyond the point it is not needed according to the moment envelope. Otherwise, the bar fully contributes along l_{bd} in resisting the moment.

The full string of beams ("continuous beam") in a frame should be designed in bending all together, combining reinforcement requirements to the right and left of interior joints. It is also recommended to combine different top or bottom bars into continuous ones, if their ends come close or overlap. To this end, few bar sizes (even a single size) should be used all along each string of beams.

The rules in Table 5.1 for DC L do not apply to deep beams, defined in Eurocode 2 as those with depth, h, less than one-third of their span. Eurocode 2 also requires skin reinforcement at the lateral sides of 1 m deep beams or deeper. For the purposes of detailing the beam longitudinal reinforcement, deep beams may be defined as deep as the lesser of 1 m, or one-third of the span or deeper. Eurocode 2 prescribes an orthogonal reinforcement mesh per lateral side of a deep beam, with maximum bar spacing which is the lesser of 300 mm or twice the web thickness and cross-sectional area per side and direction not less than 150 mm^2/m or 0.05% of the concrete area (i.e., 0.1% total for both sides). Much more demanding are the requirements of Eurocode 2 for skin reinforcement placed to control cracking at the web of 1 m deep beams or deeper (see next Section). Note that the minimum ratio of tension steel, ρ_{min}, in row 2 of Table 5.1 is also to control potential cracking throughout the tension zone. However, if it is concentrated just at the tension chord, its effectiveness in that role is reduced at points further away. So, if the beam is deeper than 1 m, Eurocode 2 assigns that role to skin reinforcement distributed over the entire tension zone. In that case, placing at the tension chord a quantity, $A_{s,min}$, of minimum reinforcement equal to ρ_{min} times the full effective cross-sectional area, *bd*, would be not just duplication, but a waste. To avoid it, it is recommended here for deep beams:

– to determine the minimum reinforcement concentrated at the tension chord, $A_{s,min}$, as ρ_{min} from row 2 of Table 5.1, times the product of *b* and a depth of 1 m;
– to distribute over the depth of the section horizontal skin reinforcement at a steel ratio of ρ_{min}; that reinforcement should also be dimensioned for crack control in the web, according to the next Section.

5.3.3 Serviceability requirements in EC2 - Impact on beam longitudinal reinforcement

5.3.3.1 Introduction

Eurocode 2 includes important Serviceability Limit State (SLS) requirements concerning the level of stresses in steel or concrete and the crack width under service loads, as well as the amount and form of reinforcement necessary to control cracking due to non-quantified imposed deformations and other ill-defined, often random, causes. These requirements are relevant to beams, but have very little to do with seismic design; moreover, they are normally met by default in ordinary beams designed and detailed for earthquake resistance. So, strictly speaking, they are outside the scope of this book. However, they often control the longitudinal reinforcement in oversized beams, such as deep foundation beams, especially those which double as perimeter walls of basements. As a matter of fact, in Chapter 7 they are applied to such elements of the example building and found to control their longitudinal reinforcement. For all these reasons, and because there is still a gap for this topic in literature concerning the application of Eurocode 2, these SLS requirements are highlighted here, alongside guidance on how to apply them to beams. They are relevant to those regions of a beam where tension may build up under service conditions: normally the top flange at the end sections of beams in the superstructure and the bottom one in the span; the reverse in foundation beams.

For a sample application of Sections 5.3.2 and 5.3.3 to the beams and columns of the 7-storey example building, see Section 7.6.2.1

5.3.3.2 Stress limitation SLS

The SLS of stress limitation imposes stress limits on concrete and steel under service conditions. The limits are Nationally Determined Parameters (NDPs) with recommended values:

- under the "characteristic" gravity loads, $G+Q$:
  - concrete stress, $\sigma_{c,G+Q} \leq 0.6 f_{ck}$;
  - steel stress, $\sigma_{s1,G+Q} \leq 0.8 f_{yk}$
- under the quasi-permanent gravity loads, $G+\psi_2 Q$:
  - concrete stress, $\sigma_{c,G+\psi 2Q} \leq 0.45 f_{ck}$.

Once the amount of tension, compression and web reinforcement in the beam section is determined on the basis of Sections 5.3.1 and 5.3.2, etc, the above limits are checked as follows:

$$\sigma_{s1,G+Q} = E_s \frac{M_{G+Q}}{EI_{b,eff}} (1-\xi) d \leq 0.8 f_{yk} \tag{5.20}$$

$$\sigma_{c,G+Q} = E_{cm} \frac{M_{G+Q}}{EI_{b,eff}} \xi d \leq 0.6 f_{ck} \tag{5.21a}$$

$$\sigma_{c,G+\psi 2Q} = E_{cm} \frac{M_{G+\psi 2Q}}{EI_{b,eff}} \xi d \leq 0.45 f_{ck} \tag{5.21b}$$

where E_s and E_{cm} are the elastic moduli of steel and concrete (mean values) and $EI_{b,eff}$, ξ are determined from Eqs. (5.6)-(5.8). As the actions causing these stresses are (almost fully) long term, it makes sense to use in Eqs. (5.6)-(5.8) the value $\alpha = (1+\varphi_\infty)E_s/E_{cm}$, with φ_∞ the final value of the creep coefficient. This is safe-sided for σ_{s1}, but reduces the estimate of σ_c.

5.3.3.3 Crack width SLS

The characteristic value of crack width, w_k, under the quasi-permanent gravity loads, $G+\psi_2Q$, is checked against an upper limit value, w_{max}, which is an NDP, with a recommended value of 0.3 mm for the common environmental exposure classes in buildings. According to Eurocode 2, for long-term loading (such as the quasi-permanent gravity loads), w_k may be computed as:

$$w_k = 1.7\left(2(c_{nom} + d_{bw}) + 0.1\frac{d_{bL,mean}}{\rho_{eff}}\right)\frac{\max\left(\left(\sigma_{s1,G+\psi 2Q} - 0.4 f_{ctm}\frac{1+\alpha\rho_{eff}}{\rho_{eff}}\right); 0.6\sigma_{s1,G+\psi 2Q}\right)}{E_s} \tag{5.22}$$

Coefficient 1.7 in Eq. (5.22) converts the mean estimate of the crack width to a characteristic value; the term that follows is a semi-empirical best estimate of the crack spacing; the last term is an estimate of the difference in mean tensile strains of steel and concrete between adjacent cracks. Concerning symbols, c_{nom} is the nominal concrete cover of the stirrup (minimum required for durability, plus a tolerance of 10 mm), d_{bw} the diameter of the stirrup, $d_{bL,mean}$ the mean longitudinal bar diameter in the tension zone and $\sigma_{s1,G+\psi 2Q}$ the steel stress due to the quasi-permanent gravity loads, computed from Eq. (5.20), using $M_{G+\psi 2Q}$ instead of M_{G+Q}. The tension steel ratio:

$$\rho_{eff} = A_{s1}/A_{c,eff} \tag{5.23}$$

refers to the effective area of concrete in tension surrounding the tension reinforcement, A_{s1}. For sections in bending, with rectangular tension zone having width b_w, Eurocode 2 defines $A_{c,eff}$ as:

$$A_{c,eff} = \min\left(2.5d_1; \frac{h-\xi d}{3}\right) b_w \tag{5.24a}$$

where d_1 is the centroidal distance of A_{s1} from the extreme tension fibres and $h\text{-}\xi d$ the depth of the tension zone. If A_{s1} is spread in a well defined T- or L-shaped tension zone with flange width b and thickness t, and if b_w denotes the width of the web, then (Fig. 5.1):

$$A_{c,eff} = \min\left(2.5d_1; \frac{h-\xi d}{3}\right) b_w + \min\left(t; 2.5d_1; \frac{h-\xi d}{3}\right)(b-b_w) \tag{5.24b}$$

Note that, strictly speaking, the case of Eq. (5.24a) is not that of the effective flange width in tension introduced in Section 5.2.2, which contributes to the hogging moment resistance of the beam's end section with its slab bars. That concept is introduced in Eurocode 8, not in Eurocode 2, and refers to the ULS, not the SLS. By the same token, the slab bars in that effective width in tension are not included in A_{s1} for the purposes of Eq. (5.23). A representative case where Eq. (5.24b) does apply is the strip footing of the deep foundation beam of Fig. 7.42, with all eight of its longitudinal bars included in A_{s1}.

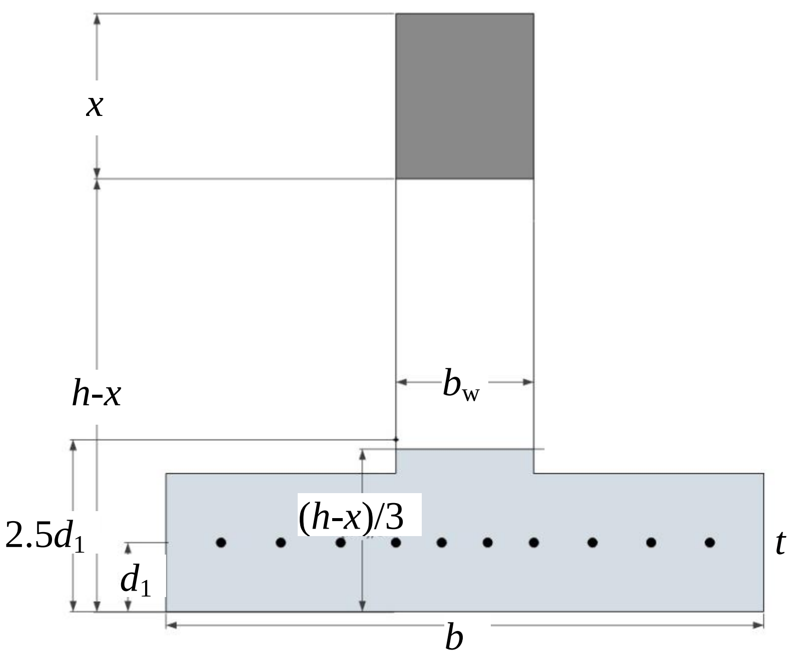

Fig. 5.1 Effective concrete area in tension for crack control

Eurocode 2 defines α for use in Eq. (5.22) as $\alpha = E_s/E_{cm}$. However, as the crack width is computed for the quasi-permanent loads, it makes more sense to use the value $\alpha = (1+\varphi_\infty)E_s/E_{cm}$, as for Eqs. (5.20), (5.21).

Eurocode 2 differentiates the estimation of crack width in the web of deep beams where skin reinforcement is placed (see the last parts of Section 5.3.2 above and 5.3.3.4 below): the coefficient in front of $d_{bL,mean}/\rho_{eff}$ in the term representing the mean crack spacing is 0.2, instead of 0.1; moreover, a mean value of $\sigma_{s1,G+\psi 2Q}$ over the web is used, equal to one-half the maximum steel stress computed over the section.

5.3.3.4 Minimum steel for crack control

Should cracking occur due to non-quantified imposed deformations or another ill-defined, possibly often random, cause, the steel crossing the crack should be able to keep the crack width below the applicable limit value, w_{max}. To this end, its cross-sectional area in the tension zone, $A_{s,min}$, should be sufficient to resist the tensile force released when the part of the so far uncracked section which is in tension cracks, and, indeed, developing a steel stress, σ_s, which is low enough to keep the resulting crack width below w_{max}. According to Eurocode 2, the usual limit value $w_{max} = 0.3$ mm is achieved, if $A_{s,min}$, develops a stress, σ_s, which depends on the mean bar diameter, $d_{bL,mean}$, as:

– if 8 mm $< d_{bL,mean} \leq$ 12 mm: $\sigma_s\,(\text{MPa}) = 280 + 20\times(12 - d_{bL,mean})$ (5.25a)

– if 12 mm $< d_{bL,mean} \leq$ 16 mm: $\sigma_s\,(\text{MPa}) = 240 + 10\times(16 - d_{bL,mean})$ (5.25b)

– if 16 mm $< d_{bL,mean} \leq$ 25 mm: $\sigma_s\,(\text{MPa}) = 200 + (40/9)\times(25 - d_{bL,mean})$ (5.25c)

– if 25 mm $< d_{bL,mean} \leq$ 32 mm: $\sigma_s\,(\text{MPa}) = 160 + (40/7)\times(32 - d_{bL,mean})$ (5.25d)

If the tension zone in the uncracked section is rectangular, with width that of the web, b_w, and depth, $y_{cg,t}$, equal to the distance of the centroid of the uncracked section to the extreme tension fibres, then, the minimum reinforcement of beams in flexure per Eurocode 2 is:

$$A_{s,\min} = 0.4 k_h b_w y_{cg,t} \frac{f_{ctm}}{\sigma_s} \tag{5.26a}$$

where k_h reflects the reduction of the net tensile force in deep sections due to non-uniform self-equilibrating stresses:

– if $h \leq 0.3$ m: $k_h = 1.0$ (5.27a)

– if $0.3 \text{ m} < h \leq 0.8$ m: $k_h = 1.21 - 0.7h$(m) (5.27b)

– if $0.8 \text{ m} < h$: $k_h = 0.65$ (5.27c)

If the tension zone in the uncracked beam has a T- or L-shape, and b and t denote the width and the thickness of the tension flange, while b_w still stands for the width of the web, then:

$$A_{s,\min} = \left[0.4 k_h b_w y_{cg,t} + \max\left(0.5; 0.9 k_b \left(1 - \frac{t}{2 y_{cg,t}} \right) \right) (b - b_w) t \right] \frac{f_{ctm}}{\sigma_s} \tag{5.26b}$$

where k_b is the counterpart of k_h for a wide tension flange:

– if $(b\text{-}b_w) \leq 0.3$ m: $k_b = 1.0$ (5.28a)

– if $0.3 \text{ m} < (b\text{-}b_w) \leq 0.8$ m: $k_b = 1.21 - 0.7(b\text{-}b_w)$ (m) (5.28b)

– if $0.8 \text{ m} < (b\text{-}b_w)$: $k_b = 0.65$ (5.28c)

The rules in Eurocode 2 concerning the minimum skin reinforcement for crack control in deep beams allow taking $\sigma_s = f_{yk}$ and $k_h = 0.5$, giving a minimum ratio of horizontal web reinforcement:

$$\rho_{h,\min} = 0.5 k_c \frac{f_{ctm}}{f_{yk}} \tag{5.29}$$

where k_c reflects the distribution of stresses within the tributary area of the skin reinforcement - it is the counterpart of 0.4 in Eqs. (5.26a) and of $0.9k_b(1\text{-}0.5t/y_{cg,t})$ in (5.26b). The most adverse condition is a uniform stress distribution, as in pure tension; then $k_c = 1.0$. This gives the same minimum steel ratio as listed at the second row of Table 5.1 for DC M or H beams, but this time distributed over the sides of the web. Eurocode 2 points out that, if the target is to control the crack width in the web to w_{max} = 0.3 mm, the value of σ_s which corresponds to the diameter of the skin reinforcement according to Eqs. (5.25) should be used in Eq. (5.29), instead of f_{yk}.

5.3.4 Beam moment resistance at the end sections

After dimensioning and detailing the beam longitudinal bars at the two end sections, the design values of beam moment resistance at these sections are computed from the final cross-sectional areas of its reinforcement. If there is only top and bottom reinforcement in the section, A_{s1} and A_{s2}, the design values of moment resistance in hogging or sagging bending, respectively, may be estimated as:

$$M^{-}_{Rd,b} = \min(A_{s1}, A_{s2}) f_{yd} (h - d_1 - d_2) + \max[0, (A_{s1} - A_{s2})] f_{yd} [h - d_1 - 0.5(A_{s1} - A_{s2}) f_{yd} / (b_w f_{cd})] \quad (5.30a)$$

$$M^{+}_{Rd,b} = A_{s2} f_{yd} \max[(h - d_2 - 0.5 A_{s2} f_{yd} / (b_{eff} f_{cd})), (h - d_1 - d_2)] \quad (5.30b)$$

where:

- d_1, d_2 are the centroidal distances of A_{s1}, A_{s2}, from the top or bottom of the beam section, respectively,
- b_w, b_{eff} are the effective widths in compression of the bottom flange (normally that of the web) and the top flange, respectively.

Often a simpler option is considered to provide sufficient accuracy:

$$M^{-}_{Rd,b} = zA_{s1} f_{yd}; \quad M^{+}_{Rd,b} = zA_{s2} f_{yd} \quad (5.30c)$$

where the internal lever arm, z, may be taken equal to 0.9d.

The values from Eqs.(5.30) are used in the "capacity design" of columns in flexure, Eq.(5.31), and for the "capacity design" shears in the beam itself and the columns connected to it (see Eqs. (5.42) and (5.44), respectively). Eurocode 8 stresses that, whenever Eq.(5.30a) is used for these "capacity design" purposes, the area, $\Delta A_{s,slab}$, of all slab bars which are: a) parallel to the beam, b) placed on each side of it within the effective flange width in tension per Eurocode 8 (given in Section 5.2.2) and c) well anchored within the joint or beyond, should be included in A_{s1}, no matter whether they are relied upon to provide the tension reinforcement area, A_s, required for the ULS in flexure under the extreme hogging moment according to Section 5.3.1 (see also second paragraph from the end of that section).

The moment resistance of deep beams, having uniformly distributed reinforcement between its top and bottom ones, A_{s1} and A_{s2}, may be determined from Section 5.4.3, applicable to asymmetrically reinforced column sections with uniformly distributed reinforcement along the lateral sides, by setting the axial load equal to zero.

5.4 Detailed design of columns in flexure

5.4.1 Strong column-weak beam capacity design

To pursue the desired global ductility, Eurocode 8 promotes beam-sway mechanisms and takes measures to prevent a soft-storey (cf. Sections 2.2, 4.5.2, 5.4.2). A soft-storey mechanism (Fig. 2.9(a)) develops in a frame system when the top and bottom ends of (all) the columns in a storey yield in opposite bending and start undergoing unrestrained flexural rotations there, without a notable increase of their bending moments beyond the corresponding moment resistance, M_{Rc} (this is, in fact, how a flexural "plastic hinge" is defined). The way to prevent soft-storeys in frames is by forcing flexural plastic hinges out of the columns and into the beams, so that a beam-sway mechanism develops (Fig. 2.9(b) and (c)). To this end, within any vertical plane in which a soft-storey is to be prevented, the two columns framing into a beam-column joint from above and below are dimensioned to be jointly stronger by 30% than the (one, two or more) beams connected to the same joint from any side (Fig. 5.2):

$$\sum M_{Rd,c} \geq 1.3 \sum M_{Rd,b} \tag{5.31}$$

where:

– $M_{\mathrm{Rd,c}}$: design value of column moment resistance at the face of the joint, in the vertical plane of bending in which a soft-storey is to be prevented (i.e., with moment vector at right angles to that plane), with the sum referring to the column sections above and below the connection;

– $M_{\mathrm{Rd,b}}$: design value of beam moment resistance at the face of the joint, with the sum extending to all beam ends connected to the joint; beams which are not in the vertical plane in which a soft-storey is to be prevented but at an angle α to it, enter Eq. (5.31) with their $M_{\mathrm{Rd,b}}$-value multiplied by cosα.

Normally Eq. (5.31) is checked within two orthogonal vertical planes. For the usual columns with section composed of rectangular parts (including L- or T-sections, etc), these vertical planes are chosen parallel to the column sides, facilitating the calculation of $M_{\mathrm{Rd,c}}$. In the most common case where the beams connected to the column at the joint are parallel to the column sides, they have $\alpha = 0$ in one of the two horizontal directions in which Eq. (5.31) is checked and $\alpha = 90^{\circ}$ in the orthogonal one.

