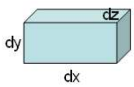
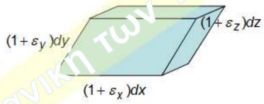
Ανηγμένη διόγκωση και μέτρο διόγκωσης



Λόγω : $\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zy}\end{pmatrix}$

! οι διατμητικές δεν επιφέρουν μεταβολή όγκου παρά μόνο σχήματος !



$$\Delta V=(1+\varepsilon_x)dx\cdot(1+\varepsilon_y)dy\cdot(1+\varepsilon_z)dz-dxdydz\approx(\varepsilon_x+\varepsilon_y+\varepsilon_z)\underbrace{dxdydz}_{V}$$

$$(\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)<<$$

$$e=\frac{\Delta V}{V}=\varepsilon_x+\varepsilon_y+\varepsilon_z$$

Ανηγμένη μεταβολή όγκου

$$\varepsilon_x=\frac{\sigma_x}{E}-\nu\frac{\sigma_y}{E}-\nu\frac{\sigma_z}{E}$$
$$\varepsilon_y=-\nu\frac{\sigma_x}{E}+\frac{\sigma_y}{E}-\nu\frac{\sigma_z}{E}$$
$$\varepsilon_z=-\nu\frac{\sigma_x}{E}-\nu\frac{\sigma_y}{E}+\frac{\sigma_z}{E}$$

Volumetric strain

$$e=\varepsilon_x+\varepsilon_y+\varepsilon_z=\frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z)$$

! η διόγκωση του υλικού είναι ανάλογη του αθροίσματος των ορθών τάσεων !

$$e=\varepsilon_x+\varepsilon_y+\varepsilon_z=\frac{1-2\nu}{E}(\sigma_x+\sigma_y+\sigma_z)$$

"υδροστατική" πίεση: $-p=\sigma_x=\sigma_y=\sigma_z$

$$e=-\frac{3(1-2\nu)}{E}p$$

Bulk modulus

Μέτρο διόγκωσης : $K=\frac{-p}{e}=\frac{E}{3(1-2\nu)}$

Το **μέτρο διόγκωσης** του υλικού ορίζεται ως ο λόγος της υδροστατικής πίεσης προς την ανηγμένη μεταβολή όγκου.

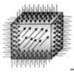
$$\Sigma=\begin{pmatrix}\sigma_x&\tau_{xy}&\tau_{xz}\\ \tau_{yx}&\sigma_y&\tau_{yz}\\ \tau_{zx}&\tau_{zy}&\sigma_z\end{pmatrix}\qquad E=\begin{pmatrix}\varepsilon_x&\frac{\gamma_{xy}}{2}&\frac{\gamma_{xz}}{2}\\ \frac{\gamma_{yx}}{2}&\varepsilon_y&\frac{\gamma_{yz}}{2}\\ \frac{\gamma_{zx}}{2}&\frac{\gamma_{zy}}{2}&\varepsilon_z\end{pmatrix}$$

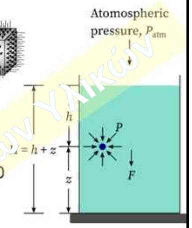
I_1^{Σ} = Trace (Σ) = const.

I_1^{E} = Trace (E) = const.

Bulk modulus

Μέτρο διόγκωσης : $K=\boxed{\frac{-p}{e}}=\frac{E}{3(1-2\nu)}$





$$\boxed{e=\frac{\Delta V}{V}<0}\Rightarrow K>0\Rightarrow\frac{E}{3(1-2\nu)}>0\Rightarrow$$
$$\Rightarrow 3(1-2\nu)>0\Rightarrow(1-2\nu)>0$$
$$\Rightarrow \nu<0.5$$

