CHAPTER 9

DESIGN MODELS FOR FRP-STRENGTHENED MASONRY

9.1 General

In this chapter we present simple models for the design of masonry strengthened with unidirectional composites. The relevant formulations will be given for the cases of out-of-plane flexure, in-plane flexure, in-plane shear and confinement.

9.2 Out-of-plane flexure combined with axial force

Let us consider a masonry element of length ℓ and thickness t at the ultimate limit state in out-of-plane flexure, with a moment resistance $M_{Rd,o}$ corresponding to an axial force N_{Rd}. The FRP reinforcement is assumed uniformly distributed in the tension side, with a total cross sectional area A_f within the length ℓ.

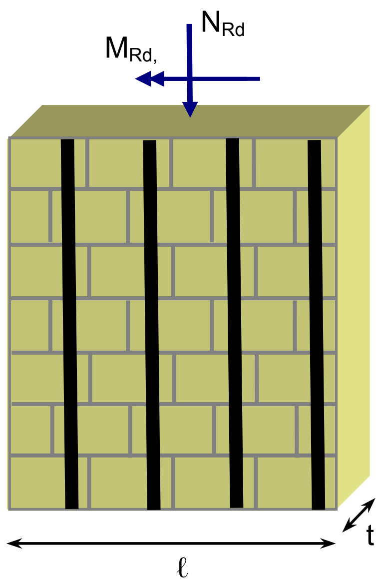


Fig. 9.1 Masonry element subjected to out-of-plane flexure combined with axial force.

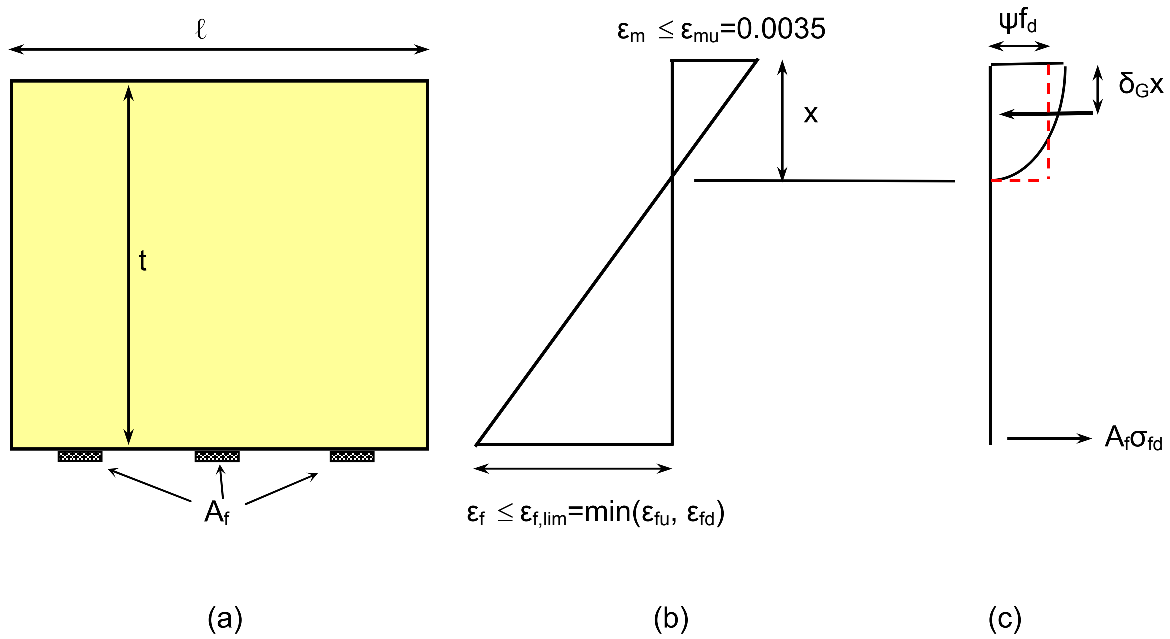


Fig. 9.2 Cross section analysis at the ultimate limit state in out-of-plane flexure: (a) geometry, (b) strain distribution, (c) internal force distribution.