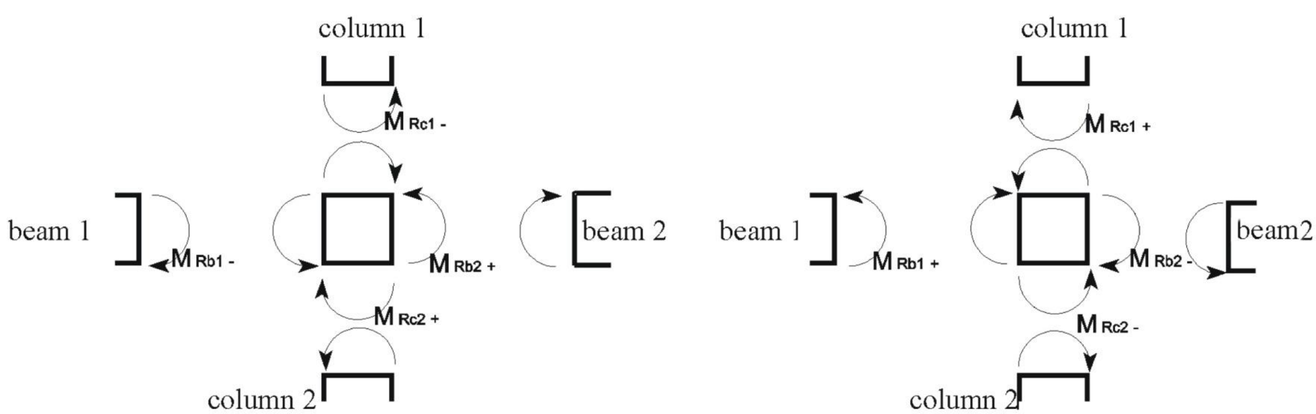

Fig. 5.2 Sense of action of column and beam moment resistances around a joint in the capacity design check of the column

The check of Eq.(5.31) takes place twice in each of the two vertical planes considered: first with both column moments, $M_{\mathrm{Rd,c}}$, acting clockwise on the joint in the direction about the normal to that plane and then counterclockwise (Fig. 5.2). Beam moment resistances, $M_{\mathrm{Rd,b}}$, are taken to act on the joint in the opposite sense with respect to those of the columns. The values of $M_{\mathrm{Rd,b}}$ may be calculated from Eqs.

(5.30); the beams connected to one side of the joint with respect to the normal of the vertical plane are hogging and their $M_{Rd,b}$-value is computed from Eq.(5.30a); those connected to the opposite are sagging and Eq. (5.30b) is used for them.

For the application of Eq. (5.31), see Examples 5.1 and 5.2 at the end of the Chapter.

The calculation of column moment resistance, $M_{Rd,c}$, for known column reinforcement and given axial force, N, is addressed in Section 5.4.3. The value of N to be used in this calculation should be the most safe-sided for the fulfillment of Eq. (5.31), notably the minimum compressive or maximum tensile force in the range of values derived from the analysis for the "seismic design situation". In general, this extreme value of N is obtained by subtracting from the axial load due to the quasi-permanent gravity loads, G+ψ_2Q, the value of N due to the design seismic action. However, the application of this general rule should be physically consistent with the sense of action (clockwise or counterclockwise) of $\sum M_{Rd,b}$ in Eq. (5.31), and hence its value. Section 5.8.6 deals in more detail with the value of N in capacity design calculations.

Eq. (5.31) is called "capacity design" of columns in flexure, because the demand for the required (design value of) column moment resistance, $M_{Rd,c}$, is not an action effect from the analysis, but the (design values of the) "capacities", $M_{Rd,b}$, at the locations where plastic hinges are allowed (even promoted), in this case at the beam ends. This design rule employs only equilibrium (of moments) and is independent of the magnitude of the design seismic action; so, it achieves its goal for any earthquake, no matter how strong it is. Note at this point that, although the equilibrium of moments is meant to refer to the "centre" of the joint, where the beam and column axes theoretically intersect, the transfer of $M_{Rd,c}$, $M_{Rd,b}$, from the faces to the centre of the joint is omitted in Eq. (5.31) for convenience. This is safe-sided, provided that $1.3h_b/h_c > H_{cl}/L_{cl}$, with h_b, h_c denoting the cross-sectional depths of the beam and columns, respectively, and L_{cl}, H_{cl} the average clear span of the beams on either side of the joint, or the average clear storey height above and below it, respectively, all in the vertical plane in which Eq. (5.31) is checked (see Fardis (2009)).

Eurocode 8 exempts the following cases of columns from the enforcement of Eq. (5.31):

1. In the horizontal direction(s) where walls take at least 50% of the elastic base shear, i.e., the system qualifies as a wall system or a wall-equivalent dual one; the reason is that a wall (be it with the minimum length-to-thickness ratio of 4 per Eurocode 2) is very unlikely to yield in counterflexure at both the top and bottom sections in a storey (Fig. 2.9(d), (f)); so, if there are plenty of them in a horizontal direction, they prevent soft-storey mechanisms.
2. Around the joints of the top floor, no matter the structural system; one reason is that it makes little difference for the plastic mechanism whether the plastic hinge forms at the top of a top storey column, or at the ends of the beams connected to it; another reason is the good ductility of the top storey columns thanks to their low axial load; note also that it is hard to meet Eq. (5.31) with only one column section at the left-hand-side.
3. In two-storey buildings of any structural system, provided that none of the ground storey columns has axial load ratio, $\nu_d = N_d/(A_c f_{cd})$, above 0.3, for the maximum column axial load, N_d, in any combination of the seismic design situation (design seismic action plus concurrent gravity loads, G+ψ_2Q); such columns have sufficient ductility to withstand concentration of the entire deformation demand in one storey instead of two, with consequent doubling of the ductility demand in ground storey columns.
4. In one-out-of-four columns per plane frame with columns of similar size, in a horizontal direction not exempted from Eq. (5.31) on the basis of 1 above; it is worth profiting from this exemption at interior joints rather than at exterior ones, where a beam frames from one side only and Eq. (5.31) is easily met.

Eurocode 8 presumes that a plastic hinge will form at any column end where Eq. (5.31) is not checked thanks to the exemptions above and requires to detail these plastic hinge regions so that they can develop significant inelastic deformations after plastic hinging. In fact, the same detailing rules apply in these regions as those applied at the base of the column – where a plastic hinge is allowed anyway.

5.4.2 Dimensioning of column vertical reinforcement for action effects from the analysis

The base section at the bottom storey of a column (the connection to the foundation), as well as all columns exempted from the capacity-design rule, Eq.(5.31), are dimensioned for the ULS in biaxial flexure with axial force, using triplets M_y-M_z-N from the analyses for the combination of the design seismic action with the quasi-permanent gravity loads, G+ψ_2Q. This is combination (b) in Section 5.3.1 for the seismic design situation; combination (a) is normally not critical for the dimensioning of primary columns and may be ignored.

The column sections right above and right below a beam-column joint are served by the same vertical bars. Besides, as pointed out in Section 5.2.3.1, it is good practice to avoid changing the column section from one storey to the next. So, these two sections are dimensioned as a single one, for all M_y-M_z-N triplets that the analysis gives for them in the seismic design situation, each triplet being the single triplet due to the quasi-permanent gravity loads G+ψ_2Q plus a seismic one (see Section 5.8 for the number and composition of the seismic triplets, depending on the analysis method and the use of Eq. (3.99) or (3.100)). Most critical of all triplets is the one giving the largest amount of reinforcement in one of the two sections; however, it is not easy to screen out non-critical ones. Generally, for the usual range of values of the dimensionless axial load, $N_d/A_c f_{cd}$, most critical among triplets with similar biaxial moments is the one having the lowest axial compression.

There are several iterative algorithms for the ULS verification of sections with any shape and amount and layout of reinforcement for a combination M_y-M_z-N. They employ section analysis and the σ-ε laws used for design (elastic-perfectly plastic for steel, normally parabolic-rectangular for concrete) to find the strain distribution which satisfies equilibrium. It is checked then whether, in that strain distribution, the conventional ultimate strain of concrete, $\varepsilon_{cu,2}$, is exceeded at the corners of the section. However, there is no general algorithm for the direct calculation of the section reinforcement for a given M_y-M_z-N triplet. The traditional manual approach with design charts is not practical for the large number of columns of a real building; it is also very restrictive for the bar layout and the steel grade. A practical, yet approximate, step-by-step computational procedure is proposed in the following paragraphs for the direct dimensioning of symmetrically reinforced rectangular sections under a set of M_y-M_z-N triplets.

1. The mechanical reinforcement ratio, $\omega_{1d} = A_{s1}/(bd)\cdot(f_{yd}/f_{cd})$, of the steel bars placed along each one of two opposite sides of the section of length b, is estimated under uniaxial moment, M, with axial force, N, neglecting the orthogonal moment component; d·is the effective depth at right angles to the vector of M (cf. Eqs. (5.13)); each layer of bars with cross-sectional area A_{s1} is at centroidal distance d_1 from the nearest side of the section of length b. M, N and d_1 are normalised as:

$$\mu_d = M/\left(bd^2 f_{cd}\right), \quad \nu_d = N/\left(bdf_{cd}\right), \quad \delta_1 = d_1/d \tag{5.32}$$

Section analysis is used, with the material σ-ε laws and criteria adopted in Eurocode 2 for the ULS design:

   - elastic-perfectly plastic steel, with a yield stress of f_{yd} and unlimited strain capacity;
   - parabolic-rectangular σ-ε law for concrete, with design strength f_{cd} at strain ε_{c2}, with ultimate strain ε_{cu2} (for $f_{ck} \leq 50$ MPa, $\varepsilon_{c2} = 0.002$, $\varepsilon_{cu2} = 0.0035$).

Depending on the value of the dimensionless axial load, ν_d, there are three possible cases:

(i) The most usual case is to have the tension and the compression reinforcement past yielding:

$$\delta_1 \frac{\varepsilon_{cu2} - \varepsilon_{c2}/3}{\varepsilon_{cu2} - \varepsilon_{yd}} \equiv \nu_2 \leq \nu_d < \nu_1 \equiv \frac{\varepsilon_{cu2} - \varepsilon_{c2}/3}{\varepsilon_{cu2} + \varepsilon_{yd}} \tag{5.33a}$$

where $\varepsilon_{yd} = f_{yd}/E_s$. Then, the neutral axis depth, x, normalised to d as ξ=x/d, is:

$$\xi = \frac{\nu_d}{1 - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}} \tag{5.34a}$$

to be substituted in terms of ν_d in the following equation, to be solved directly for ω_{1d}:

$$(1-\delta_1)\omega_{1d} = \mu_d - \xi\left[\frac{1-\xi}{2} - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2} - \xi + \frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right] \tag{5.35a}$$

(ii) The second commonest case is to have the tension bars yielding, but the compression ones elastic; this happens if ν_d is less than ν_2, as given at the left-hand-side of Eq. (5.33a):

$$\nu_d \le \delta_1 \frac{\varepsilon_{cu2} - \varepsilon_{c2}/3}{\varepsilon_{cu2} - \varepsilon_{yd}} \equiv \nu_2 \tag{5.33b}$$

Then ξ and ω_1 are related to the dimensionless axial force and moment through:

$$\left[1 - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\right]\xi^2 - \left[\nu_d + \omega_{1d}\left(1 - \frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)\right]\xi - \omega_{1d}\frac{\varepsilon_{cu2}\delta_1}{\varepsilon_{yd}} = 0 \tag{5.34b}$$

$$\omega_{1d}\frac{(1-\delta_1)}{2}\left(1 + \frac{\xi - \delta_1}{\xi}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right) = \mu_d - \xi\left[\frac{1-\xi}{2} - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2} - \xi + \frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right] \tag{5.35b}$$

By replacing ω_{1d} from Eq.(5.35b) into (5.34b), a strongly nonlinear equation is obtained for ξ, to be solved numerically-iteratively; ω_{1d} is then determined from Eq.(5.35b).

(iii) The most rare (and undesirable) case is to have yielding compression bars and the tension ones elastic; this happens if ν_d exceeds ν_1, given by the right-hand-side of Eq.(5.33a):

$$\nu_1 \equiv \frac{\varepsilon_{cu2} - \varepsilon_{c2}/3}{\varepsilon_{cu2} + \varepsilon_{yd}} \le \nu_d \tag{5.33c}$$

Then ξ and ω_{1d} are related to each other and to ν_d, μ_d, through:

$$\left[1 - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\right]\xi^2 - \left[\nu_d - \omega_{1d}\left(1 + \frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)\right]\xi - \omega_{1d}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}} = 0 \tag{5.34c}$$

$$\omega_{1d}\frac{(1-\delta_1)}{2}\left(1+\frac{1-\xi}{\xi}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)=\mu_d-\xi\left[\frac{1-\xi}{2}-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2}-\xi+\frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right] \tag{5.35c}$$

Substituting for ω_{1d} in Eq. (5.35b) the expression from (5.35c), a highly nonlinear equation results for ξ, to be solved numerically-iteratively; ω_{1d} is then determined from Eq. (5.35c).

2. The procedure in step 1 above is applied first with all M_y-N pairs in the set of M_y-M_z-N combinations, with b the side length parallel to the vector of M_y and dimensions d, d_1 at right angles to it. The most critical pair gives the total area of reinforcement, A_{sy}, along each side parallel to the M_y-vector. This is repeated with all M_z-N pairs and the roles reversed, to find the total area of reinforcement, A_{sz}, along each one of the two other sides – those parallel to the M_z-vector. As the M_y-N pair from which A_{sy} is derived most likely does not belong in the same M_y-M_z-N combination as the pair M_z-N giving A_{sz}, these reinforcement requirements are superimposed on the section and translated into a bar layout meeting the Eurocode 8 detailing rules in Table 5.2 for column vertical bars, with the corner ones counting to both sides (Fig. 5.3).

Table 5.2 EC8 detailing rules for vertical bars in primary columns (in secondary ones: as in DC L)

	DC H	**DC M**	**DC L**
$\rho_{min} = A_{s,min}/A_c$	1%		$0.1N_d/A_cf_{yd}$, 0.2% (1)
$\rho_{max} = A_{s,max}/A_c$	4%		4% (1)
diameter, d_{bL}	≥8mm		
number of bars per side	≥3		≥2
spacing along the perimeter of bars restrained by a tie corner or hook	≤150mm	≤200mm	-
distance along perimeter of unrestrained bar to nearest restrained one	≤150mm		
lap splice length (2)	$l_0 = 1.5[1-0.15(c_d/d_{bL}-1)]a_{tr}(d_{bL}/4)f_{yd}/(2.25f_{ctd})$ (3), (4), (5)		

(1)NDP (Nationally Determined Parameter) per EC2; the value recommended in EC2 is given here.
(2) Anchorage length in tension is reduced by 30% if the bar end extends by $\geq 5d_{bL}$ beyond a bend $\geq 90^o$.
(3) c_d: minimum of: concrete cover of lapped bar and 50% of clear spacing to adjacent lap splice.
(4) $a_{tr} = 1-k(2n_wA_{sw}-A_{s,t,min})/A_s$, with k = 0.1 if the bar is at a corner of a hoop or tie, k = 0.05 otherwise; A_{sw}: cross-sectional area of a column tie; n_w: number of ties in the cover of the lapped bar over the outer third of the length l_0; $A_s = \pi d_{bL}^2/4$ and $A_{s,t,min}$ is specified in EC2 as equal to A_s.
(5) $f_{ctd}=f_{ctk,0.05}/\gamma_c = 0.7f_{ctm}/\gamma_c = 0.21f_{ck}^{2/3}/\gamma_c$: design value of 5%-fractile tensile strength of concrete.

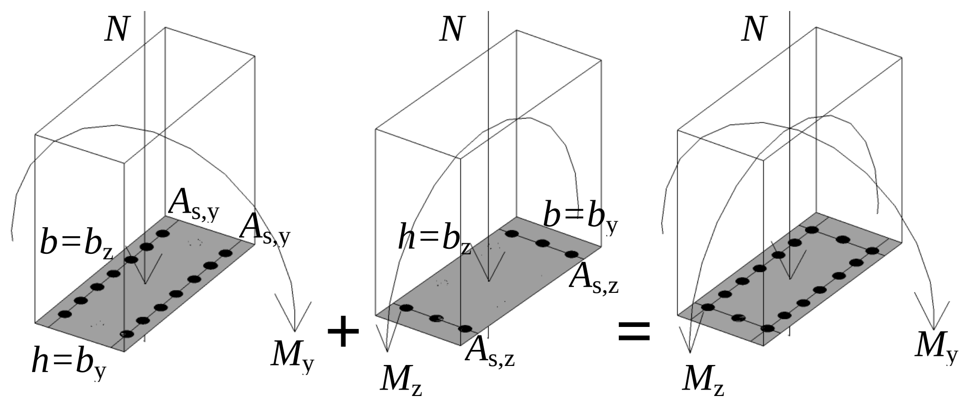

Fig. 5.3 Dimensioning of reinforcement in column section for biaxial moments with axial force

3. If available, an iterative algorithm may be used in the end to verify that the section with the selected layout of reinforcement satisfies the ultimate strain of concrete, ε_{cu2}, under any one of the M_y-M_z-N triplets. If it does not, one bar may be added to each side, till the section meets the verification criteria.

The procedure above can be applied to sections composed of more than one rectangular parts, orthogonal to each other (L, T, etc.). In Step 1 such a section is replaced by an equivalent rectangular, having cross-sectional area the same as the actual one and the same effective depth at right angles to the vector of the uniaxial bending moment considered. The reinforcement areas, A_{sy} and A_{sz}, coming out of this exercise are distributed along the corresponding extreme tension and compression fibres of the section, while meeting the detailing rules in Table 5.2. If Step 3 is carried out, it should be done for the actual cross-sectional shape and bar layout.

"Capacity design" through Eq.(5.31) normally governs over the ULS verification in biaxial bending and axial force using the action effects from the analysis for the seismic design situation. So, if Eq.(5.31) should be met at a joint, it makes sense to use it from the outset as the basis for dimensioning the column vertical reinforcement, instead of the analysis results. To this end, in Step 1 above, each uniaxial moment from the analysis is replaced by half the maximum value of the right hand side of Eq. (5.31) for clockwise or counterclockwise beam moments on the joint in the vertical plane of bending of interest, i.e., at right angles to the pair of column sides where the so-computed reinforcement is arranged. Step 1 is repeated with half the maximum value of the right hand side of Eq. (5.31) in a vertical plane of bending orthogonal to the first one, to determine the reinforcement along the other pair of sides of the section. As pointed out in Section 5.4.1, the value of the column axial force to be used is the minimum compressive or the maximum tensile one over the combinations in the seismic design situation which produce N-values consistent with the sense of action of the $\sum M_{\mathrm{Rd,b}}$ on the joint – clockwise or counterclockwise; it is normally different for the two directions of bending.

To avoid the onerous ULS design/verification of sections in biaxial bending with axial force, Eurocode 8 allows to replace it with separate uniaxial verifications, but under pairs (M_y/0.7)-N and (M_z/0.7)-N, i.e., with the moments increased by 43%. Unless $M_y \approx M_z$, this simplification is too conservative (especially if seismic action effects are obtained from Eq. (3.100) in Section 3.1.7). So, if the computational capability of a truly biaxial verification is available, there is no need to resort to this uniaxial approximation. Note also that, normally, the detailing rules in Table 5.1 and/or rounding up of the reinforcement required to meet the analysis results for the beams produce a value of $\sum M_{\mathrm{Rd,b}}$ in Eq. (5.31) which exceeds by more than 10% the beam capacity strictly necessary according to the analysis. So, if "capacity design", Eq. (5.31), is met for the most safe-sided (minimum compressive or maximum tensile) column axial force, the simplified uniaxial ULS verification per Eurocode 8 is also met automatically; there is no reason to do both.

If "capacity design", through Eq. (5.31), applies at the top section of a column in the bottom storey, it may give more vertical reinforcement there than at the base section of the same column, which is not subject to "capacity design" and is dimensioned only for the ULS in bending with axial force under the action effects from the analysis for the seismic design situation. If so, it is good practice to place at the base section the same reinforcement as at the top. Indeed, this is required in Eurocode 8 for DC H buildings. This ensures that, after plastic hinging at the base of the column, the moment at the top will not increase to much larger values than at the bottom.

5.4.3 Calculation of the column moment resistance for given reinforcement and axial load

This section is about the design values of the ULS moment resistance of a column, $M_{Rd,c}$, to be used in Eq. (5.31) and in the other "capacity design" calculations of Section 5.5.

In rectangular sections $M_{Rd,c}$ refers to centroidal axes parallel to the sides. The assumptions made in the previous Section apply. The same for the notation introduced there, including Eqs. (5.32). Additional notation is introduced here for the reinforcement (the same as the one used in Section 5.2.3.4 for Eq. (5.6)):

- The mechanical reinforcement ratio $\omega_{1d} = A_{s1}/(bd)\cdot(f_{yd}/f_{cd})$ refers to the tension flange only; for generality, the compression flange may have different reinforcement, A_{s2}, with mechanical ratio $\omega_{2d}=A_{s2}/(bd)\cdot(f_{yd}/f_{cd})$; its centroidal distance from the extreme compression fibres is still d_1.
- There are intermediate bars between the tension and compression reinforcement, uniformly distributed along the length (h-2d_1) of the cross-sectional depth h; their total cross-sectional area, A_{sv}, is taken smeared along that length, with mechanical reinforcement ratio:

$$\omega_{vd} = A_{sv}/(bd)\cdot(f_{yd}/f_{cd}) \tag{5.36}$$

  Only one-half of each corner bar is included in A_{s1} or A_{s2}; the other half counts as part of A_{sv}.