$\nu=0.0\quad e=-\frac{3}{E}p\quad K=\frac{E}{3}$

$\nu=0.5\quad e=0\quad K=\infty$

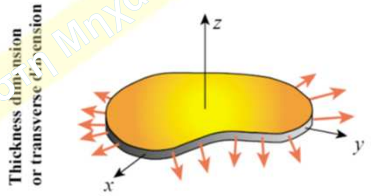
Απολύτως ασυμπίεστα υλικά – Perfectly incompressible materials
(perfectly incompressible isotropic material deformed elastically at small strains; e.g. rubber)

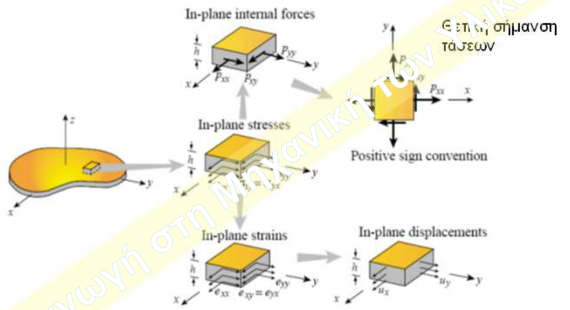
Επίπεδη εντατική κατάσταση
(Plane Stress)

Μηδενικές οι ορθές και οι διατμητικές τάσεις που αναπτύσσονται κάθετα ως προς το επίπεδο του σώματος.

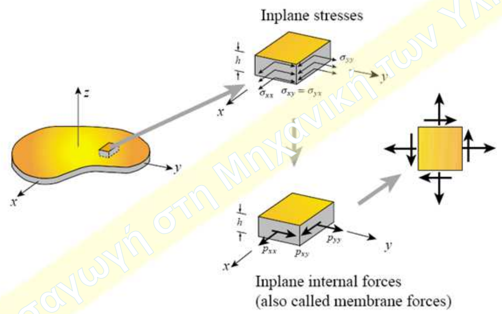
Plate in Plane Stress

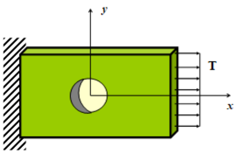
$\sigma_z=\tau_{xz}=\tau_{yz}=0$
$\varepsilon_z\neq0$





αλλά και $\varepsilon_z\neq0$





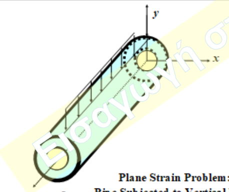
Plane Stress Problem:
Plate with a Hole

Plane Stress Problem:
Plate with a Fillet

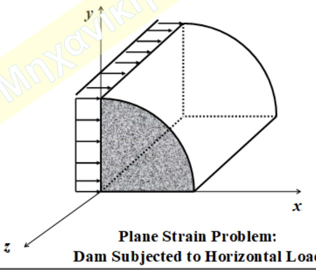
Επίπεδη παραμόρφωση
(Plane Strain)

Μηδενικές οι ορθές και οι διατμητικές παραμορφώσεις που αναπτύσσονται κάθετα ως προς το επίπεδο του σώματος.

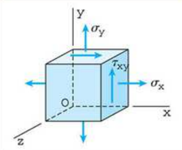
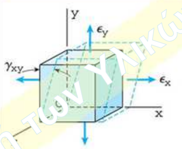
$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$
$\sigma_z \neq 0$



Plane Strain Problem:
Pipe Subjected to Vertical Load



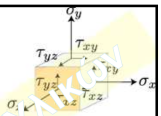
Plane Strain Problem:
Dam Subjected to Horizontal Load

	Plane stress	Plane strain
		
Stresses	$\sigma_z = 0$ $\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, and τ_{xy} may have nonzero values	$\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, σ_z, and τ_{xy} may have nonzero values
Strains	$\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, ϵ_z, and γ_{xy} may have nonzero values	$\epsilon_z = 0$ $\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, and γ_{xy} may have nonzero values