There are three cases (i)-(iii), analogous to those in Section 5.4.2, but generalised to accommodate the more general reinforcement layout:

(i) Tension and compression reinforcement yield, if the normalised axial load is in the range:

$$\omega_{2d}-\omega_{1d}+\frac{\omega_{vd}}{1-\delta_1}\left(\delta_1\frac{\varepsilon_{cu2}+\varepsilon_{yd}}{\varepsilon_{cu2}-\varepsilon_{yd}}-1\right)+\delta_1\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}-\varepsilon_{yd}}\equiv\nu_2\le\nu_d<$$
$$\nu_1\equiv\omega_{2d}-\omega_{1d}+\frac{\omega_{vd}}{1-\delta_1}\left(\frac{\varepsilon_{cu2}-\varepsilon_{yd}}{\varepsilon_{cu2}+\varepsilon_{yd}}-\delta_1\right)+\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}+\varepsilon_{yd}} \tag{5.37a}$$

The design value of moment resistance is:

$$\frac{M_{Rd,c}}{bd^2f_{cd}}=$$
$$\xi\left[\frac{1-\xi}{2}-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2}-\xi+\frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right]+\frac{(1-\delta_1)(\omega_{1d}+\omega_{2d})}{2}+\frac{\omega_{vd}}{1-\delta_1}\left[(\xi-\delta_1)(1-\xi)-\frac{1}{3}\left(\xi\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)^2\right] \tag{5.38a}$$

with the normalised neutral axis depth computed from:

$$\xi=\frac{(1-\delta_1)(\nu_d+\omega_{1d}-\omega_{2d})+(1+\delta_1)\omega_{vd}}{(1-\delta_1)\left(1-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\right)+2\omega_{vd}} \tag{5.39a}$$

(ii) The tension bars yield, the compression ones are elastic, if ν_d is less than ν_2 from Eq. (5.37a):

$$\nu_d\le\omega_{2d}-\omega_{1d}+\frac{\omega_{vd}}{1-\delta_1}\left(\delta_1\frac{\varepsilon_{cu2}+\varepsilon_{yd}}{\varepsilon_{cu2}-\varepsilon_{yd}}-1\right)+\delta_1\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}-\varepsilon_{yd}}\equiv\nu_2 \tag{5.37b}$$

Then the design value of the moment resistance is:

$$\frac{M_{Rd,c}}{bd^2f_{cd}}=\xi\left[\frac{1-\xi}{2}-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2}-\xi+\frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right]+\frac{(1-\delta_1)}{2}\left(\omega_{1d}+\omega_{2d}\frac{\xi-\delta_1}{\xi}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)+$$
$$\frac{\omega_{vd}}{4(1-\delta_1)}\left[\xi\left(1+\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)-\delta_1\right]\left[1+\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\left(\frac{\xi-\delta_1}{\xi}\right)\right]\left[1-\frac{\delta_1}{3}-\frac{2}{3}\xi\left(1+\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)\right] \tag{5.38b}$$

with ξ the positive root of the equation:

$$\left[1-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}+\frac{\omega_{vd}}{2(1-\delta_1)}\frac{(\varepsilon_{cu2}+\varepsilon_{yd})^2}{\varepsilon_{cu2}\varepsilon_{yd}}\right]\xi^2-\left[\nu_d+\omega_{1d}-\omega_{2d}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}+\frac{\omega_{vd}}{1-\delta_1}\left(1+\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\delta_1\right)\right]\xi-\left[\omega_{2d}-\frac{\omega_{vd}\delta_1}{2(1-\delta_1)}\right]\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\delta_1=0$$

(5.39b)

(iii)The third case is to have the compression bars yielding and the tension ones elastic; this happens if ν_d exceeds ν_1 at the right-hand-side of Eq. (5.37a):

$$\nu_1 \equiv \omega_{2d} - \omega_{1d} + \frac{\omega_{vd}}{1-\delta_1}\left(\frac{\varepsilon_{cu2} - \varepsilon_{yd}}{\varepsilon_{cu2} + \varepsilon_{yd}} - \delta_1\right) + \frac{\varepsilon_{cu2} - \varepsilon_{c2}/3}{\varepsilon_{cu2} + \varepsilon_{yd}} \leq \nu_d \tag{5.37c}$$

Then:

$$\frac{M_{Rd,c}}{bd^2 f_{cd}} = \xi\left[\frac{1-\xi}{2} - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2} - \xi + \frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right] + \frac{(1-\delta_1)}{2}\left(\omega_{1d}\frac{1-\xi}{\xi}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}} + \omega_{2d}\right) + \frac{\omega_{vd}}{4(1-\delta_1)}\left[1-\xi\left(1-\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)\right]\left[1+\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\left(\frac{1-\xi}{\xi}\right)\right]\left[\frac{1}{3} - \delta_1 + \frac{2}{3}\xi\left(1-\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)\right] \tag{5.38c}$$

with ξ the positive root of the equation:

$$\left[1-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}} - \frac{\omega_{vd}}{2(1-\delta_1)}\frac{(\varepsilon_{cu2}-\varepsilon_{yd})^2}{\varepsilon_{cu2}\varepsilon_{yd}}\right]\xi^2 + \left[\omega_{2d} + \omega_{1d}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}} - \nu_d + \frac{\omega_{vd}}{1-\delta_1}\left(\frac{\varepsilon_{cu2}}{\varepsilon_{yd}} - \delta_1\right)\right]\xi - \left[\omega_{1d} + \frac{\omega_{vd}}{2(1-\delta_1)}\right]\frac{\varepsilon_{cu2}}{\varepsilon_{yd}} = 0 \tag{5.39c}$$

Note that ν_1 at the right-hand-side of Eq. (5.37a) and the left-hand-side of (5.37c) is the dimensionless "balance load". For that value of ν_d, Eqs. (5.38a) and (5.38c) give the maximum possible value of $M_{Rd,c}$ that the section can develop. As we will see in Section 5.5, this moment resistance at "balance load" is of interest for the capacity design shears of beams and columns.

For an application of the above to a rectangular column section, see Example 5.3 at the end of the Chapter.

If the cross-section consists of more than one rectangular parts in two orthogonal directions (L-, T- or C-sections, etc), it is convenient to compute the moment resistance, $M_{Rd,c}$, with respect to centroidal axes parallel to the two orthogonal directions of the sides, since the beams connected to the column are parallel or normal to the sides of the rectangular parts. If an iterative algorithm of the type mentioned in Section 5.4.2 (e.g., at Step 3) is available for the ULS verification of sections with any shape and reinforcement layout under any M_y-M_z-N combination, it can be used for the calculation of $M_{Rd,c}$, by setting the strain all-along the extreme fibres of the compression flange equal to ε_{cu2} and searching for a neutral axis depth that equilibrates the axial load N. If such an algorithm is not available, $M_{Rd,c}$ may be estimated considering the section as rectangular, with width *b* being that of the compression flange. This is acceptable, if the width of the compression zone is constant between the neutral axis and the extreme compression fibres (i.e., the compression zone lies within a single one of the rectangular parts of the section).

5.5 Detailed design of beams and columns in shear

5.5.1 Capacity design shears in beams or columns

The monotonic or cyclic behaviour of concrete members in flexure is fairly ductile: after flexural yielding they can sustain significant inelastic deformations (i.e., rotations), with little loss of moment resistance. Their inherent flexural ductility can easily be improved further, by using dense, closed stirrups, or similar types of transverse reinforcement, which laterally confine the concrete and prevent the longitudinal bars from buckling. By contrast, concrete members are inherently brittle in shear, whether monotonic or cyclic: if they reach their shear resistance before yielding in flexure, they suffer a drastic and often sudden drop in resistance right after its peak. For this reason, shear failure of members before flexural yielding should be prevented by all means. Eurocode 8 accomplishes this goal by enforcing "capacity design" of all members in shear. Capacity design of beams or columns in shear is a more straightforward and effective application of the "capacity design" concept than that in Section 5.4.1 for columns in flexure. The present Section will show that, once plastic hinges are presumed to form at the relevant member ends, equilibrium of moments suffices to establish the maximum possible shear force in the member which is physically permitted by the "capacities", M_{Rd}, of the plastic hinges. By designing against this "capacity design" shear, instead of the shear force from the analysis for the seismic design situation, we preclude shear failure of the beams or the column not only before flexural yielding, but also afterwards; indeed indefinitely, for any level of earthquake.

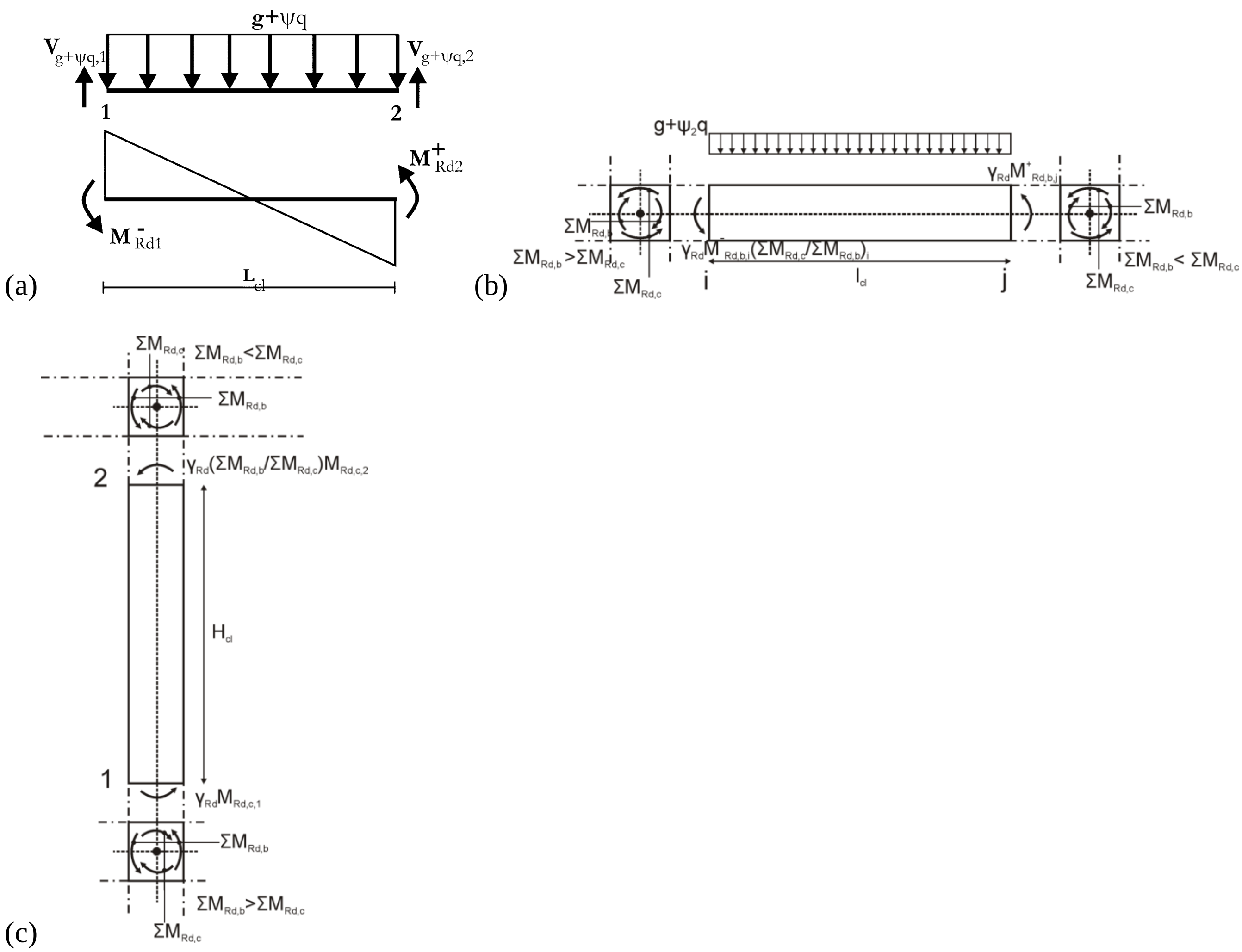

Fig. 5.4 End moments considered for the capacity design in shear of: (a) an isolated beam; (b) a beam connected to columns with plastic hinging around the joint at the beam or the column ends; (c) a column connected to beams with plastic hinging around the joint at the column or the beam end

With the internal forces at beam ends in Fig. 5.4(a) taken as positive, equilibrium of moments about one end gives the shear force at the other:

$$V_1 = V_{g+\psi q,1} + \frac{M_1 + M_2}{l_{cl}} \tag{5.40a}$$

$$V_2 = V_{g+\psi q,2} - \frac{M_1 + M_2}{l_{cl}} \tag{5.40b}$$

where $V_{g+\psi q,1}$ and $V_{g+\psi q,2}$ are the moments of the transverse load acting between the two ends with respect to end 2 or 1, respectively, divided by the clear span of the beam, l_{cl} (i.e., the reactions to this load when the beam is simply supported). The maximum value of V_1 develops when M_1 and M_2 in the sum M_1+M_2 both attain their maximum possible positive values; when M_1 and M_2 attain their algebraically minimum negative values, V_2 reaches its minimum possible value.

If the beam is connected at both ends to stronger columns, which satisfy Eq.(5.31) without the 1.3 factor, the maximum possible positive values of M_1 and M_2 are the corresponding moment resistances, taken for convenience as equal to their design values, M_{Rd}, times an overstrength factor, $\gamma_{Rd} \geq 1.0$. Accordingly, in Eq.(5.40a) we take:

$$M_1 = \gamma_{Rd} M^-_{Rd,b1}\,,\; M_2 = \gamma_{Rd} M^+_{Rd,b2} \tag{5.41a}$$

and in (5.40b):

$$M_1 = -\gamma_{Rd} M^+_{Rd,b1}\,,\; M_2 = -\gamma_{Rd} M^-_{Rd,b2} \tag{5.41b}$$

Using in Eqs. (5.40a), (5.40b) the values of M_1, M_2 from Eqs. (5.41a), (5.41b), respectively, we obtain the maximum possible ("capacity design") shears at ends 1 and 2, respectively, of a beam which is weaker than the columns it is connected to.

Beams connected to weaker columns (i.e. not satisfying Eq.(5.31) without the 1.3 factor) will most likely not develop plastic hinges at their ends before the columns. Assuming that at end i (: 1, 2) of the beam the beam moment is negative and that the sum of the beam design moment resistances around the joint exceeds that of the columns in the sense associated with negative beam moment at that end, $M^-_{Rd,bi}$ in Eq.(5.41a) should be replaced by the beam moment at column hinging above and below the joint at end i. Assuming that the moment input from the yielding columns to the elastic beams is shared by the two beams connected to the joint in proportion to their own moment resistance, the beam moment at end i after the columns yield can be taken equal to $M^-_{Rd,bi}[\Sigma M_{Rd,c}/\Sigma M_{Rd,b}]_i < M^-_{Rd,bi}$, with $\Sigma M_{Rd,b}$ referring to the sections of the beam across the joint at end i and $\Sigma M_{Rd,c}$ being the sum of moment resistances of the column above and below the joint, for bending in the vertical plane of the beam (for columns with sides at an angle ψ to that plane, the $M_{\text{Rd,c}}$-values with respect to centroidal axes parallel to the sides in $\Sigma M_{\text{Rd,c}}$ multiplied by sinψ). Similarly for the positive sense of bending of the beam at end i. So, a rational generalisation of Eqs. (5.40), (5.41) for the design value of the maximum shear at a section x in the part of the beam closer to end i (with j denoting the other end of the beam) is (see Fig. 5.4(b)):

$$\max V_{\text{i,d}}(x) = \frac{\gamma_{\text{Rd}}\left[M_{\text{Rd,bi}}^{-}\min\left(1;\frac{\sum M_{\text{Rd,c}}}{\sum M_{\text{Rd,b}}}\right)_i + M_{\text{Rd,bj}}^{+}\min\left(1;\frac{\sum M_{\text{Rd,c}}}{\sum M_{\text{Rd,b}}}\right)_j\right]}{l_{cl}} + V_{\text{g+}\psi\text{q,o}}(x) \qquad (5.42a)$$

where:

- all moments and shears enter as positive;
- if $M_{\text{Rd,bi}}$ acts on the joint in the clockwise direction, so does $(\Sigma M_{\text{Rd,b}})_{\text{i}}$, but $(\Sigma M_{\text{Rd,c}})_{\text{i}}$ acts on the joint in the counterclockwise direction;
- $V_{\text{g+}\psi\text{q,o}}(x)$ is the shear force at cross-section x due to the quasi-permanent gravity loads, with the beam simply supported (index: o); if the gravity load is not uniformly distributed along the beam, it may be conveniently computed from the results of the analysis of the full structure for these gravity loads alone: then $V_{\text{g+}\psi\text{q,o}}(x)$ may be taken as the shear force $V_{\text{g+}\psi\text{q}}(x)$ at cross-section x in the full structure, corrected for the shear force $(M_{\text{g+}\psi\text{q,1}}-M_{\text{g+}\psi\text{q,2}})/l_{\text{cl}}$ due to the bending moments $M_{\text{g+}\psi\text{q,1}}$ and $M_{\text{g+}\psi\text{q,2}}$ at the beam end sections 1 and 2 in the full structure;
- $\gamma_{\text{Rd}} = 1.2$ for beams of DC H and $\gamma_{\text{Rd}} = 1$ for DC M.

With $V_{\text{g+}\psi\text{q,o}}(x)$ taken positive at sections x in the part of the beam closer to end i, the minimum shear in that section is:

$$\min V_{\text{i,d}}(x) = -\frac{\gamma_{\text{Rd}}\left[M_{\text{Rd,bi}}^{+}\min\left(1;\frac{\sum M_{\text{Rd,c}}}{\sum M_{\text{Rd,b}}}\right)_i + M_{\text{Rd,bj}}^{-}\min\left(1;\frac{\sum M_{\text{Rd,c}}}{\sum M_{\text{Rd,b}}}\right)_j\right]}{l_{cl}} + V_{\text{g+}\psi\text{q,o}}(x) \qquad (5.42b)$$

The moments and shears at the right-hand-side of Eq.(5.42b) being positive, its outcome may be positive or negative. If it is negative, the shear at section *x* may change sense of action during the seismic response (from down- to upwards, or vice versa). We will see in Section 5.5.3 that, in the dimensioning of the transverse reinforcement of beams in DC H buildings, Eurocode 8 uses the ratio:

$$\zeta_i = \frac{\min V_{\mathrm{i,d}}(x_i)}{\max V_{\mathrm{i,d}}(x_i)} \tag{5.43}$$

as a measure of the reversal of shear at end *i* (similarly at end *j*).

Eq. (5.42a) gives safe-sided results even in a situation when the positive plastic hinge develops not at the very end *j* of the beam but at a point nearby, where the available moment resistance in positive bending is exhausted for the first time by the demand moment under the combination of quasi-permanent gravity loads and the seismic action that causes beam or column yielding – whichever occurs first – around the joint at end *i*.

Owing to the transverse gravity loads on the beam and the – in general – different longitudinal reinforcement of its two ends, the value of the capacity design shear from Eq. (5.42a) varies along the beam. The absolutely maximum value of shear at a certain cross-section *x*, $\max V_{\mathrm{i,d}}(x)$ from Eq. (5.42a), is in the same direction (down- or upwards) as the shear at *x* due to the quasi-permanent gravity loads in the simply supported beam, $V_{\mathrm{g+\psi q,o}}(x)$; the absolutely minimum, $\min V_{\mathrm{i,d}}(x)$ from Eq. (5.42b), is in the opposite direction; $\max V_{\mathrm{i,d}}(x)$ and $\min V_{\mathrm{i,d}}(x)$ take place when the beam exhausts at end *i* its moment resistance in hogging or sagging bending, respectively.

If a beam is not connected at end *i* to a beam-column joint, but is ("indirectly") supported on another beam or girder, it is not expected to develop sizeable seismic moments there, let alone its moment resistance under seismic loading. The capacity design shear along the beam may then be estimated by replacing $M^{+}_{\mathrm{Rd,bi}}$ or $M^{-}_{\mathrm{Rd,bi}}$ in Eqs. (5.42) with the moment at end *i* under the quasi-permanent gravity loads alone, $M_{\mathrm{g+\psi q,i}}$ (taken positive if it induces tension to the same flange of the beam as the moment resistance it replaces, or negative otherwise).

The picture is much simpler in columns, as there is no transverse load between the two ends; so the capacity design shear is constant all along the column. The design shear force parallel to a set of sides of a column, having clear height H_{cl} within the plane of bending (in general, equal to the distance of the top of the beam or slab at the base of the column to the soffit of the beam at the top), symmetric cross-section and reinforcement (so that $M_{Rd,c}$ is the same clockwise or counterclockwise) and ends indexed by 1 and 2, is given by a parallel to Eq.(5.42a):

$$V_{CD,c} = \frac{\gamma_{Rd}\left[M_{Rd,c1}\min\left(1;\frac{\sum M_{Rd,b}}{\sum M_{Rd,c}}\right)_1 + M_{Rd,c2}\min\left(1;\frac{\sum M_{Rd,b}}{\sum M_{Rd,c}}\right)_2\right]}{H_{cl}} \tag{5.44}$$

$M_{Rd,c1}$ and $M_{Rd,c2}$ are moment resistances with respect to centroidal axes at right angle to the shear force being computed. The possibility of having plastic hinges at the end(s) of the column itself or in the beams connected to it is taken into account (Fig. 5.4(c)); $\Sigma M_{Rd,c}$ refers to the sections of the column above and below the joint and $\Sigma M_{Rd,b}$ to the beam sections on opposite sides of it (for a beam at an angle α to the column shear force being calculated, $M_{Rd,b}$ enters $\Sigma M_{Rd,b}$ multiplied by $\cos\alpha$); the sense of action of $\Sigma M_{Rd,c}$ on the joint is the same as that of $M_{Rd,ci}$, while that of $\Sigma M_{Rd,b}$ is opposite.

Eurocode 8 specifies $\gamma_{Rd} = 1.3$ for columns in buildings of DC H and $\gamma_{Rd} = 1.1$ for those of DC M.

For the largest absolute value of the beam capacity design shear from Eq. (5.42a) and the algebraically minimum value of ζ in Eq. (5.43), the values of $\Sigma M_{Rd,c}$ to be used in Eqs. (5.42) should be the maximum ones within the range of fluctuation of the column axial load from the analysis for all combinations of the design seismic action with the quasi-permanent gravity loads. The maximum moment resistance, $M_{Rd,c}$, is normally obtained from the maximum compressive force in that range of N, i.e., the value of N due to the quasi-permanent gravity loads, G+ψ_2Q, plus the value due to the design seismic action. However, if that sum exceeds the "balanced load" $v_1 bd f_{cd}$ (see right-hand-side of Eq. (5.37a) or left-hand-side of (5.37c) in Section 5.4.3 for v_1) $M_{\mathrm{Rd,c}}$ is taken equal to the moment resistance at "balance load" mentioned after Eq. (5.39c).

Concerning the capacity design shear of columns, $V_{\mathrm{CD,c}}$ from Eq. (5.44), as we will see in Section 5.5.4, the shear capacity of a column per Eurocode 2 increases with increasing axial compression. To find which one is the most critical shear verification of the column, we may have to consider more than one possible axial force values for $M_{\mathrm{Rd,ci}}$ (i = 1, 2) in Eq. (5.44), namely:

1. The minimum compression, which normally minimises the demand, $V_{\mathrm{CD,c}}$, and the capacity, $V_{\mathrm{Rd,c}}$.
2. The maximum compression, which maximises $V_{\mathrm{Rd,c}}$ and most often $V_{\mathrm{CD,c}}$ as well (except in case 3 below).
3. The "balanced load" mentioned in the previous paragraph, if it is less than the maximum compression in 2 above; this "balanced load" maximises $V_{\mathrm{CD,c}}$ and gives an intermediate value of $V_{\mathrm{Rd,c}}$.

More detailed guidance concerning the extreme values of N due to the design seismic action is given in Section 5.8.6.

For an application of this subsection to a beam, see Example 5.4 at the end of the Chapter. For sample applications of the sub-section and the rest of Section 5.5 to the beams and columns of the 7-storey example building, see Sections 7.6.2.2 and 7.6.2.3.

5.5.2 Dimensioning of beams for the ULS in shear

Eurocode 2 uses for the ULS resistance in shear the variable strut inclination truss model: a model with angle of inclination, θ, of the compression stress field in the web with respect to the member axis which varies in the range:

$$0.4 \le \tan\theta \le 1 \; (22^{\mathrm{o}} \le \theta \le 45^{\mathrm{o}}) \tag{5.45}$$

According to this model:

1. Transverse reinforcement with design value of yield stress f_{ywd} and geometric ratio $\rho_w = A_{sh}/b_w s_h$ (where A_{sh} is the total area of transverse reinforcement with spacing s_h along the beam) contributes a shear resistance equal to:

$$V_{Rd,s} = \rho_w b_w z f_{ywd} \cot\theta \tag{5.46}$$

2. The shear resistance cannot exceed the following limit value, without failure of the web in diagonal compression:

$$V_{Rd,\max} = 0.3 b_w z \left(1 - \frac{f_{ck}(MPa)}{250}\right) f_{cd} \sin 2\theta \tag{5.47}$$

The design shear force at section x along the beam, $V_{Ed}(x)$, is the maximum of the two values:

– from capacity design, Eq. (5.42a);

– from the analysis for the gravity loads in the "persistent and transient design situation".

The general procedure for dimensioning in shear a section x of a beam is the following:

1. $V_{Ed}(x)$ is set equal to $V_{Rd,max}$ and Eq. (5.47) is inverted for a value of θ.
2. In the very unlikely case that $V_{Rd,max}$ is less than $V_{Ed}(x)$ even for $\theta = 45^o$, the width of the web is increased so that $\theta \leq 45^o$.
3. In the very usual case when the condition $V_{Ed}(x) = V_{Rd,max}$ gives a θ-value below the lower limit in Eq. (5.45), θ is set equal to that limit.
4. The shear reinforcement is dimensioned by setting: $V_{Ed}(x) = V_{Rd,s}$ for the final value of θ.
5. Dimensioning of the shear reinforcement starts at a section at a distance d from the face of a supporting column; the so-dimensioned shear reinforcement at the section is maintained to the face of the column.
6. Apart from point 5 above, a reverse "shift rule" applies to the shear reinforcement determined at section x: it can be maintained constant over a distance $z\cot\theta$ in the direction of increasing shears, i.e., toward the nearest support.

The above apply both to the "seismic" design situation and the "persistent and transient" one.

The shear reinforcement chosen should respect the detailing rules prescribed in Eurocodes 2 and 8, summarised in Table 5.3.

Table 5.3 EC8 detailing rules for the transverse reinforcement of primary beams (in secondary ones: as in DC L)

	DC H	**DC M**	**DC L**
	outside critical regions		
spacing, $s_h \leq$	$0.75d$		
$\rho_w = A_{sh}/b_w s_h \geq$	$(0.08\sqrt{f_{ck}(\mathrm{MPa})})/f_{yk}(\mathrm{MPa})$ (1)		
	in critical regions		
diameter, $d_{bw} \geq$	6mm		
spacing, $s_h \leq$	$6d_{bL}$ (2), $h/4$, $24d_{bw}$, 175mm	$8d_{bL}$ (2), $h/4$, $24d_{bw}$, 225mm	-

(1) NDP (Nationally Determined Parameter) per EC2; the value recommended in EC2 is given here.
(2) d_{bL}: minimum diameter of all top and bottom longitudinal bars within the critical region.

Apart from the special dimensioning rules for DC H beams highlighted in Section 5.5.3, the only difference that design against seismic actions per Eurocode 8 or non-seismic ones per Eurocode 2 makes for beams in shear is the special detailing prescribed in Eurocode 8 for the stirrups in the end regions where plastic hinges are likely to form. These are termed "critical regions" and a conventional length is specified for them. The prescribed maximum stirrup spacing as a multiple of the longitudinal bar diameter aims at preventing buckling of these bars.

The stirrup diameter and spacing are constant within each "critical region", obeying the relevant detailing rules in Table 5.3 and determined from the condition $V_{Ed}(x) = V_{Rd,s}$ at a distance $x = d$ from the column face. A practical implication of the different detailing of "critical regions" is that shear reinforcement in the rest of the beam is dimensioned from the condition $V_{Ed}(x) = V_{Rd,s}$ at a distance, x, from the column face equal to the "critical region" length plus $z\cot\theta$. It is normally kept constant between the "critical regions", as controlled by the most demanding section beyond a distance of $z\cot\theta$ from their ends.

5.5.3 Special rules for seismic design of critical regions in DC H beams for the ULS in shear

In DC H beams, additional Eurocode 8 rules differentiate further the dimensioning of "critical regions" in shear from the rest of the beam. For the dimensioning of these regions in the "seismic design situation", Eurocode 8 sets in Eqs. (5.46), (5.47) the strut inclination, θ, equal to 45^o. This choice (i.e., tanθ = 1) gives the minimum value of $V_{Rd,s}$ in the range of θ per Eurocode 2, Eq. (5.45). It amounts to a classical Mörsch-Ritter 45^o-truss for the design in shear without a concrete contribution term. The reason of this choice is that in plastic hinges the shear resistance due to the transverse reinforcement decreases with increasing inelastic cyclic deformations (Biskinis et al, 2004); the magnitude of these deformations is significant in beams of DC H. Despite this apparently large penalty on $V_{Rd,s}$, the density of beam stirrups in the "critical regions" of DC H beams is usually controlled by the detailing requirements at the last row of Table 5.3.

Another aspect where shear design in "critical regions" of DC H beams deviates from the Eurocode 2 rules in Section 5.5.2 is the use of inclined bars against shear sliding at the end section of a beam at an instant in the response when the end section is cracked through its depth and the shear force is high. This may happen if the shear force has large reversals and a high peak value. Because a through-cracked section is not crossed by stirrups, Eurocode 8 requires for it inclined bars against sliding shear, if, with ζ from Eq.(5.43), both of the following criteria are met:

$$-1 \leq \zeta < -0.5 \tag{5.48}$$

$$\max V_{i,d} > (2+\zeta) f_{ctd} b_w d \tag{5.49}$$

where max$V_{i,d}$ is the maximum design shear force from Eq. (5.42a) at the end section of the beam "critical region" at end i; the design value of the 5%-fractile of the tensile strength of concrete is $f_{ctd} = f_{ctk,0.05}/\gamma_c = 0.7 f_{ctm}/\gamma_c = 0.21 f_{ck}^{2/3}/\gamma_c$ (MPa).The limit shear at the right-hand-side of Eq. (5.49) is from one-third to one-half the value of $V_{Rd,max}$ for θ = 45^o.

If both Eqs. (5.48) and (5.49) are met, the end section should be crossed by inclined bars at an angle $\pm\alpha$ to the beam axis. These bars, with total cross-sectional area A_s, should resist with the vertical components $A_s f_{yd}\sin\alpha$ of their yield force – in tension and compression – one-half of $\max V_{i,d}$ from Eq.(5.42a). The other half should be resisted by stirrups, according to the recommendation in Eurocode 2 to take at least one-half of the design shear force with shear links:

$$A_s f_{yd}\sin\alpha \geq 0.5\max V_{i,d} \tag{5.50}$$

If the beam is short, inclined bars are conveniently placed along its two diagonals in elevation (see coupling beam of Fig. 5.5); then $\tan\alpha \approx (d\text{-}d_1)/L_{cl}$. If it is not short, the inclination of its diagonals to the beam axis is small and inclined bars placed along them are not efficient; in such cases it is more cost-effective to place two sets of shear links: one at an angle $\alpha = 45^o$ to the beam axis, the other at $\alpha = -45^o$. However, constructability and reinforcement congestion hamper this option. Normally there is neither risk from sliding shear nor a need for inclined reinforcement, if we avoid beams which are short and not loaded by significant gravity loads (in such beams the first term in the right-hand-side of Eq. (5.42b) is large and the second one small.

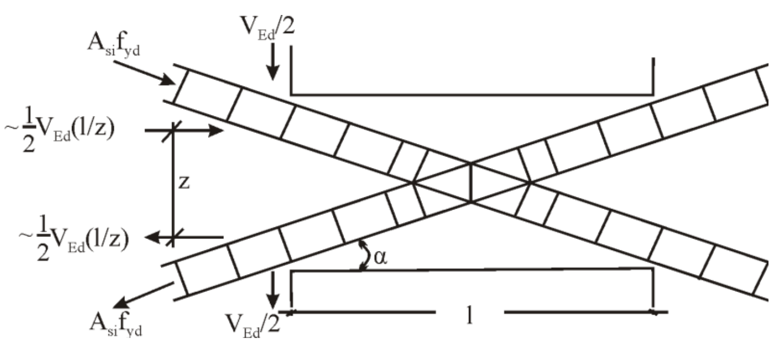

Fig. 5.5 Coupling beam with diagonal reinforcement per EC8

5.5.4 Dimensioning of columns for the ULS in shear

In columns designed to Eurocode 8, plastic hinging under the design seismic action is the exception. If it does take place, it leads to lower ductility demands than in DC H beams and hence to a smaller reduction of shear resistance. So, Eurocode 8 neglects this reduction for columns.

Columns are subjected to almost full shear reversals, while their capacity design shears from Eq. (5.44) normally exceed the limit at the right-hand-side of Eq. (5.49) for $\zeta = -1$. Nevertheless, Eurocode 8 does not require for them inclined bars against shear sliding, trusting their axial force to close through-cracks of the end section against the low plastic strains that may build up in the vertical bars. Sliding is also resisted by clamping and dowel action of the large diameter intermediate bars between the corners, which remain elastic when the peak shear and moment occur in the column. So, the dimensioning of columns in shear takes place according to the Eurocode 2 alone, i.e., taking into account the effect of axial load on shear resistance as follows:

1. A compressive axial force, N_d, increases the shear resistance, $V_{Rd,s}$, due to the transverse reinforcement by the transverse component of the strut which carries N from the compression zone at the top section of the column to that of the bottom at an inclination of z/H_{cl} to the column axis:

$$V_{Rd,s} = \frac{z}{H_{cl}} N_d + \rho_w b_w z f_{ywd} \cot\theta \tag{5.46a}$$

2. Eurocode 2 introduces in $V_{Rd,max}$ an empirical multiplicative factor which is a function of $\nu_d = N_d/A_c f_{cd}$ and takes into account: a) the contribution of N_d to shear resistance, at the same time as b) the burden placed on the inclined compression field accompanying the tension in the transverse reinforcement by the normal stress component in the strut due to N for $\nu_d > 0.5$:

$$V_{Rd,\max} = 0.3\min\left(1.25;\, 1+\nu_d;\, 2.5(1-\nu_d)\right) b_w z \left(1 - \frac{f_{ck}(MPa)}{250}\right) f_{cd} \sin 2\theta \tag{5.47a}$$

With these modifications in the shear resistance formulas, steps 1 to 4 of the general procedure in Section 5.5.2 for dimensioning beams in shear are also applicable to columns, using all along the column $V_{CD,c}$ from Eq. (5.44), instead of $V_{Ed}(x)$. This procedure is followed separately in the two transverse directions of the column, using the corresponding values of $V_{CD,c}$ as design shears. In rectangular columns, the side length at right angles to the plane of bending is used as b_w in Eqs. (5.46a), (5.47a) and 90% of the effective depth, d, in the other direction as z.

If the section comprises more than one rectangular parts along two orthogonal directions, it is simpler and safe-sided to assign the design shear of each transverse direction only to the longest part of the section in that direction (i.e., to one leg per direction in a T- or L-section). That part plays the role of the web; only the stirrup legs in it which are parallel to the design shear contribute to the area of transverse reinforcement per unit height of the column, A_{sh}/s_h, in the direction considered.

Column sides longer than about 250 mm in DC H or 300 mm in DC M should have intermediate vertical bars engaged at a corner of a stirrup or by the hook of a cross-tie (see relevant rule in Table 5.2, row 3 from the bottom). The legs of these intermediate stirrups or cross-ties contribute to the shear resistance per Eq. (3.46a) at right angles to the column side; their cross-sectional area enters in $\rho_w = A_{sh}/b_w s_h$ multiplied by $\cos\alpha$, where α is the angle between the leg and the direction of the shear force. Although the cross-sectional area and/or spacing of intermediate stirrups or cross-ties may well differ from those of the perimeter hoops, they are usually chosen the same, for simplicity.

The transverse reinforcement should respect the detailing rules in Table 5.4. Except those concerning the effective mechanical ratio $a\omega_{wd}$ of stirrups, which have a fundamental basis explained in Sections 5.7.3, 5.7.5, these rules are empirical. As in beams, the rule prescribing the maximum stirrup spacing in "critical regions" as a multiple of the diameter of longitudinal bars aims at preventing buckling.

If the stirrup diameter and/or spacing are not controlled by the design shear, $V_{CD,c}$, which is constant along the column, but by the detailing rules, which are different in "critical regions" and outside, the transverse reinforcement may be chosen different in each "critical region" in a storey and in-between these regions. For simplicity, the transverse reinforcement is often chosen the same throughout the storey, as controlled by the most demanding of the two "critical regions".

Table 5.4 EC8 detailing rules for transverse reinforcement in primary columns (secondary ones: as in DC L)

	DC H	DC M	**DC L**
critical region length (1) $\geq$	$1.5h_c$, $1.5b_c$, 0.6m, $H_{cl}/5$	h_c, b_c, 0.45m, $H_{cl}/6$	h_c, b_c,
Outside the critical regions			
diameter, $d_{bw} \geq$	6mm, $d_{bL}/4$		
spacing, $s_w \leq$	$20d_{bL}$, h_c, b_c, 400mm		
at lap splices of bars with d_{bL}>14mm, $s_w \leq$	$12d_{bL}$, $0.6h_c$, $0.6b_c$, 240mm		
In critical regions (2)			
diameter, $d_{bw} \geq$ (3)	6mm, $0.4\sqrt{(f_{yd}/f_{ywd})}d_{bL}$	6mm, $d_{bL}/4$	
spacing, $s_w \leq$ (3), (4)	$6d_{bL}$, $b_o/3$, 125mm	$8d_{bL}$, $b_o/2$, 175mm	as outside critical regions
mechanical ratio $\omega_{wd} \geq$ (5)	0.08	-	
effective mechanical ratio $a\omega_{wd} \geq$ (4), (5), (6), (7)	$30\mu_\phi^* \nu_d \varepsilon_{yd}\, b_c/b_o$ - 0.035	-	
In the critical region at the base of the column (at the connection to the foundation)			
mechanical ratio $\omega_{wd} \geq$	0.12	0.08	-
effective mechanical ratio $a\omega_{wd} \geq$ (4), (5), (6), (8), (9)	$30\mu_\phi \nu_d \varepsilon_{yd} b_c/b_o$ -0.035		-

(1) h_c, b_c, H_{cl}: column sides and clear length.

(2) For DC M: If a value of $q \leq 2$ is used for the design, the transverse reinforcement in critical regions of columns with an axial load ratio $\nu_d \leq 0.2$ may follow only the rules for DCL columns.

(3) For DC H: In the two lower storeys of the building, the requirements on d_{bw}, s_w apply over a distance from the end section not less than 1.5 times the critical region length.

(4) Index c denotes the full concrete section; index o the confined core to the centreline of the perimeter hoop; b_o is the smaller side of this core.

(5) ω_{wd}: volume ratio of confining hoops to confined core (to centreline of perimeter hoop) times f_{ywd}/f_{cd}.

(6) $a = (1-s/2b_o)(1-s/2h_o)(1-\{b_o/[(n_h-1)h_o]+h_o/[(n_b-1)b_o]\}/3)$: confinement effectiveness factor of rectangular hoops at spacing s, with n_b legs parallel to the side of the core with length b_o and n_h legs parallel to the side of length h_o.

(7) For DCH: at column ends protected from plastic hinging through the capacity design check at beam-column joints, μ_ϕ^* is the value of the curvature ductility factor that corresponds per Eqs. (5.64) to 2/3 of the basic value, q_o, of the behaviour factor applicable to the design; at the ends of columns where plastic hinging is not prevented, because of the exemptions from the application of Eq. (5.31), μ_ϕ^* is taken equal to μ_ϕ defined in Note (8) (see also Note (9)); $\varepsilon_{yd} = f_{yd}/E_s$.

(8) μ_ϕ: curvature ductility factor corresponding per Eqs. (5.64) to the basic value, q_o, of the behaviour factor applicable to the design.

(9) For DCH: The requirement applies also in the critical regions at the ends of columns where plastic hinging is not prevented, because of the exemptions from the application of Eq. (5.31).