Γενικευμένος νόμος του Hooke –

Γενικευμένη τριοδιάστατη εντατική κατάσταση

3D Stress State



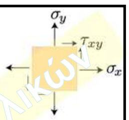
$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$
$$\gamma_{yz} = \frac{1}{G}\tau_{yz}$$
$$\gamma_{zx} = \frac{1}{G}\tau_{zx}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)]$$
$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z)]$$
$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)]$$
$$\tau_{xy} = G\,\gamma_{xy}$$
$$\tau_{yz} = G\,\gamma_{yz}$$
$$\tau_{zx} = G\,\gamma_{zx}$$

Γενικευμένος νόμος του Hooke –

Επίπεδη εντατική κατάσταση

Plane Stress



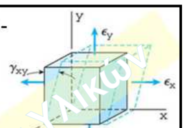
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$
$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$
$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$
$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$
$$\tau_{xy} = G\,\gamma_{xy}$$

Γενικευμένος νόμος του Hooke -

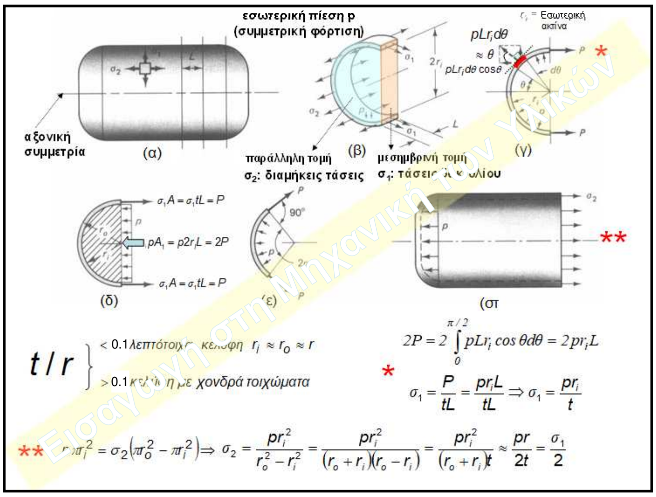
Επίπεδη εντατική κατάσταση

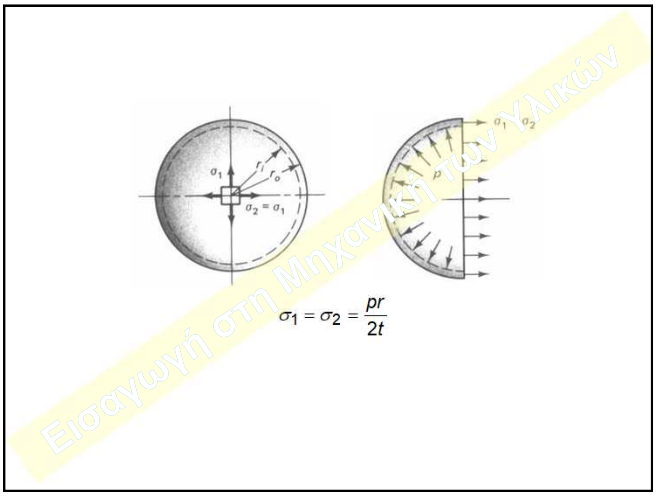
Plane Strain

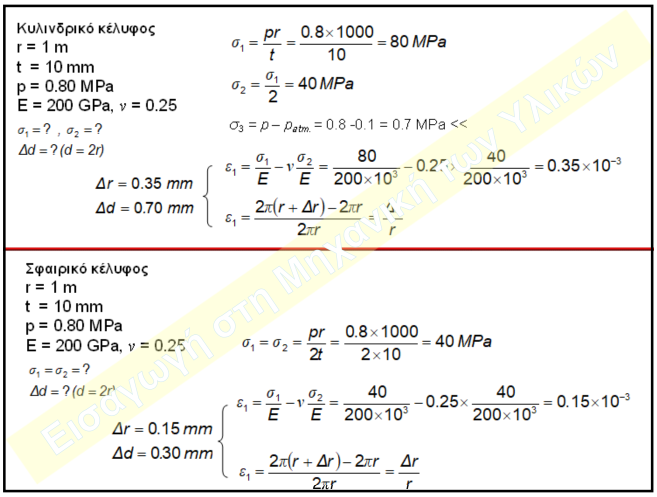