5.6 Detailed design of ductile walls in flexure and shear

5.6.1 Design of ductile walls in flexure

5.6.1.1 Design moments of ductile walls

To ensure that flexural plastic hinging is limited to the wall base and the wall stays elastic above it, despite higher mode response after the plastic hinge develops at the base, Eurocode 8 requires designing in flexure the rest of the wall height for a linear envelope of the positive and negative wall moments derived from the analysis for the design seismic action. The linear envelope is shown schematically in Fig. 5.6, for simplicity without the tension shift; real examples of wall moment diagrams from the analysis are depicted in the upper half of Fig. 7.44 in Chapter 7, alongside the design envelopes fitted to them per Eurocode 8 and shifted upwards by the tension shift. Thanks to the resulting flexural overstrength, the rest of the wall does not need to be specially detailed for flexural ductility, nor to be designed in shear accounting for the cyclic decay of shear resistance in plastic hinges; so, its design and construction are much simpler and possibly less costly.

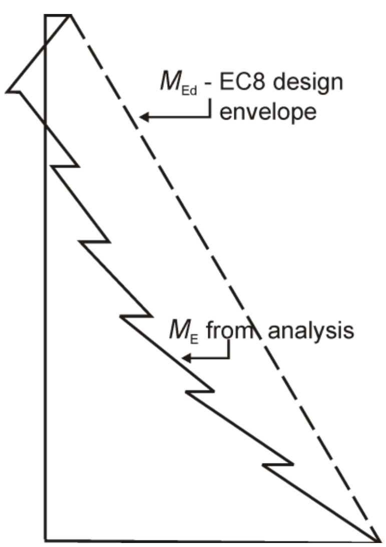

Fig. 5.6 Bending moment diagram of a wall from the analysis and moment-envelope per EC8 for the design of a ductile wall in flexure.

A wall flange longer than 4-times its thickness qualifies itself as a wall in the orthogonal direction (walls with T-, L-, H- or C-section). Then its design moments in that direction are obtained from a linear envelope as depicted in Fig. 5.6 and not directly from the analysis.

5.6.1.2 Dimensioning and detailing of vertical reinforcement in ductile walls

The detailed design of a wall starts with dimensioning its vertical reinforcement at the base section for the normal action effects (moment(s) and axial force) derived from the analysis for the seismic design situation, per the Eurocode 2 criteria and rules for the ULS in flexure with axial force. The present section describes the dimensioning procedure, after an introduction about the distribution of vertical reinforcement over a wall section.

A wall differs from a frame column in the shape of its seismic moments diagram from the analysis. It differs from an isolated column, cantilevering from the foundation without connection to any floor beam, only in the shape of the cross-section, which, in a wall, comprises one or more elongated rectangular parts – conventionally per Eurocodes 2 and 8 with ratio of sides above 4.0. If it consists of a single elongated rectangular part, the wall develops essentially uniaxial moments and shears (in a vertical plane of bending in the long direction of the section), even when the seismic response is equally strong in the two horizontal directions. The main impact of the section geometry on the wall design, though, even for sections with two or more elongated rectangular parts (L-, T-, I-, C-sections, etc), is the clear separation of the two ends of the section in the long direction. These end regions provide most of the moment resistance through vertical stresses – tensile at one end, compressive at the other – and play the prime role for flexural ductility: it is only them which are enclosed in steel hoops for concrete confinement and antibuckling restraint of vertical bars. In that respect, they resemble the top and bottom "flanges" of a beam. Another common point with beams is that the part of the section between the longitudinally (and heavily) reinforced "flanges" resists the shear, acting as a "web". A third commonality is, of course, the (essentially) uniaxial bending, parallel to the "web". By contrast, a column (even a big one behaving as a vertical cantilever) works in both transverse directions and requires vertical bars and confinement all around the section.

Like a deep beam, a wall has longitudinal reinforcement in the web as well, to control the width of flexural or shear cracks in that part of the wall too. This reinforcement is placed in two curtains (one near each face of the web, see Fig. 5.7 and examples in Figs. 7.45-7.47 of Chapter 7) and is normally chosen on the basis of the prescriptive minimum requirements of Eurocodes 2 and 8, listed in Table 5.5 under "Web" and "vertical bars". Suppose that the detailing rules concerning the minimum steel ratio and bar diameter and the maximum bar spacing in each curtain give a ratio of vertical web reinforcement $\rho_v = A_{sv}/bs_v$, where A_{sv} is the cross-sectional area of two web bars (one per curtain), s_v the bar spacing along the length, l_w, of the wall section and b the width of the compression flange (this ρ_v is normalised to the compression flange width, b, whereas the minimum and maximum web reinforcement ratios in Table 5.5 are normalised to the actual thickness of the web, $b_{wo} \leq b$). The corresponding mechanical ratio is $\omega_{vd} = \rho_v f_{yd}/f_{cd}$.

Table 5.5 EC8 detailing rules for ductile walls

	DC H	DC M	DC L
critical region height, h_{cr}	$\geq \max(l_w, H_w/6)$ (2) $\leq \min(2l_w, h_{storey})$ if wall ≤6 storeys $\leq \min(2l_w, 2h_{storey})$ if wall > 6 storeys		-
Boundary elements			
a) in critical height region:			
- length l_c from wall edge ≥	$0.15l_w$, $1.5b_w$, part of the section where $\varepsilon_c > 0.0035$		-
- thickness b_w over l_c ≥	0.2m; $h_{st}/15$ if $l_c \leq \max(2b_w, l_w/5)$, $h_{st}/10$ otherwise		-
- vertical reinforcement:			
ρ_{min} over $A_c = l_c b_w$	0.5%		0.2% (1)
ρ_{max} over A_c	4% (1)		

spacing along perimeter of bars restrained by tie corner or cross-tie hook	≤150mm	≤200mm	-
- confining hoops (index w) (3):			
diameter, $d_{bw} \geq$	6mm, $0.4\sqrt{(f_{yd}/f_{ywd})}d_{bL}$	6mm,	wherever $\rho_L >$ 2% in section: as over rest of the wall (see case b below)
spacing, $s_w \leq$ (4)	$6d_{bL}$, $b_o/3$, 125mm	$8d_{bL}$, $b_o/2$, 175mm	
$\omega_{wd} \geq$ (3)	0.12	0.08	
$\alpha\omega_{wd} \geq$ (4), (5)	$30\mu_\phi(\nu_d+\omega_v)\varepsilon_{yd}b_w/b_o - 0.035$		
b) over the rest of the wall height:	Wherever in the section ε_c>0.2%: $\rho_{v,min}$ = 0.5%; elsewhere: 0.2% In parts of the section where ρ_L > 2%: - distance of unrestrained bar in compression zone to nearest restrained bar ≤ 150mm; - hoops with $d_{bw} \geq$ max(6mm, $d_{bL}/4$), spacing $s_w \leq$ min($12d_{bL}$, $0.6b_{wo}$, 240mm) (1) till distance $4b_w$ above or below floor slab /beam; $s_w \leq$ min($20d_{bL}$, b_{wo}, 400mm) (1) beyond that distance		
	Web		
thickness, $b_{wo} \geq$	max(150mm, $h_{storey}/20$)		-
vertical bars (index: v):			
$\rho_v = A_{sv}/b_{wo}s_v \geq$	0.2%, but 0.5% wherever in the section ε_c>0.002		0.2% (1)
$\rho_v = A_{sv}/b_{wo}s_v \leq$	4%		
$d_{bv} \geq$	8mm	-	
$d_{bv} \leq$	$b_{wo}/8$	-	
spacing, $s_v \leq$	min($25d_{bv}$, 250mm)	min($3b_{wo}$, 400mm)	
horizontal bars (index: h):			
$\rho_{h,min}$	0.2%	max(0.1%, $0.25\rho_v$) (1)	
$d_{bh} \geq$	8mm	-	
$d_{bh} \leq$	$b_{wo}/8$	-	
spacing, $s_h \leq$	min($25d_{bh}$, 250mm)	400mm	
$\rho_{v,min}$ at construction joints (6)	max(0.25%; $\frac{1.3f_{ctd} - N_{Ed}/A_c}{f_{yd} + 1.5\sqrt{f_{cd}f_{yd}}}$)	-	

(1) NDP (Nationally Determined Parameter) per EC2; the value recommended in EC2 is given here.
(2) l_w: long side of rectangular wall section or rectangular part thereof; H_w: total height of wall; h_{storey}: storey height.
(3) (In DC M only) The DCL rules apply to the confining reinforcement of boundary elements, if: under the maximum axial force in the wall from the analysis for the seismic design situation, the wall axial load ratio $\nu_d = N_{Ed}/A_cf_{cd}$ is ≤ 0.15; or, if $\nu_d \leq 0.2$ but the q-value used in the design is ≤ 85% of the q-value allowed when the DC M confining reinforcement is used in boundary elements.
(4) Notes (4), (5), (6) of Table 5.4 apply for the confined core of boundary elements.
(5) μ_ϕ: value of the curvature ductility factor corresponding through Eqs. (5.64) to the product of the basic value q_o of the behaviour factor times the ratio M_{Edo}/M_{Rdo} of the moment at the wall base from the analysis for the seismic design situation to the design value of moment resistance at the wall base for the axial force from the same analysis; $\varepsilon_{yd} = f_{yd}/E_s$; ω_{vd}: mechanical ratio of vertical web reinforcement.
(6) N_{Ed}: minimum axial load from the analysis for the seismic design situation (positive for compression); $f_{ctd} = f_{ctk,0.05}/\gamma_c = 0.7f_{ctm}/\gamma_c = 0.21f_{ck}^{2/3}/\gamma_c$: design value of 5%-fractile tensile strength of concrete.

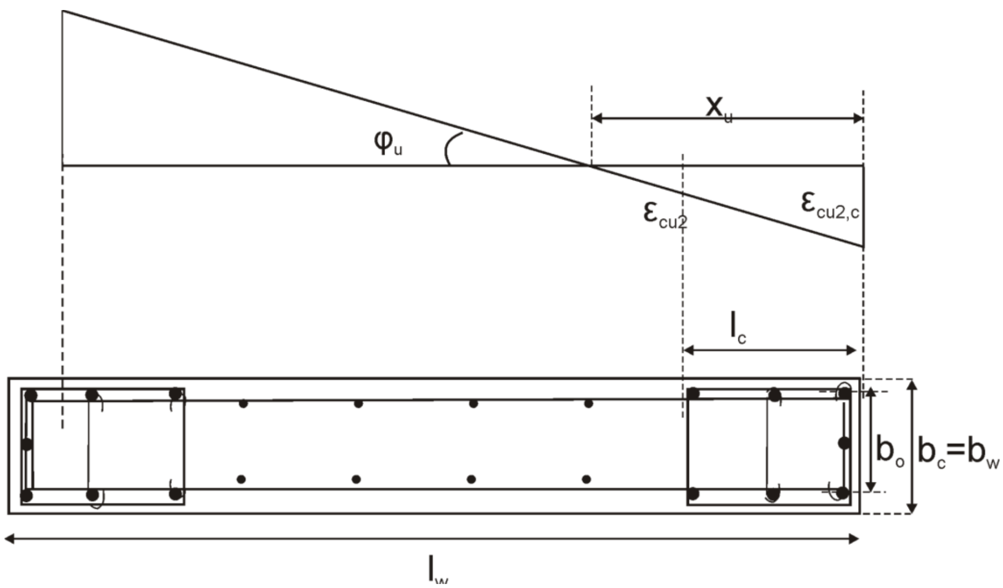

Fig. 5.7 Schematic arrangement of vertical reinforcement in a ductile wall section and determination of boundary element length

The vertical reinforcement which is concentrated near the tension edge is considered for the present purposes as lumped at the centroid of its cross-sectional area, A_{s1}, at an effective depth d; we define its mechanical ratio as $\omega_{1d} = A_{s1}/(bd)\cdot(f_{yd}/f_{cd})$. We assume that the vertical reinforcement concentrated near the compression edge has the same cross-sectional area as the tension one, $A_{s2} = A_{s1}$, and is lumped at its centroid at a distance d_1 from the extreme compressive fibres; its mechanical ratio is $\omega_{2d} = \omega_{1d}$.

Using as a basis Eqs. (5.37)-(5.39) in Section 5.4.3, which take into account the web reinforcement with a uniform mechanical ratio, ω_{vd}, between the tension and the compression reinforcement, we can modify the procedure proposed in Section 5.4.2 for rectangular columns with symmetric tension and compression reinforcement only, to calculate $\omega_{1d} = \omega_{2d}$, for known ω_{vd} and given dimensionless parameters as defined by Eq. (5.32) in Section 5.4.2. Note that walls have a low axial load ratio; in fact for walls Eurocode 8 sets an upper limit of 0.35 for DC H and of 0.4 for DC M to the ratio of the maximum axial load from the analysis for the seismic design situation to $A_c f_{cd}$ – which exceeds ν_d as defined in Eq. (5.32). We then have only cases (i) and (ii), as follows.

(i) The tension and the compression reinforcement both yield, if ν_d is in the range:

$$\frac{\omega_v}{1-\delta_1}\left(\delta_1\frac{\varepsilon_{cu2}+\varepsilon_{yd}}{\varepsilon_{cu2}-\varepsilon_{yd}}-1\right)+\delta_1\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}-\varepsilon_{yd}}\equiv\nu_2\le\nu_d<\nu_1\equiv\frac{\omega_v}{1-\delta_1}\left(\frac{\varepsilon_{cu2}-\varepsilon_{yd}}{\varepsilon_{cu2}+\varepsilon_{yd}}-\delta_1\right)+\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}+\varepsilon_{yd}} \quad (5.51a)$$

Then, with a normalised neutral axis depth computed as:

$$\xi=\frac{(1-\delta_1)\nu_d+(1+\delta_1)\omega_v}{(1-\delta_1)\left(1-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\right)+2\omega_v} \quad (5.52a)$$

we find the symmetric edge reinforcement, $\omega_{1d}=\omega_{2d}$, from:

$$(1-\delta_1)\omega_{1d}=\mu_d-\xi\left[\frac{1-\xi}{2}-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2}-\xi+\frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right]-\frac{\omega_{vd}}{1-\delta_1}\left[(\xi-\delta_1)(1-\xi)-\frac{1}{3}\left(\frac{\xi\varepsilon_{yd}}{\varepsilon_{cu2}}\right)^2\right] \quad (5.53a)$$

(ii) The tension steel yields but the compression one is elastic; ν_d is less than ν_2 from Eq. (5.51a):

$$\nu_d\le\frac{\omega_{vd}}{1-\delta_1}\left(\delta_1\frac{\varepsilon_{cu2}+\varepsilon_{yd}}{\varepsilon_{cu2}-\varepsilon_{yd}}-1\right)+\delta_1\frac{\varepsilon_{cu2}-\varepsilon_{c2}/3}{\varepsilon_{cu2}-\varepsilon_{yd}}\equiv\nu_2 \quad (5.51b)$$

Then ξ is the positive root of the equation:

$$\left[1-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}+\frac{\omega_{vd}}{2(1-\delta_1)}\frac{(\varepsilon_{cu2}+\varepsilon_{yd})^2}{\varepsilon_{cu2}\varepsilon_{yd}}\right]\xi^2-\left[\nu_d+\omega_{1d}\left(1-\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)+\frac{\omega_{vd}}{1-\delta_1}\left(1+\frac{\varepsilon_{cu2}\delta_1}{\varepsilon_{yd}}\right)\right]\xi-\left[\omega_{1d}-\frac{\omega_{vd}\delta_1}{2(1-\delta_1)}\right]\frac{\varepsilon_{cu2}\delta_1}{\varepsilon_{yd}}=0$$

(5.52b)

We can replace ω_{1d} in Eq. (5.52b) in terms of ξ and the moment from:

$$\omega_{1d}\frac{(1-\delta_1)}{2}\left(1+\frac{\xi-\delta_1}{\xi}\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\right)=\mu_d-\xi\left[\frac{1-\xi}{2}-\frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}\left(\frac{1}{2}-\xi+\frac{\varepsilon_{c2}}{4\varepsilon_{cu2}}\xi\right)\right]-$$
$$\frac{\omega_{vd}}{4(1-\delta_1)}\left[\xi\left(1+\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)-\delta_1\right]\left[1+\frac{\varepsilon_{cu2}}{\varepsilon_{yd}}\left(\frac{\xi-\delta_1}{\xi}\right)\right]\left[1-\frac{\delta_1}{3}-\frac{2}{3}\xi\left(1+\frac{\varepsilon_{yd}}{\varepsilon_{cu2}}\right)\right] \quad (5.53b)$$

The resulting very nonlinear equation is solved numerically for ξ; ω_{1d} is then found from Eq. (5.53b).

The edge reinforcement area, $A_{s1}=\omega_{1d}(bd)\cdot(f_{cd}/f_{yd})$, from Eqs. (5.53) is implemented as a number of bars near the edge of the section, normally spread over a certain distance, l_c, from it, e.g., along a "boundary element" (see Fig. 5.7 and examples in Figs. 7.45-7.47 of Chapter 7). The minimum l_c value specified by

Eurocode 8 within the critical region at the base of the wall is given at the top of the "boundary elements" part of Table 5.5. The distance d_1 of this reinforcement from the section edge refers to the centroid of these bars. Note that, because ω_{vd} is considered uniform between the centroids of ω_{2d} and ω_{1d}, a fraction $(l_c/d-\delta_1)/(1-\delta_1)$ of the total web reinforcement area, $\rho_v bd$, falls within the distance l_c over which the edge reinforcement is spread and should be added to $A_{s1} = \omega_{1d}(bd)\cdot(f_{cd}/f_{yd})$ before translating it into an edge reinforcement area.

The minimum web reinforcement continues to the top of the wall. There are two ways to decide at which levels the edge reinforcement placed at the base section is curtailed:

1. We take away from each edge region one pair of bars at a time (on opposite long faces of the wall), or even two such pairs or more. As long as the distance from the wall base is less than the critical region height, h_{cr}, given at the top of Table 5.5, the length l_c of the "boundary element" still applies; the bars removed are chosen from the unrestrained ones along the perimeter of the "boundary element", preferably far from the extreme fibres. Above the critical region height, the pair of bars removed is the one further away from the extreme fibres; the size of the "boundary element" shrinks accordingly, below the minimum specified by Eurocode 8 for the critical region; the minimum web reinforcement extends over the freed space in the section. The remaining moment resistance of the section is computed using Eqs. (5.37)-(5.39) from Section 5.4.3 and compared to the linear M-envelope per Eurocode 8 (Fig. 5.6), in order to find the level where the reduced edge reinforcement suffices. Note that this level should be consistent with the value of the wall axial force, N, used in Eqs. (5.37)-(5.39) with the reduced amount of reinforcement. The process continues with further pairs of bar removed from each edge, to the top of the wall.
2. The second approach lends itself better to systematic dimensioning within an integrated computational environment. It presumes that bars start at floor level of each storey and serve the

bottom section of the storey above; at floor level of that storey the bars are lap-spliced with some of the edge bars starting there, or are continued for anchorage if they are not needed anymore. The dimensioning procedure described above for the wall base (Eqs. (5.51)-(5.53)) is repeated at the bottom section of each storey, with the values of the moment and axial force applying there, to dimension the edge reinforcement which should come from the storey below, in order to cover the requirements in flexure with axial force at the bottom section of the storey. In all storeys whose base falls within the critical region height, h_{cr}, in Table 5.5, the layout of the bars placed near each edge follows that in the "boundary element" of the base, as far as the outline and the location of restrained bars along the perimeter of the "boundary element" are concerned. Above the critical region, little care is taken to follow the same pattern as in the critical region or to place the bars very close to those coming from the storey below; the overriding consideration is to spread the bars over a distance l_c from the extreme fibres, so that the maximum steel ratio is not violated within the area $A_c = l_c b_w$.

If the wall section comprises two or more elongated rectangular parts at right angles to each other (as in T-, L-, C- or H-sections), it should be designed in flexure as a whole, for the M_y-M_z-N triplet of the entire section, assuming that it remains plane. The 3-step procedure proposed in Section 5.4.2 for dimensioning the vertical reinforcement of columns, rectangular or not, under M_y-M_z-N triplets, may be adapted to Eqs. (5.51)-(5.52), to account for web bars distributed between the two edges in the direction considered. The so-modified procedure normally gives a safe-sided estimate of the vertical reinforcement near the corners of the non-rectangular section. The full vertical reinforcement placed over the section should also meet the detailing rules in Table 5.5 for boundary elements, web minimum reinforcement, etc. Note that the size of any boundary elements needed around the non-rectangular section may be estimated from the strain profile(s) obtained in the course of Step 3 of the procedure, namely through the iterative algorithm for the ULS verification of sections with any shape and layout of reinforcement for any combination M_y-M_z-N.

Strictly speaking, even a rectangular wall is subjected to biaxial bending with axial force, M_y-M_z-N. So, although this is rarely done for rectangular walls, after the vertical reinforcement is estimated and placed according to the pertinent detailing rules, the base section of each storey may be verified for the ULS in bending with axial force for all M_y-M_z-N combinations from the analysis for the seismic design situation. The moment in the strong direction of the wall, let's say M_y, is obtained from the linear M-envelope in Fig. 5.6; the value of M_z is that from the analysis.

The edge bars curtailed according to procedure 1 or 2 above – or any alternative – should extend vertically above the level where they are not needed anymore for the ULS in bending with axial force by a length equal to $z\cot\theta/2$, according to the "shift rule" per EC2, where θ is the value of the strut inclination used at that level in the design of the wall in shear (see Section 5.6.2). They are extended by their anchorage length, only if the inclination of the moment envelope to the vertical (which is constant up the wall, see Fig. 5.6) exceeds the bar yield force, $f_{yd}\pi d_{bL}^2/4$, times the ratio of the internal lever arm, z, to l_{bd} (i.e., $(2.25\pi f_{ctd})d_{bL}z/\{a_{tr}[1\text{-}0.15(c_d/d_{bL}\text{-}1)]\}$ with the notation of Table 5.2).

5.6.2 Design of ductile walls in shear

5.6.2.1 Design shears in ductile walls

"Ductile walls", designed to develop a flexural plastic hinge only at the base, are protected by Eurocode 8 from shear failure throughout their height. The design value of moment resistance at the wall's base section, M_{Rdo}, and equilibrium alone do not suffice for the derivation of the maximum seismic shears that can develop at various levels of the wall, because, unlike in the cases of Fig. 5.4, the forces applied on the wall at intermediate levels are unknown and vary during the seismic response. It is reasonable to assume that, if M_{Rdo} exceeds the bending moment at the base from the elastic analysis for the design seismic action, M_{Edo}, the seismic shears at any level of the wall exceed those from the same elastic analysis in proportion to M_{Rdo}/M_{Edo}. This amounts to multiplying the shear forces from the elastic analysis for the design seismic action, V'_{Ed}, by a capacity-design magnification factor ε:

$$\varepsilon = \frac{V_{Ed}}{V'_{Ed}} = \gamma_{Rd}\left(\frac{M_{Rdo}}{M_{Edo}}\right) \leq q \qquad (\text{if } h_w/l_w \leq 2) \tag{5.54a}$$

where γ_{Rd} covers overstrength at the base, e.g. due to steel strain hardening. If the wall is rectangular, $M_{Rd,o}$ can be estimated according to Section 5.4.3, Eqs. (5.37)-(5.39). The last paragraph of Section 5.4.3 applies to non-rectangular walls.

Eurocode 8 adopts Eq. (5.54a) with γ_{Rd}=1.2, for DC H walls with ratio of wall height to horizontal dimension, $h_w/l_w \leq 2$ ("squat"). If a DC H wall has $h_w/l_w > 2$ ("slender"), Eurocode 8 amplifies further the shear forces from the elastic analysis, V'_{Ed}, to account for an increase of shears after plastic hinging at the base due to higher modes. It follows in this respect the approach in (Eibl and Keintzel 1988, Keintzel 1990). That approach essentially presumes that:

1. M_{Edo} and V'_{Ed} are computed via "lateral force" elastic analysis, with a first mode period T_1.
2. The behaviour factor, q, should be applied only to the first mode results; higher mode response is elastic – at least as far as the wall shears are concerned.
3. Higher mode periods lie in the constant-spectral-acceleration plateau of the elastic spectrum; their spectral acceleration is equal to $S_a(T_C)$, where T_C is the upper corner period of the plateau.
4. The ratio of the sum-of-the-squares of higher mode participation factors to the square of the participation factor of the first mode is equal to 0.1 – i.e. a very safe-sided estimate.

These considerations lead to an increase of ε in DC H walls with $h_w/l_w > 2$ per Eurocode 8:

$$\varepsilon = \frac{V_{\mathrm{Ed}}}{V'_{\mathrm{Ed}}} = \sqrt{\left(\gamma_{\mathrm{Rd}}\frac{M_{Rdo}}{M_{Edo}}\right)^2 + 0.1\left(q\frac{S_a(T_C)}{S_a(T_1)}\right)^2} \leq q \quad (\text{if } h_{\mathrm{w}}/l_{\mathrm{w}} > 2) \tag{5.54b}$$

where T_1 is the first mode period in the horizontal direction closest to that of the wall shear force.

Eq. (5.54b) gives very safe-sided (i.e., high values), especially if M_{Edo} and V'_{Ed} are computed via a "modal response spectrum" elastic analysis (Antoniou et al, 2014).

The value of $M_{\mathrm{Rdo}}/M_{\mathrm{Edo}}$, and hence of ε in Eqs.(5.54), may well exceed 1.0 if:

- The wall base is oversized with respect to the seismic demand, M_{Edo}, and has the minimum vertical reinforcement in its web and – sizeable – boundary elements. To reduce this type of overstrength, the wall should not be so thick as to have the minimum requirements per Table 5.5 control its vertical reinforcement.
- The analysis for the design seismic action produces a high axial force at the wall base. The vertical reinforcement at the wall base is governed by the sign of the design seismic action giving – alongside the moment, M_{Edo} – the minimum axial compression from the analysis for the seismic design situation. When the sign of the design seismic action reverses, we have the maximum axial compression but the same moment, M_{Edo}, producing a large overstrength: $M_{\mathrm{Rdo}} >> M_{\mathrm{Edo}}$. Such an overstrength is acute in walls placed near the corners in plan of high-rise buildings, in piers of coupled walls, etc.

As a shield to excessive values of ε from Eqs. (5.54) due to the above reasons, Eurocode 8 sets for DC H walls the ceiling of q to its value, so that the final design shear, V_{Ed}, does not exceed the value qV'_{Ed} corresponding to a fully elastic response.

To avoid the intrinsic complexity and conservatism of Eqs. (5.54), Eurocode 8 allows to DC M walls the following simplification:

$$\varepsilon = \frac{V_{Ed}}{V'_{Ed}} = 1.5 \tag{5.55}$$

Note that, unlike Eqs. (5.54) which may be overconservative, Eq.(5.55) gives a very low safety margin (if any) against flexural overstrength at the base or inelastic higher mode effects.

Higher mode effects on inelastic shears are larger at the upper storeys of the wall, especially in dual (frame-wall) systems. In such systems the frames restrain the walls at the upper storeys, to the extent that the wall shears in the top storey or the one below from the “lateral force” analysis reverse sign and are opposite to the total seismic shear applied. In general, the elastic analysis gives very small wall shears in the upper storeys, which will not come anywhere close to the relatively high values that may develop there owing to higher modes, even after multiplication by the factor ε of Eqs. (5.54) to (5.55) (see Fig. 5.8, where dotted curves represent the shear force from the analysis and its value multiplied by ε). To deal with this problem, Eurocode 8 sets a minimum for the design shear force at the top of the ductile walls in dual (frame-wall) systems, equal to half the magnified shear at the base, increasing linearly to the magnified value of the shear, $\varepsilon V'_{Ed}$, at one third of the wall height from the base (Fig. 5.8). For a sample application of the part of this sub-section and of the rest of Section 5.6 referring to DC M walls, see Section 7.6.2.4 for the walls of the 7-storey example building,.

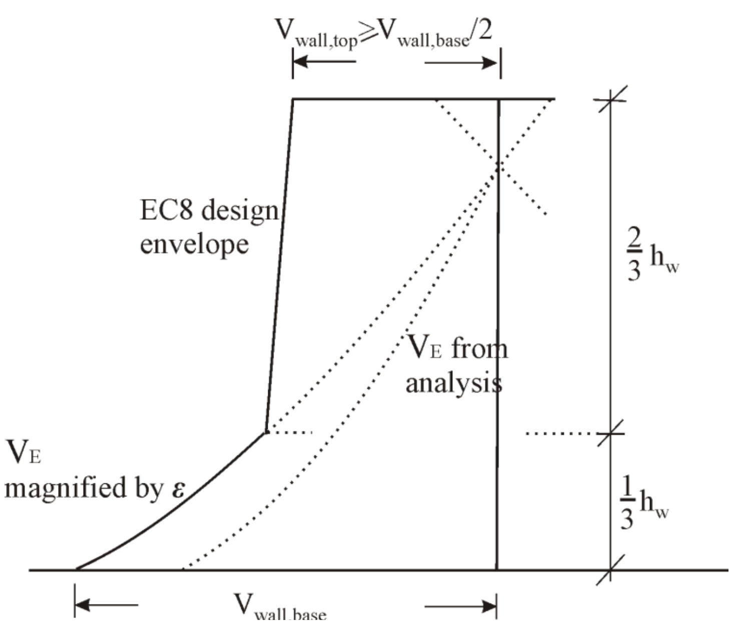

Fig. 5.8 Design shear forces per EC8 up a ductile wall in a dual (wall-frame) system.

5.6.2.2 Verification of ductile walls in shear - Special rules for critical regions of DCH walls

The general Eurocode 2 rules highlighted in Section 5.5.2 for beams and 5.5.4 for columns apply to the verification in shear of DC M walls throughout their height and of DC H ones outside their critical height. The contribution of the axial load to $V_{\mathrm{Rd,s}}$ is small; so, Eq. (5.46) may be applied. The internal lever arm, z, may be taken equal to 80% of the length, l_{w}, of the wall section. The web reinforcement should also meet the prescriptive detailing rules in Table 5.5.

Three types of special rules apply in the critical region of DC H walls:

1. The design value of shear resistance, as controlled by diagonal compression in the web, $V_{\mathrm{Rd,max}}$, is taken as 40% of the value given by Eq. (5.47) per Eurocode 2. A value of θ is not fixed in the range of Eq. (5.45), but, with such a drastic reduction in $V_{\mathrm{Rd,max}}$, it makes sense to take $\theta = 45^{\mathrm{o}}$. This large reduction is fully supported by the available cyclic test data (Biskinis et al, 2004, Fardis, 2009). It has not been extended to DC M walls as well, because, if applied alongside the magnification of design shears per Section 5.6.2.1 (Eq. (5.55)), it might be prohibitive to use ductile walls for earthquake resistance in Eurocode 8. However, caution should be exercised in exhausting the liberal $V_{\mathrm{Rd,max}}$-value of Eq. (5.47a) in ductile walls of DC M.
2. Unlike columns (but like DC H beams), ductile walls of DC H should be explicitly verified against sliding, because their axial load level is too low to mobilise sufficient friction and the web bars are of smaller diameter and more sparse than in columns. The base of every storey within the critical height of the wall should be verified in sliding shear. The design value of the resisting shear against horizontal sliding along the base section in a storey is given by Eurocode 8 as:

$$V_{\mathrm{Rd,SLS}} = V_{\mathrm{fd}} + V_{\mathrm{dd}} + V_{\mathrm{id}} \tag{5.56}$$

i.e., as the sum of:

– the friction resistance, V_{fd}, of the compression zone:

$$V_{\mathrm{fd}}=\min\left(\mu_{\mathrm{f}}\left[\left(\sum A_{\mathrm{sj}}f_{\mathrm{yd}}+N_{\mathrm{Ed}}\right)\xi+\frac{M_{Ed}}{z}\right];0.3\left(1-\frac{f_{\mathrm{ck}}(MPa)}{250}\right)f_{\mathrm{cd}}b_{\mathrm{wo}}\left(\xi d\right)\right) \tag{5.57}$$

with:

- μ_{f}: friction coefficient, equal to 0.7 for rough interfaces or to 0.6 for smooth ones;
- ΣA_{sj}: total area of vertical bars in the web and of those placed in the boundary elements specifically against shear sliding without counting in the ULS for bending;
- M_{Ed}, N_{Ed}: values from the analysis for the seismic design situation; and
- ξ: normalised neutral axis depth, from Eqs.(5.39) in Section 5.4.3;

– a design value of the dowel resistance, V_{dd}:

$$V_{\mathrm{dd}}=\sum A_{\mathrm{sj}}\min\left(1.3\sqrt{f_{\mathrm{cd}}f_{\mathrm{yd}}};0.25f_{\mathrm{yd}}\right) \tag{5.58}$$

for concrete class above C20/25, the second term governs in Eq. (5.58); it reflects yielding of the dowel in pure shear without axial force, with a safety margin of about 2.3;

– the horizontal projection, V_{id}, of the design resistance of any inclined bars, with a total area (in both directions) ΣA_{si}, placed at an angle $\pm\varphi$ to the base of the wall:

$$V_{\mathrm{id}}=\sum A_{\mathrm{si}}f_{\mathrm{yd}}\cos\varphi \tag{5.59}$$

Inclined bars should preferably cross the base section at about mid-length, to avoid affecting – through the couple of vertical components of their tension and compression forces – the moment resistance, M_{Rdo}, used in Eqs. (5.54) for the design shear, V_{Ed}, or the location of the plastic hinge; they should extend at least to a level of $l_{\mathrm{w}}/2$ above the base section, making an inclination $\varphi = 45^{\mathrm{o}}$ not only convenient, but also very cost-effective. Although inclined bars are needed only if $V_{\mathrm{fd}} + V_{\mathrm{dd}}$ is less than the design shear, V_{Ed}, Eurocode 8 requires placing them always at the base of "squat" walls (i.e., those with $h_{\mathrm{w}}/l_{\mathrm{w}} \leq 2$) of DC H, at a quantity sufficient to resist, through V_{id}, at least

$0.5V_{Ed}$. In such walls Eurocode 8 requires inclined bars at the base of all storeys, at a quantity sufficient to resist at least 25% of the storey design shear.

Note that the minimum clamping reinforcement required across construction joints (i.e., at the base section of each storey) in a DC H wall according to the last row in Table 5.5 counts also against shear sliding. The second, non-prescriptive term in this requirement comes from the condition that cohesion, plus friction, plus dowel action at such a joint exceeds the shear stress at shear cracking of the concrete cast right above the joint.

3. A special rule applies for dimensioning the web reinforcement ratios, horizontal ρ_h, and vertical ρ_v, in those storeys of DC H walls where the maximum shear span ratio, $\alpha_s = M_{Ed}/(V_{Ed}l_w)$ (normally at the base of the storey) is less than 2. This rule is a modification of the Eurocode 2 rule for the calculation of shear reinforcement in members with $0.5 < \alpha_s < 2$:

$$V_{Rd,s} = V_{Rd,c} + \rho_h b_{wo}\left(0.75 l_w \alpha_s\right) f_{yhd} = V_{Rd,c} + \rho_h b_{wo}\left(0.75\frac{M_{Ed}}{V_{Ed}}\right) f_{yhd} \tag{5.60}$$

where:

- ρ_h is the ratio of the horizontal reinforcement, normalised to the web width, b_{wo}, and f_{yhd} its design yield strength;
- $V_{Rd,c}$ is the design shear resistance of members without shear reinforcement per Eurocode 2 (in kN):

$$V_{Rd,c} = \left\{ \min\left[\frac{180}{\gamma_c}(100\rho_L)^{1/3},\ 35\sqrt{1+\sqrt{\frac{0.2}{d}}}\ f_{ck}^{1/6}\right]\left(1+\sqrt{\frac{0.2}{d}}\right) f_{ck}^{1/3} + 0.15\min\left(\frac{N_{Ed}}{A_c},\, 0.2\frac{f_{ck}}{\gamma_c}\right)\right\} b_{wo} d \tag{5.61}$$

where ρ_L is the tension reinforcement ratio, γ_c the partial factor for concrete, f_{ck} is in MPa, b_{wo} and d are in meters, the wall gross cross-sectional area, A_c, is in m^2 and N_{Ed} in kN (in the critical region of the wall, $V_{Rd,c} = 0$ if N_{Ed} is tensile).

The ratio of vertical web reinforcement, ρ_v, is then dimensioned to provide a 45^o inclination of the concrete compression field in the web, together with the horizontal reinforcement and the vertical compression in the web due to the minimum axial force in the seismic design situation, minN_{Ed}:

$$\rho_h f_{ydh} b_{wo} z \leq \rho_v f_{ydv} b_{wo} z + \min N_{Ed} \tag{5.62}$$

Note that M_{Ed} in α_s comes from the M-envelope of Fig. 5.6, which does not exhibit inflection points in any storey, and V_{Ed} comes from Eqs. (5.54) – and the envelope in Fig. 5.8, if the wall belongs to a dual (frame-wall) system. So, α_s may be less than 2 at upper storeys of walls with large l_w, giving unduly large web reinforcement. Judgment should be used in such cases, as these expressions are meant for the base region of "squat" walls (those with $h_w/l_w \leq 2$); besides, owing to the very limited knowledge and data at the time on the cyclic behaviour and shear failure of squat walls , Eqs. (5.60), (5.62) are conservative (safe-sided).

The special rules for the critical region of DC H walls (especially no. 1 above concerning $V_{Rd,max}$), in conjunction with the magnification of shear forces from the analysis per Eqs. (5.54) in Section 5.6.2.1, make it difficult to verify DC H walls in shear. It is normally not very effective to increase the web thickness, b_{wo}, in order to meet this verification: this will increase proportionally the value of $V_{Rd,max}$, but will also increase, albeit less than proportionally, the seismic shear force from the analysis. It is much more effective to keep the ratio $M_{Rd,o}/M_{Ed,o}$ at the wall base as low as possible, preferably close to 1.0 (see discussion after Eq. (5.54b) in Section 5.6.2.1).

5.7 Detailing for ductility

5.7.1 "Critical regions" in ductile members

Of the two constituents of reinforced concrete, steel is ductile in tension but not in compression, as bars may buckle, shedding their force and risking fracture. Concrete is brittle, unless its lateral expansion is well restrained by confinement. So, the only way to build a RC member which is ductile and can reliably dissipate energy during inelastic seismic response is by combining:

- reinforcing bars in the direction where tensile principal stresses are expected to develop; and
- concrete and reinforcement in the direction of compressive principal stresses, with dense ties to laterally confine the concrete and restrain the bars against buckling.

This is feasible wherever principal stresses and strains develop during the seismic response invariably in the directions where reinforcement can be conveniently placed. In one-dimensional RC members (beams, columns, slender walls), it is convenient to place the reinforcement in the longitudinal and transverse directions. Cyclic flexure indeed produces at the extreme fibres of a RC member principal stresses and strains in the longitudinal direction and allows effective use of reinforcement, both to take up directly the tension and to restrain the concrete and the compression bars transverse to their compressive stresses. Flexure is the only mechanism of force transfer in such a member which allows using to advantage and reliably the ductility of tension reinforcement and effectively enhancing the ductility of concrete and of compression bars through lateral restraint. The regions of the member dominated by flexure under seismic loading are its ends, where the seismic moments take their maximum value. After flexural yielding of the end section, a flexural plastic hinge develops there, dissipating energy in alternate positive and negative bending. Eurocode 8 calls this region “critical region”, which has a more conventional connotation than the term “dissipative zone”, used also in Eurocode 8 for the part of a member or connection of any material where energy dissipation takes place by design.

A “critical region” is a conventionally defined part of a primary RC member, up to a certain distance from:

1. The base section of a ductile wall, i.e., at the connection to the foundation or the top of a rigid

basement.

2. An end of a column or beam connected to a beam or a vertical element, respectively, no matter whether the relative magnitude of the moment resistances of the members around the connection show that a plastic hinge at that end is likely. Cantilevering beams not designed for a vertical seismic action, or a beam end supported on a girder at a distance from a joint of the girder with a vertical member, cannot develop large seismic moments; so there is no beam "critical region" in those cases.
3. A beam section where the hogging moment from the analysis for the seismic design situation attains its maximum value along the span; often that section is at the beam end or nearby and the "critical region" coincides with one of those described under 2 above.

The length of "critical regions" prescribed for RC members by Eurocode 8 is given at the top of Tables 5.1, 5.4 and 5.5. These tables give the special detailing rules – mostly prescriptive – that apply in these regions. Sections 5.7.4 to 5.7.5 below focus on and elaborate the application of those detailing rules which have a rational basis.

5.7.2 Material requirements

Ductility depends not only on the detailing of RC members, but also on the ductility and quality of their materials. So the requirements of Eurocode 8 on concrete and steel increase with DC.