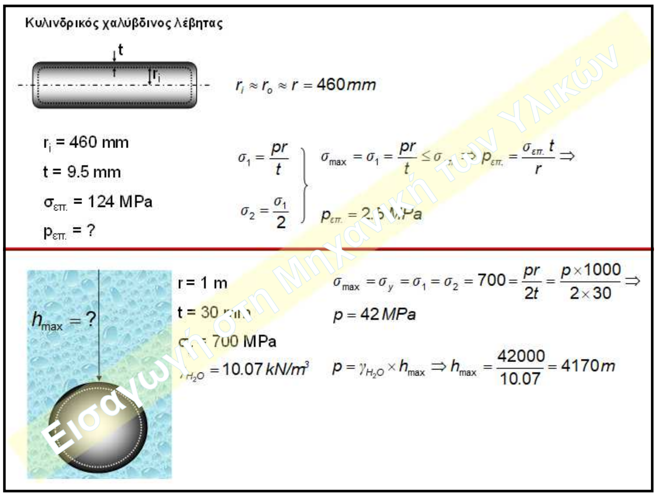
$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

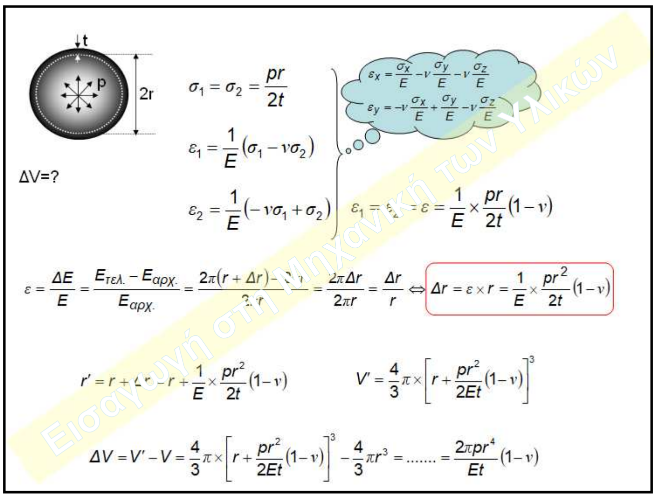
$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu\varepsilon_y]$$
$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_y + \nu\varepsilon_x]$$
$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}[\nu(\varepsilon_x + \varepsilon_y)]$$
ή $\sigma_z = \nu(\sigma_x + \sigma_y)$
$$\tau_{xy} = G\,\gamma_{xy}$$


εσωτερική πίεση p (συμμετρική φόρτιση)
αξονική συμμετρία
(α)
παράλληλη τομή
σ_2: διαμήκεις τάσεις
(β)
μεσημβρινή τομή
(γ)
(δ)
(ε)
$\sigma_1 A = \sigma_1 tL = P$
$pA_1 = p2r_iL = 2P$
t/r < 0.1 λεπτότοιχο κέλυφος $r_i \approx r_o \approx r$
> 0.1 κέλυφος με χονδρά τοιχώματα
$2P = 2\int_0^{\pi/2} pLr_i \cos\theta d\theta = 2pr_iL$
$\sigma_1 = \frac{P}{tL} = \frac{pr_iL}{tL} \Rightarrow \sigma_1 = \frac{pr_i}{t}$
$p\pi r_i^2 = \sigma_2(\pi r_o^2 - \pi r_i^2) \Rightarrow \sigma_2 = \frac{pr_i^2}{r_o^2 - r_i^2} = \frac{pr_i^2}{(r_o + r_i)(r_o - r_i)} = \frac{pr_i^2}{(r_o + r_i)t} \approx \frac{pr}{2t} = \frac{\sigma_1}{2}$