Eurocode 8 sets a lower limit of 16 MPa on the nominal cylindrical strength of concrete (concrete class C16/20) in primary elements of DC M buildings, or of 20 MPa (concrete class C20/25) in those of DC H. These limits are at the low end of what is normally used in buildings in Europe. Neither Eurocode 8 nor Eurocode 2 set a lower limit on concrete strength in DC L buildings. All concrete classes foreseen in Eurocode 2 are allowed by Eurocode 8: there is no upper limit for any DC.

The requirements of Eurocode 8 on reinforcing steel are summarised in Table 5.6. The lower limits on the 10%-fractile of the hardening ratio, f_t/f_y, and of the strain at maximum stress (also called tensile strength, f_t), ε_{su}, ensure a minimum extent of the flexural plastic hinge and a minimum curvature ductility, respectively. The aim of the upper limits on f_t/f_y and on the 95%-fractile of the actual yield stress is to avoid flexural overstrength at plastic hinges beyond what is covered by the overstrength factors γ_{Rd} in Eqs. (5.31), (5.42), (5.44) and (5.54) and avoid jeopardising the capacity design of columns in flexure and of beams, columns or walls in shear, respectively.

The requirements for DC M or L buildings are met by steel bars of Class B or C per Eurocode 2. The requirements for DC H in the last two rows of Table 5.6 are met by steel of Class C of Eurocode 2, but not by class B. That on $f_{yk,0.95}/f_{yk}$ for DC H comes from Eurocode 8 and is not automatically met by steels of class C (let alone B); however, steel types with special ductility produced and used in the most seismic-prone part of Europe do meet this additional requirement. Note that this requirement is sometimes violated, when a quantity of steel, which is originally produced for a certain nominal yield strength but fails the minimum criteria on its f_{yk}-value as 5%-fractile, is then re-classified and marketed as steel of lower nominal yield strength.

Table 5.6 EC8 requirements for reinforcing steel in primary members

Ductility Class	**DC L or M**	**DC H**
5%-fractile of yield strength (: nominal yield strength), f_{yk}	400 to 600 MPa	
95%-fractile of actual yield strength to nominal, $f_{yk,0.95}/f_{yk}$	-	$\leq$1.25
10%-fractile of the ratio of tensile strength (maximum stress) to the yield strength, $(f_t/f_y)_{k,0.10}$	$\geq$1.08	$\geq$1.15 $\leq$1.35
10%-fractile of strain at maximum stress, $\varepsilon_{su,k,0.10}$	$\geq$ 5%	$\geq$ 7.5%

Example 5.5 illustrates the categorisation of steel according to the Eurocode 8 criteria for DC L, M or H, on the basis of statistics of the steel properties from samples of bars.

The requirements in Table 5.6 for DC L apply throughout the length of any primary element. Strictly speaking, those for DC M or H apply only within “critical regions”. However, the whole length of a primary member of DC M or H should meet the requirements in the second column of the table, because its local ductility may not be inferior to that of a DC L member in any respect. The additional requirements in the last column apply only to the “critical regions” in DC H buildings. However, it is not practical to implement different material specifications in the “critical region” than over the rest of the element length. So, in practice the requirements on steel in “critical regions” are applied over the whole of a primary element of DC M or H, including the slab it may be working with (as they apply to the slab bars which are parallel to a primary beam and fall within its effective flange width in tension defined in Sect. 5.2.2).

5.7.3 Curvature ductility demand in “critical regions”

Eurocode 8 links the local deformation demand for which a plastic hinge should be detailed to the basic value of the behaviour factor, q_o, applicable to the building's DC and structural system per Table 4.1 in Section 4.6.3. Only in few cases are the values of q_o in Eurocode 8 discrete: for inverted pendulum or torsionally flexible systems and for wall systems of DC M; in all other DC M or H systems, q_o is proportional to α_u/α_1 (see Section 4.6.3), hence takes values in a continuous range. Therefore, it is not feasible to specify discrete values of the curvature ductility factor, μ_φ, for these other structural systems. So, Eurocode 8 gives μ_φ as an algebraic function of q_o (see Eqs. (5.64), below). This expression is derived from:

1. The q-μ-T relation between the global displacement ductility factor, μ_δ, the ductility dependent part of the behaviour factor, q_μ, and the period, T, of a SDOF oscillator adopted in Eurocode 8 (Eqs. (3.119), (3.120) in Section 3.2.3);
2. The approximate equality, $\mu_\theta \approx \mu_\delta$, of μ_δ to the local ductility factor of chord rotation, μ_θ, at those member ends where plastic hinges form in a beam-sway mechanism imposed on the structural system by a stiff/strong spine provided by the walls of wall- or wall-equivalent dual systems, or by

the strong columns of frame- or frame-equivalent dual systems (see Section 4.5.2 and Figs 2.9(b) to (e)).

3. A safe-sided approximation of the curvature ductility factor at the member's end section, μ_φ, in terms of μ_θ, underlain by the Eurocode 2 model for confined concrete and a safe-sided average plastic hinge length, L_{pl}, equal to 18.5% of the shear span (M/V-ratio), L_s, at the end section of a typical RC member in buildings:

$$\mu_\theta = 1 + 3\frac{L_{pl}}{L_s}\left(1 - \frac{L_{pl}}{2L_s}\right)(\mu_\varphi - 1) \approx 1 + 0.5(\mu_\varphi - 1) \quad \rightarrow \quad \mu_\varphi = 2\mu_\theta - 1 \tag{5.63}$$

4. A safe-sided assumption that the full basic value of the behaviour factor is due to ductility, neglecting overstrength: $q_o = q_\mu$.

By combining 1 to 4 above, Eurocode 8 gives the following relation between q_o and μ_φ:

$$\mu_\varphi = 2q_o - 1 \qquad \text{if } T \geq T_C \tag{5.64a}$$

$$\mu_\varphi = 1 + 2(q_o - 1)\frac{T_C}{T} \quad \text{if } T < T_C \tag{5.64b}$$

where T is the first mode period in the vertical plane where bending of the member being detailed takes place and T_C the upper corner period of the constant-spectral-acceleration plateau of the elastic spectrum – cf. Eq.(5.54b). Eqs. (5.64) use the basic value, q_o, instead of the final value, q, of the behaviour factor; q may be less than q_o owing to irregularity in elevation, or other features which may the global ductility capacity for given local ductility capacities (e.g., because of non-uniform distribution of ductility in heightwise irregular buildings).

In ductile walls designed to Eurocode 8, the lateral force resistance – which is the quantity directly related to the q-factor – depends only on the moment capacity of the base section. The ratio M_{Rdo}/M_{Edo} captures the wall overstrength (where M_{Edo} is the moment at the wall base from the analysis for the design seismic action and M_{Rdo} the design value of moment resistance under the corresponding axial force from the analysis). So the behaviour factor value utilised by the wall is $q/(M_{Rdo}/M_{Edo})$. As a result, Eurocode 8 allows to compute μ_φ at the base of individual ductile walls using in Eqs. (5.64) the value of q_o divided by the minimum value of the wall M_{Rdo}/M_{Edo}-ratio in all combinations of the seismic design situation.

Because a less ductile steel of Class B per Eurocode 2 used as longitudinal reinforcement in the “critical region” of a primary element (as indeed allowed in DC M, see Table 5.6) may reduce its flexural ductility, Eurocode 8 requires to use for the detailing of members with Class B steel a value of μ_φ increased by 50% over the one resulting from Eqs. (5.64).

5.7.4 Upper and lower limit on longitudinal reinforcement ratio of primary beams

If the beam cross-section is large, the longitudinal reinforcement may fracture when the concrete cracks, unless it can resist the cracking moment without yielding. In other words, the yield moment should exceed the cracking moment. This condition gives the minimum steel ratio listed at the row 2 of the requirements in Table 5.1 for DC M or H beams (about double the minimum ratio for DC L beams per Eurocode 2). Although the minimum steel ratio applies only to the tension side of the beam, it is prudent to implement it at both top and bottom of every section, because the magnitude of seismic moments and their distribution along the beam are very uncertain.

Recalling the lower limit listed in Table 5.6 for the 10%-fractile margin between the tensile strength, f_t, and the yield stress, f_y, of steel and taking into account that the mean yield stress, f_{ym}, normally exceeds the nominal, f_{yk}, by about 15%, the minimum steel ratio for DC M and H beams in Table 5.1 gives a safety margin against potential steel fracture due to overstrength of the concrete in tension (the 95%-fractile of the concrete tensile strength exceeds f_{ctm} by about 30%, but increases with age much less than the compressive strength).

The upper limit to the steel ratio for DC M or H beams listed at the third row of requirements in Table

5.1 aims at ensuring that the value of μ_φ from Eqs. (5.64) is achieved at the end section. It is derived from:

- the definition of μ_φ as the ratio of: (a) the curvature, φ_u, when the extreme compression fibres reach the ultimate concrete strain per Eurocode 2, $\varepsilon_{cu2} = 0.0035$, to (b) the curvature at yielding, φ_y, taken equal to the semi-empirical value $\varphi_y = 1.54\varepsilon_y/d$ fitted to tests of beams or columns (Fardis 2009); and
- the calculation of φ_u as $\varepsilon_{cu2}/(\xi_u d)$, with ξ_u taken from Eq. (5.39a) in Sect. 5.4.3 for $\omega_{vd} = 0$, $\nu_d = 0$, $\varepsilon_{cu2} = 0.0035$ and $\varepsilon_{c2} = 0.002$.

Example 5.6 at the end of the Chapter illustrates the application of Eqs. (5.64) alongside the calculation of the maximum reinforcement ratio in beams.

The physical meaning behind the maximum top reinforcement ratio is the following: The most likely failure mode of the plastic hinge is crushing of the narrow compression zone at the bottom, in its effort to balance the tension force of the top reinforcement due to hogging bending (see Fig. 2.22(c)). The compression zone is assisted in this task by the bottom reinforcement, with which it shares the force to be balanced. So, the lower the difference between top and bottom reinforcement, the less the burden falling on the concrete (mathematically, the lower the value of ξ_u from Eq. (5.39a)) and its risk of failure.

The upper limit on the top reinforcement ratio is very restrictive at the supports of DC M and H beams, especially if the value of μ_φ is high (notably for the high q_o-values of DC H). The amount of top steel reinforcement which the beam needs in order to satisfy the ULS in bending at the supports in the seismic design situation and EN1990's "persistent and transient design situation" (i.e., under factored gravity loads) is fixed. To accommodate it, without excessively increasing the width of the beam to reduce the top steel ratio, the bottom reinforcement ratio, ρ', should preferably be increased beyond the prescriptive minimum values given at the second row of requirements in Table 5.1 and at its last two rows.

5.7.5 Confining reinforcement in "critical regions" of primary columns

Columns normally have symmetric longitudinal reinforcement: $\omega_{1d} = \omega_{2d}$. Besides, the compression zone also has to resist the compression force, ν_d. So, in a flexural plastic hinge of a column the value of μ_φ from Eqs. (5.64) cannot be achieved in the same way as in a beam, i.e., by reducing ξ_u from Eq. (5.39a) through a reduction in (ρ_1 - ρ_2). Instead, the extreme concrete fibres are allowed to reach their ultimate strain, ε_{cu2} = 0.0035, and spall; the plastic hinge relies thereafter on the enhanced ultimate strain of the confined concrete core inside the hoops, to provide the required value of μ_φ through confinement. The effective mechanical volumetric ratio of confining reinforcement, $a\omega_{wd}$, required in plastic hinges of DC M or H columns is given at the last row of Table 5.4 (see also Notes (4)-(6) thereof) and is repeated here for convenience:

$$a\omega_{wd} = 30\mu_\varphi \varepsilon_{yd} \nu_d \frac{b_c}{b_o} - 0.035 \qquad (5.65a)$$

The mechanical volumetric ratio ω_{wd} is defined as $(\rho_h+\rho_b)f_{ywd}/f_{cd}$, with the transverse reinforcement ratios, ρ_h, ρ_b, normalised to the sides of the confined core to the centreline of the perimeter hoop:

$$h_o = h_c - 2(c+d_{bw}/2), \qquad b_o = b_c - 2(c+d_{bw}/2) \qquad (5.66)$$

where h_c, b_c are the external depth and width of the column section, respectively, c the concrete cover to the outside of the hoop and d_{bw} the hoop diameter. The confinement effectiveness factor, a, is a product of:

- one component, a_s, reflecting the variation of confinement along a column with discrete stirrups, and
- a_n, expressing the assumption that there is no confinement over the part of the section outside parabolic arcs emerging from the centres of adjacent vertical bars laterally restrained at tie corners or cross-tie hooks, at an angle of 45^o to the chord connecting these two bar centres.

For a rectangular section with a perimeter hoop at a centreline spacing of *s*, the confinement effectiveness factor, *a*, is:

$$a = \left(1 - \frac{s}{2b_o}\right)\left(1 - \frac{s}{2h_o}\right)\left(1 - \frac{\sum b_i^2/6}{b_o h_o}\right) \tag{5.67}$$

where the product of the first two terms is a_s and the third term is a_n (with b_i denoting the spacing along the perimeter of adjacent laterally restrained vertical bars, the summation extending over all pairs of such bars and the denominator being the area enclosed by the polygonal line connecting the laterally restrained bar centres, see Fig. 5.9).

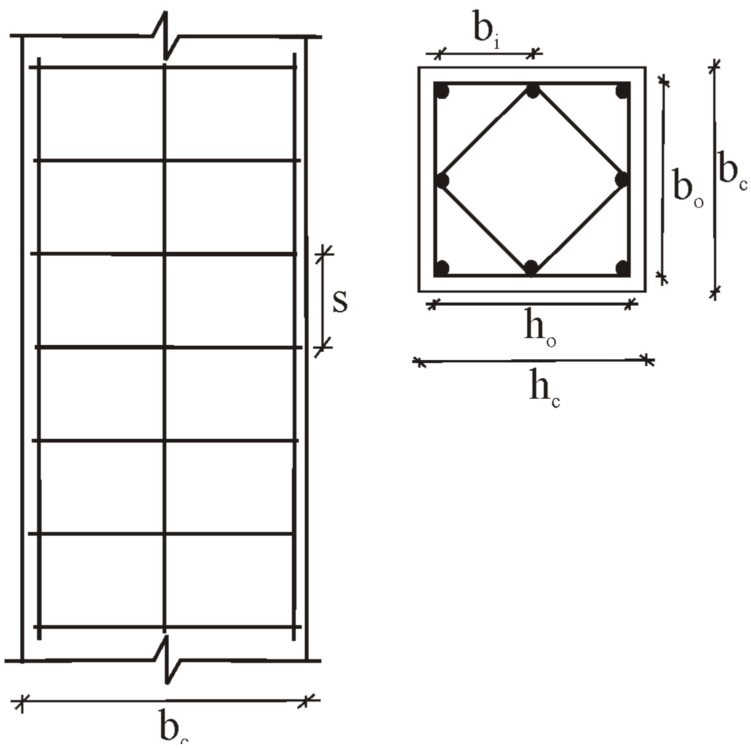

Fig. 5.9 Definition of geometric terms for the confinement of a rectangular column

Example 5.7 at the end of this Chapter illustrates the application of Eqs. (5.64), (5.65a), (5.66), (5.67), while Examples 5.8 and 5.9 demonstrate the definition and calculation of a_n in non-rectangular sections. Finally, Example 5.10 shows alternative layouts of confining reinforcement and compares them in terms of cost-effectiveness.

Although they appear so different, the expression for $a\omega_{wd}$ in Eq. (5.65a) and the maximum steel ratio in beams at the third row of requirements in Table 5.1 are derived similarly; the differences from the two bullet points in Section 5.7.4 are that:

- the full section depth, h, is used, instead of the effective one, d, in nondimensional values, such as $\xi_u = x_u/h$, $\nu_d = N_d/(bhf_{cd})$, etc., in the semi-empirical expression: $\varphi_y = 1.75\varepsilon_y/h$, etc.;
- φ_u is calculated for the confined core, with the ultimate strain of its extreme compression fibres given as a function of $a\omega_{wd}$ according to the confinement model in Eurocode 2, with $\xi_u = x_u/h$ computed from Eq. (5.39a) for $\nu_d = N_d/(bhf_{cd})$, $\omega_{1d} = \omega_{2d}$ and ω_{vd} neglected, compared to ν_d.

The confinement reinforcement per Eq. (5.65a) is required by Eurocode 8 not indiscriminately in every "critical region" of a column, but only where a plastic hinge may form by design, namely at the base of DC M or H columns – at the connection to the foundation or at the top of a rigid basement. In all other "critical regions" of DC M columns, only the prescriptive detailing rules in Table 5.4 for the minimum ω_{wd}-value, the maximum spacing, s_w, and the minimum diameter, d_{bw}, of stirrups apply. In DC H buildings, however, Eurocode 8 requires confining reinforcement per the last row of Table 5.4 in the "critical regions" at all column ends which are not checked per Eq. (5.31) – i.e., those falling into the exemptions from Eq. (5.31) per Eurocode 8, listed herein in Section 5.4.2 – see Notes (7) and (9) in Table 5.4. Besides, some confining reinforcement is also required even in the "critical regions" at the ends of DC H columns which are protected from plastic hinging by meeting Eq. (5.31) in both horizontal directions. That confining reinforcement is computed from Eq. (5.65a), but for a μ_φ-value (denoted in Table 5.4 by μ_φ^*) which is obtained using in Eqs. (5.64) two-thirds of the basic q-factor value, q_o, applicable for the design, instead of the full q_o-value (see Note (7) in Table 5.4).

Wherever it is required, the confining reinforcement should be computed separately in the two directions of bending, using the values of q_o (and hence of μ_φ) applying to the structural system in these two directions and the most unfavourable (i.e., maximum) value of the axial force from the analysis for the seismic design situation. The largest value from these two separate calculations should be used for ω_{wd}. It should be implemented as the sum of the mechanical reinforcement ratios in both transverse directions, $(\rho_h+\rho_b)f_{ywd}/f_{cd}$, providing however about equal transverse reinforcement ratios in both: $\rho_h \approx \rho_b$. If the value of $a\omega_{wd}$ comes out as negative for $b_o = b_c$, then the target value of μ_φ can be achieved by the unspalled section without confinement. In that case the stirrups in the "critical region" may just follow the prescriptive detailing rules of the corresponding DC concerning their minimum ω_{wd}-value, maximum spacing s_w, minimum diameter d_{bw}, etc (see Table 5.4).

For a sample application of this Section at the base of the columns of the 7-storey example building, see Section 7.6.2.2.

5.7.6 Confinement of "boundary elements" at the edges of a wall section

It was pointed out in the second paragraph of Section 5.6.1.2 that the ULS-design and the detailing of a wall as RC member differ from those of columns: the moment resistance of a wall is provided by "tension and compression chords" or "flanges" at the edges of its section; its shear resistance by the "web" in-between. The wall's vertical reinforcement is concentrated in "boundary elements" at the two edges of the section; confinement of concrete is also limited there (see Fig. 5.7 and examples in Figs. 7.45-7.47 of Chapter 7).

Table 5.5 gives in separate sections the detailing rules of Eurocode 8 for the "boundary elements" and the "web". The first part of the "boundary elements" section refers to the critical region; the second, to the rest of the wall height. The rows before the last one in the first part give prescriptive rules for the geometry, the vertical bars and the confining reinforcement of boundary elements; the last row specifies the effective mechanical volumetric ratio of confining reinforcement, $a\omega_{wd}$, in the boundary elements of DC H or M walls as a function of the value of μ_ϕ which corresponds, via Eqs. (5.64), to the product of q_o

times the ratio M_{Edo}/M_{Rdo} at the wall base (see second to last paragraph of Sect. 5.7.3):

$$a\omega_{wd} = 30\mu_{\varphi}\varepsilon_{yd}\left(\nu_d + \omega_{vd}\right)\frac{b_w}{b_o} - 0.035 \tag{5.65b}$$

b_w appears in Eq. (5.65b) instead of b_c, but has the same meaning: it is the external width of the compression flange. Eq. (5.65b) has the same rationale and derivation as Eq. (5.65a) for columns, but includes also the mechanical ratio of vertical bars in the web, $\omega_{vd} = \rho_v f_{yd}/f_{cd}$, as non-negligible compared to ν_d.

Note (3) in Table 5.5 points out that, under certain conditions often met in practice, Eurocode 8 allows to determine the confining reinforcement of boundary elements in DC M walls according to the rules for walls of DC L. As a matter of fact, under the conditions outlined in that Note, Eq. (5.65a) most likely gives a negative outcome for $a\omega_{wd}$ when $b_o = b_w$, i.e., the target μ_φ-value can be achieved at the unspalled section without confinement (cf. last paragraph in Sect. 5.7.5).

Above the "critical region" of DC M or H walls, DC L rules apply for the confining reinforcement of boundary elements, as well as for their geometry and vertical bars. They come from Eurocode 2 and essentially require smaller boundary elements around any edge region of the section in which the vertical bars give a local vertical steel ratio above 2%. Such a region should be enclosed by hoops, following the prescriptive rules in Eurocode 2 concerning hoop diameter and spacing; these rules are very much relaxed compared to the ones of Eurocode 8 for the critical region of DC H or M walls.

The horizontal extent of a confined boundary element in the critical region may be limited to the part of the section where, when the wall reaches its ultimate deformation, the concrete strain exceeds the ultimate strain of unconfined concrete per Eurocode 2, i.e., $\varepsilon_{cu2} = 0.0035$. The hoop enclosing a boundary element should have a centreline length of $x_u(1-\varepsilon_{cu2}/\varepsilon_{cu2,c})$ in the direction of the wall length, l_w ($= h_c$), with the neutral axis depth after concrete spalling, x_u, estimated as :

$$x_{\mathrm{u}} = \left(\nu_{\mathrm{d}} + \omega_{\nu d}\right)\frac{h_c b_{\mathrm{c}}}{b_{\mathrm{o}}} \tag{5.68}$$

and the ultimate strain of unconfined concrete, $\varepsilon_{\mathrm{cu2,c}}$, estimated per Eurocode 2 as:

$$\varepsilon_{\mathrm{cu2,c}} = 0.0035+0.1a\omega_{\mathrm{wd}} \tag{5.69}$$

using the actual value of $a\omega_{\mathrm{wd}}$ in the boundary element. In Eq. (5.68), b_{c}, h_{c} are the same as the wall's b_{w}, l_{w}, respectively. The overall length of the confined boundary element includes the concrete cover and the perimeter hoop:

$$l_c \geq x_{\mathrm{u}}\left(1-\frac{\varepsilon_{\mathrm{cu2}}}{\varepsilon_{\mathrm{cu2,c}}}\right)+2\left(c+\frac{d_{\mathrm{bw}}}{2}\right) \tag{5.70}$$

The final value of l_{c} should respect the minimum values at the first row of part (a) "critical regions" in the "boundary elements" section of Table 5.5.

For a sample application of this Section to the critical height of one wall of the 7-storey example building, see Section 7.6.2.4

5.7.7 Confinement of wall or column sections with more than one rectangular parts

Wall or column sections often consist of several rectangular parts: sections with T-, L-, I-, U-shape, walls with "barbells" at the edges of the section, etc. For such sections ω_{wd} should be determined separately for each rectangular part of the section which may play the role of a compression flange under any direction of the seismic action. Eqs. (5.65) should first be applied using the external width of the compression flange at the extreme compression fibres as b_{c} in Eq. (5.65a), or as b_{w} in Eq. (5.65b). This applies also to the normalisation of N_{Ed}, and of the area of vertical reinforcement between the tension and compression flanges, A_{sv}, as $\nu_{\mathrm{d}} = N_{\mathrm{Ed}}/(h_{\mathrm{c}}b_{\mathrm{c}}f_{\mathrm{cd}})$, $\omega_{\mathrm{vd}} = A_{\mathrm{sv}}/(h_{\mathrm{c}}b_{\mathrm{c}})f_{\mathrm{yd}}/f_{\mathrm{cd}}$, with h_{c} denoting the maximum dimension of the unspalled section at right angles to b_{c} (as if the section were rectangular, with width b_{c} and depth h_{c}). For this to apply, the compression zone should be limited within the compression flange, whose width is b_c. To check if this is the case, the neutral axis depth, x_u, at the ultimate curvature after the concrete cover spalls at the compression flange is computed from Eq. (5.68). The outcome is then compared to the dimension of the rectangular compression flange at right angles to b_c (i.e., parallel to h_c), after reducing it by (c+d_{bw}/2) for spalling. If this reduced dimension exceeds x_u, the outcome of Eqs. (5.65) for ω_{wd} is implemented by placing stirrups in the compression flange in question. About equal stirrup ratios should preferably be provided in both directions of this compression flange. However, what mainly counts in this case is the steel ratio of the stirrup legs at right angles to b_c. If the value of x_u from Eq. (5.68) exceeds the dimension of the compression flange at right angles to b_c by much more than (c+d_{bh}/2), there are two practical options:

1. To physically increase the dimension of the rectangular compression flange at right angles to b_c, so that, after its reduction by (c+d_{bw}/2) due to spalling, it exceeds x_u from Eq. (5.68).
2. To confine the rectangular part of the section at right angles to the compression flange (the "web"), instead of the compression flange itself. This is meaningful only if the compression flange for which the neutral axis depth has first been calculated from Eq. (5.68) is shallow and not much wider than the "web". Eqs. (5.65) should be then applied with a width b_c or b_w equal to the thickness of the "web" (also in the normalisation of N_{Ed} and A_{sv} into ν_d, ω_{vd}). The outcome of Eqs. (5.65) for ω_{wd} should be implemented through stirrups in the "web". It is consistent with this choice to sacrifice the compression flange by placing in its parts outside the "web" transverse reinforcement meeting only the prescriptive rules for stirrup spacing and diameter, without confinement requirements. It is more prudent, though, to place in them the same confining reinforcement as in the "web".

Example 5.11 at the end of the Chapter demonstrates the relationship between confining steel at key points of a wall section and the available ductility factor per Eurocode 8.

What has been said so far in this Section covers both walls and columns with composite section. For walls of this type, Eurocode 8 requires a rigorous approach, namely to go to the fundamentals (equilibrium, σ-ε laws for steel and confined concrete per Eurocode 2, etc), to provide the required value of $\mu_\varphi=\varphi_u/\varphi_y$. The level of safety provided by Eqs. (5.65) should be maintained.

5.8 Dimensioning for vectorial action effects due to concurrent seismic action components

5.8.1General approaches

Beams are dimensioned in flexure or shear for scalar internal forces/action effects, i.e., bending moment or shear force, respectively. By contrast, columns and walls are dimensioned for uniaxial (or biaxial) bending with axial force and for uniaxial shear with axial force, i.e., for two or three concurrent internal forces. Let's consider these internal forces as arranged in a vector: $[M_y, M_z, N]^T$ for biaxial bending with axial force, $[M, N]^T$, or $[V, N]^T$, for uniaxial bending or shear with axial force, respectively. If it was the result of a single seismic action component, that vector would be added to and subtracted from its counterpart due to the quasi-permanent actions; dimensioning or verification would be carried out separately for the vector sum and the vector difference. The question is how do we combine the vectors of peak responses predicted through linear analysis for the individual seismic action components, notably the horizontal ones X and Y, when we know that these peak responses are not simultaneous?

Let's consider biaxial bending, with the vectors of seismic action effects produced by the horizontal components X and Y denoted as:

$$\boldsymbol{E}_X = [M_{y,X}, M_{z,X}, N_X]^T,\ \boldsymbol{E}_Y = [M_{y,Y}, M_{z,Y}, N_Y]^T \tag{5.71}$$

Eq. (3.99) gives the expected peak values of individual internal force components due to concurrent horizontal seismic action components, X and Y:

$$\pm M_{y,\max} = \pm\sqrt{M_{y,X}^2 + M_{y,Y}^2}\ ,\ \pm M_{z,\max} = \pm\sqrt{M_{z,X}^2 + M_{z,Y}^2}\ ,\ \pm N_{\max} = \pm\sqrt{N_X^2 + N_Y^2} \tag{5.72}$$

Their counterparts from Eq. (3.100) are:

$$M_{y,\max} = \pm\max\left[\left(|M_{y,X}| + \lambda|M_{y,Y}|\right);\left(\lambda|M_{y,X}| + |M_{y,Y}|\right)\right] \tag{5.73a}$$

$$M_{z,\max} = \pm\max\left[\left(|M_{z,X}| + \lambda|M_{z,Y}|\right);\left(\lambda|M_{z,X}| + |M_{z,Y}|\right)\right] \tag{5.73b}$$

$$N_{\max} = \pm\max\left[\left(|N_X| + \lambda|N_Y|\right);\left(\lambda|N_X| + |N_Y|\right)\right] \tag{5.73c}$$

It is physically implausible and over-conservative to assume that the maxima of the three internal forces in Eq. (5.72) – or Eq. (5.73) – take place concurrently as:

$$\boldsymbol{E} = [\pm M_{\mathrm{y,max}}, \pm M_{\mathrm{z,max}}, \pm N_{\mathrm{max}}]^{\mathrm{T}} \tag{5.74}$$

where $M_{\mathrm{y,max}}$, $M_{\mathrm{z,max}}$, N_{max} are given by Eqs. (5.72) or (5.73). Nevertheless, Eq. (5.74) is commonly used in practice. Alternative, more plausible combinations are described in (Fardis 2009); they depend on the type of linear analysis carried out. Another question concerns the permutations of signs among the three internal forces. Modal response spectrum analysis always gives positive results, taken with plus and minus sign. By contrast, the lateral force method gives results with signs; so, when the sign of the seismic action is reversed, all internal forces change sign: internal forces with the same sign keep having the same sign; those with opposite signs stay with opposite signs.

A simple approximation is suggested below as an alternative to the eight combinations of Eq. (5.74). Strictly speaking, it does not have a rigorous basis, but is rational and gives reasonable results, close to those from the rigorous approaches highlighted in (Fardis 2009). For the general case of biaxial bending, Eqs. (5.71), this alternative includes sixteen combinations:

$$[\pm M_{\mathrm{y,max}}, \pm\lambda M_{\mathrm{z,max}}, \pm N_{\mathrm{max}}]^{\mathrm{T}} \tag{5.75a}$$

$$[\pm\lambda M_{\mathrm{y,max}}, \pm M_{\mathrm{z,max}}, \pm N_{\mathrm{max}}]^{\mathrm{T}} \tag{5.75b}$$

The vector of internal forces due to the translational seismic action components, X and Y, is superimposed to the vector due to the torques produced by the accidental eccentricities of both horizontal components (see Section 3.1.8). As this latter vector is computed via static analysis, its components have signs, which may be reversed all together but not individually. If the combination of components retains the signs of individual action effects, the superposition takes place with signs, such that the internal force which is maximised in the vector due to the translational components is superimposed to its counterpart due to the torques from accidental eccentricities with the same sign; the signs of the other components of the vector due to the accidental eccentricities follow suit, so that they are the same or opposite to each other, in line with how they came out from the static analysis. This is illustrated in Example 5.12.

5.8.2 Implications for the column axial force values in capacity design calculations

The value of the column moment resistance, $M_{Rd,c}$, used in capacity-design calculations should be based on a safe-sided, yet meaningful value of the column axial force, N, within the range of values from the analysis for the combination of the design seismic action with the quasi-permanent loads. More specifically:

1. For the strong column-weak beam capacity design of Eq. (5.31), the minimum compressive or maximum tensile axial force in the column should be used.
2. For the capacity design shear of beams, Eqs. (5.42), we use the maximum compressive axial force in the columns connected to the beam.
3. For the capacity design shear of the column itself, Eq. (5.44), we are interested both in the maximum compressive and the maximum tensile (or minimum compressive) axial force in the column.
4. For the capacity design of the foundation system and the bearing capacity verification of the soil, both the maximum compressive and the maximum tensile (or minimum compressive) force in the column are of interest (see Section 6.3.2).

The maximum or minimum compressive axial forces in the seismic design situation come from the maximum compressive and maximum tensile axial force, respectively, from the analysis for the seismic action.

In principle, the value used for N should be consistent with the sense of action (sign) of $M_{Rd,c}$. For example, for response dominated by the first mode in a given horizontal direction, flexural plastic hinges at the base of columns normally have tension at the "windward" side of the column and compression at the opposite; the reverse normally holds in plastic hinges at column tops. On the other hand, the first mode dominated response induces tensile axial forces at the top and bottom of exterior columns of the "windward" side and compressive ones in those of the "leeward" side.

The controlling moment component is the one for which a plastic hinge forms, let's say M_y. The other moment component is not of interest. If $M_{Rd,c}$ is conventionally taken as positive, M_y is considered positive if it has the same sense of action as $M_{Rd,c}$. N is taken positive if it is compressive.

1. $\boldsymbol{E}_X$, $\boldsymbol{E}_Y$ are combined via Eq. (3.99): The maximum compressive or tensile seismic force is:

$$\pm\max N_E = \pm\sqrt{(N_X^2+N_Y^2)} \tag{5.76}$$

   These values are used if the sense (sign) of the bending moments doesn't make a difference to the value of $\sum M_{Rd,b}$. If it does, assuming that the plastic hinge forms in the direction of M_y, the outcome of Eq. (5.76) is multiplied by $M_{Rd,c}/M_{y,max}$. Another physically meaningful option is to take the magnitude of N as $\sqrt{(N_X^2+N_Y^2)}$ and use the sign of the axial force in the mode with the largest contribution to the moment in the direction of $M_{Rd,c}$ when that contribution has the same sense (sign) as $M_{Rd,c}$.

2. $\boldsymbol{E}_X$, $\boldsymbol{E}_Y$ are combined through Eq. (3.100). If modal analysis is used, or, if the lateral force method is applied but the sense (sign) of bending moments does not make a difference for the beam moment resistance sums, $\sum M_{Rd,b}$, the maximum compressive force in the column is:

$$\max N_E = \max[(|N_X|+\lambda|N_Y|); (|N_Y|+\lambda|N_X|)] \tag{5.77}$$

   The maximum tensile force is given by the same expression but with a minus sign. If $\boldsymbol{E}_X$, $\boldsymbol{E}_Y$ are computed separately by the lateral force method of analysis, and, in addition, the sense (sign) of bending moments makes a difference to the beam moment resistance sums, $\sum M_{Rd,b}$, then the maximum compressive force in the column is taken as:

$$\max N_E = \max[(\text{sign}(M_{y,X}N_X)N_X+\text{sign}(M_{y,Y}N_Y)\lambda N_Y); (\text{sign}(M_{y,Y}N_Y)N_Y+\text{sign}(M_{y,X}N_X)\lambda N_X)] \tag{5.78a}$$

while the maximum tensile (minimum compressive) force is:

$$\min N_E = \min[(\text{sign}(M_{y,X}N_X)N_X+\text{sign}(M_{y,Y}N_Y)\lambda N_Y); (\text{sign}(M_{y,Y}N_Y)N_Y+\text{sign}(M_{y,X}N_X)\lambda N_X)] \tag{5.78b}$$

The column axial forces due to the accidental eccentricities of both horizontal components (see Section 3.1.8) are added to the extreme seismic axial force determined as highlighted above, with the same sign (i.e., as tensile for minimum N, or compressive for maximum N).

5.9 "Secondary seismic elements"

5.9.1 Special design requirements for "secondary" members and implications for the analysis

The contribution of "secondary" members to lateral stiffness is meant to be neglected in the seismic response analysis from which the seismic action effects for the verification of "primary" members are computed. On the other hand, Eurocode 8 imposes two special requirements on "secondary" members, which require special calculations and verifications:

1. The total contribution to lateral stiffness of all "secondary" members must be less than or equal to 15% of that of all "primary" ones.
2. "Secondary" members must remain elastic under the displacements and deformations imposed on them in the seismic design situation.

In order to check condition no. 1, but also to estimate the deformations imposed on "secondary" members in the seismic design situation, the designer needs to carry out two linear analyses per horizontal component of the seismic action:

a. one including the contribution of "secondary" members to lateral stiffness, and

b. another one neglecting it.

For condition no. 1 to be met, the (inter)storey drifts computed from analysis (b) should be less than 1.15 times those from analysis (a). Note that it is on the basis of the results of analysis (b) that "primary" members are designed and that all the verifications per Eurocode 8 which do not concern "secondary" members are carried out (including the damage limitation checks on the basis of interstorey drifts due to the damage limitation seismic action, see Section 1.3.2). On the other hand, a structural model which includes the contribution of "secondary" members to lateral stiffness is essential for the design of these members against combinations of actions which include other lateral loadings, for example, if the building is also designed for wind. Besides, the same model can be used for the analysis under factored gravity loads ("persistent and transient design situation"). Finally, the results of an analysis of type (a) can be used to estimate the deformations imposed on "secondary" members in the seismic design situation (see next section).

5.9.2 Verification of "secondary" members in the seismic design situation

According to Eurocode 8, the design moment and shear resistances of "secondary" members at the ULS per Eurocode 2, M_{Rd} and V_{Rd}, may not be less than the internal forces (bending moments and shears) derived for these members from the deformations imposed by the rest of the system in the seismic design situation, in a seismic response analysis which neglects the contribution of "secondary" members to lateral stiffness. These internal forces are to be derived from the imposed seismic deformations using the cracked stiffness of "secondary" members (i.e., 50% of the gross, uncracked section stiffness). At an extreme limit case, "secondary" members must be designed for seismic action effects derived with a q-factor of 1/1.15=0.87! To meet this onerous requirement, the lateral stiffness of "secondary members" should indeed be very low and the global stiffness of the system of "primary" members and its connectivity to the "secondary" ones should be such that seismic deformations imposed on the latter are small.

The seismic deformation demands imposed on "secondary" members in the seismic design situation are determined according to the equal displacement rule through a multi-step procedure:

I. The elastic deformation demands in the "secondary" members due to the design seismic action are estimated from a linear seismic analysis of type (a) in the previous section, i.e., including the "secondary" members in the model. The design spectrum is used, i.e., the one divided by the behaviour factor, q, but its deformation results are back-multiplied by q, to estimate the displacements as though the structure were elastic.

II. The outcome of Step I for storey i is multiplied by the ratio of interstorey drifts in that storey from a type (b) linear analysis to those from a type (a) linear analysis. The result is the deformation estimate we seek; it is multiplied by the cracked stiffness of the "secondary" member in order to estimate its internal forces, to be compared to M_{Rd} and V_{Rd} (see Eq. 1.1).

5.9.3 Modelling of "secondary" members in the analysis

In the structural model for the analysis which neglects the contribution of "secondary" members to lateral stiffness (type (b) analysis in Section 5.9.2), "secondary" members should be included only with those of their properties which are essential for their gravity-load-bearing function:

– "Secondary" vertical elements may be included with their axial stiffness only and with zero flexural rigidity, or with moment releases (i.e., hinges) introduced between their ends and the joint they frame into. Such an approximation is acceptable, so long as the seismic axial forces in these members are small. This precludes vertical elements on the perimeter from such modelling (anyway, it is not sound engineering practice to consider such members as "secondary").

– "Secondary" beams directly supported on vertical elements and continuous over two or more spans should be modelled with their flexural stiffness as prescribed by Eurocode 8 for "primary" members (i.e., 50% of the uncracked, gross section stiffness). Their connectivity with the vertical elements depends on whether the latter are also "secondary" or not; if they are, zero flexural rigidity of these "secondary" vertical members, or moment releases (hinges) at their connections with the beam-column joint are satisfactory also for the "secondary" beams supported on them. If the vertical elements are "primary", then two separate nodes may be introduced at interior beam-column joints, with pin connection between them: one node on the beam and another on the vertical element; the beam and the vertical element that continue past the joint will resist the gravity loads or the seismic action, respectively, with their flexural stiffness per Eurocode 8 (50% of the uncracked, gross section stiffness); moment releases (hinges) in the beam may be used at joints where the beam terminates (this includes single-span "secondary" beams).
– "Secondary" beams not directly supported on vertical elements (e.g., supported on girders) may be included in the model with their full flexural stiffness and connectivity, because their seismic action effects are negligible anyway.

Note that using different structural models in the analyses of type (a) and (b) is inconvenient, if analysis and design take place in an integrated computational environment; the design modules will have to receive analysis results for the same or different members from the two types of analysis, and combine/modify them appropriately. The alternative, namely to use a single model which neglects the contribution of "secondary" members to lateral stiffness (for type (b) analysis), does not allow checking Condition 1 in Section 5.9.1, nor designing the building for other lateral actions, e.g., wind. Moreover, as the chord rotations at the ends of "secondary" members due to the seismic action are not computed from an analysis of type (a), the internal forces in "secondary members" due to their seismic deformation demands can be estimated only by ad-hoc, approximate and onerous procedures (Fardis, 2009), most likely by hand or with spreadsheets.