$\sigma_1 = \sigma_2 = \frac{pr}{2t}$


Κυλινδρικό κέλυφος
r = 1 m
t = 10 mm
p = 0.80 MPa
E = 200 GPa, ν = 0.25
$\sigma_1 = ?$, $\sigma_2 = ?$
$\Delta d = ? (d = 2r)$
$\sigma_1 = \frac{pr}{t} = \frac{0.8 \times 1000}{10} = 80\, MPa$
$\sigma_2 = \frac{\sigma_1}{2} = 40\, MPa$
$\sigma_3 = p - p_{atm} = 0.8 - 0.1 = 0.7$ MPa <<
$\Delta r = 0.35\, mm$
$\Delta d = 0.70\, mm$
$\varepsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \frac{80}{200 \times 10^3} - 0.25 \times \frac{40}{200 \times 10^3} = 0.35 \times 10^{-3}$
$\varepsilon_1 = \frac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \frac{\Delta r}{r}$
Σφαιρικό κέλυφος
r = 1 m
t = 10 mm
p = 0.80 MPa
E = 200 GPa, ν = 0.25
$\sigma_1 = \sigma_2 = ?$
$\Delta d = ? (d = 2r)$
$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{0.8 \times 1000}{2 \times 10} = 40\, MPa$
$\Delta r = 0.15\, mm$
$\Delta d = 0.30\, mm$
$\varepsilon_1 = \frac{\sigma_1}{E} - \nu\frac{\sigma_2}{E} = \frac{40}{200 \times 10^3} - 0.25 \times \frac{40}{200 \times 10^3} = 0.15 \times 10^{-3}$
$\varepsilon_1 = \frac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \frac{\Delta r}{r}$


Κυλινδρικός χαλύβδινος λέβητας
$r_i \approx r_o \approx r = 460 mm$
r_i = 460 mm
t = 9.5 mm
$\sigma_{επ.}$ = 124 MPa
$p_{επ.}$ = ?
$\sigma_1 = \frac{pr}{t}$
$\sigma_2 = \frac{\sigma_1}{2}$
$\sigma_{max} = \sigma_1 = \frac{pr}{t} \le \sigma_{επ.} \Rightarrow p_{επ.} = \frac{\sigma_{επ.} t}{r} \Rightarrow$
$p_{επ.} = 2.5\, MPa$
$h_{max} = ?$
r = 1 m
t = 30 mm
σ_y = 700 MPa
$\gamma_{H_2O} = 10.07\, kN/m^3$
$\sigma_{max} = \sigma_y = \sigma_1 = \sigma_2 = 700 = \frac{pr}{2t} = \frac{p \times 1000}{2 \times 30} \Rightarrow$
$p = 42\, MPa$
$p = \gamma_{H_2O} \times h_{max} \Rightarrow h_{max} = \frac{42000}{10.07} = 4170 m$


t
p
2r
ΔV=?
$\sigma_1 = \sigma_2 = \frac{pr}{2t}$
$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$
$\varepsilon_2 = \frac{1}{E}(-\nu\sigma_1 + \sigma_2)$
$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$
$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$
$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{1}{E} \times \frac{pr}{2t}(1 - \nu)$
$\varepsilon = \frac{\Delta E}{E} = \frac{E_{τελ.} - E_{αρχ.}}{E_{αρχ.}} = \frac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \frac{2\pi\Delta r}{2\pi r} = \frac{\Delta r}{r} \Leftrightarrow \boxed{\Delta r = \varepsilon \times r = \frac{1}{E} \times \frac{pr^2}{2t}(1 - \nu)}$
$r' = r + \Delta r = r + \frac{1}{E} \times \frac{pr^2}{2t}(1 - \nu)$
$V' = \frac{4}{3}\pi \times \left[r + \frac{pr^2}{2Et}(1 - \nu)\right]^3$
$\Delta V = V' - V = \frac{4}{3}\pi \times \left[r + \frac{pr^2}{2Et}(1 - \nu)\right]^3 - \frac{4}{3}\pi r^3 = ... = \frac{2\pi pr^4}{Et}(1 - \nu)$