From the cross section analysis of strain and stress (Fig. 9.2), in analogy to Section 4.4, assuming that failure will be governed either by masonry compressive crushing or by failure of the FRP (most likely due to debonding, but tensile fracture is a possibility), the following expressions for the neutral axis depth and the design moment resistance corresponding to a specific axial load are derived:

(1) Compression failure of masonry prior to failure of the FRP

Force equilibrium:

$$\psi f_d \ell x - A_f \sigma_{fd} = N_{Rd} \tag{9.1}$$

Strain compatibility:

$$\frac{\sigma_{fd}}{E_f} = \varepsilon_{mu} \frac{t - x}{x} \leq \varepsilon_{f,lim} \tag{9.2}$$

From the above expressions we calculate the neutral axis depth:

$$\frac{x}{t} = \frac{1}{2\psi}\left[-\omega_f + \frac{N_{Rd}}{\ell t f_d} + \sqrt{\left(\omega_f - \frac{N_{Rd}}{\ell t f_d}\right)^2 + 4\psi\omega_f} \right] \tag{9.3}$$

Hence, the design moment resistance is:

$$\frac{M_{Rd,o}}{\ell t^2 f_d} = \frac{1}{2}\omega_f \frac{\left(1-\frac{x}{t}\right)}{\frac{x}{t}} + \frac{1}{2}\psi\frac{x}{t}\left(1-2\delta_G\frac{x}{t}\right) \tag{9.4}$$

In the above expressions ψ=0.8, δ_G=0.4 and ω_f is the mechanical FRP reinforcing ratio, defined as

$$\omega_f = \frac{A_f}{\ell t}\frac{\varepsilon_{mu}E_f}{f_d} \tag{9.5}$$

The FRP limiting strain $\varepsilon_{f,lim}$ may be estimated according to the procedure described in Chapter 4 or may be taken approximately equal to 0.003. The ultimate strain of masonry, ε_{mu}, may be taken equal to 0.0035.

(2) FRP failure (fracture or debonding) prior to compression failure of masonry

From eq. (9.1) and given that $\varepsilon_f = \varepsilon_{f,lim}$, we calculate the depth of the neutral axis:

$$\frac{x}{t} = \frac{1}{\psi}\left(\omega_f\frac{\varepsilon_{f,lim}}{\varepsilon_{mu}} + \frac{N_{Rd}}{\ell t f_d}\right) \tag{9.6}$$

Next we calculate the design moment resistance:

$$\frac{M_{Rd,o}}{\ell t^2 f_d} = \frac{1}{2}\omega_f\frac{\varepsilon_{f,lim}}{\varepsilon_{mu}} + \frac{1}{2}\psi\frac{x}{t}\left(1-2\delta_G\frac{x}{t}\right) \tag{9.7}$$

In the above eqs. (9.6)-(9.7), as in the case of concrete beams, the coefficients ψ and δ_G are given as follows:

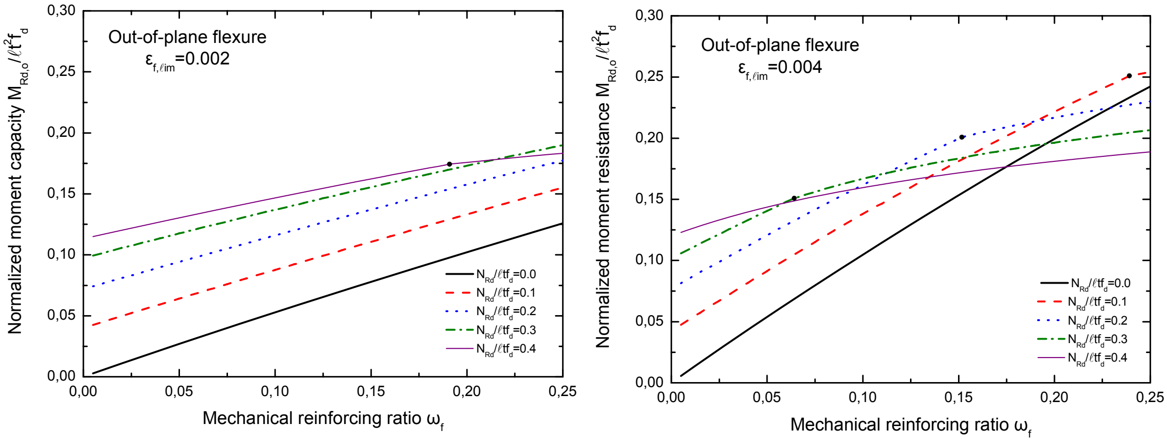
$$\psi = \begin{cases} 1000\varepsilon_m\left(0.5-\frac{1000}{12}\varepsilon_m\right) \text{if } \varepsilon_m \le 0.002 \\ 1-\frac{2}{3000\varepsilon_m} \text{ if } 0.002 \le \varepsilon_m \le 0.0035 \end{cases} \tag{9.8}$$

$$\delta_G = \begin{cases} \frac{8-1000\varepsilon_m}{4(6-1000\varepsilon_M)} & \text{if } \varepsilon_m \le 0.002 \\ \frac{1000\varepsilon_m(3000\varepsilon_m-4)+2}{2000\varepsilon_m(3000\varepsilon_m-2)} & \text{if } 0.002 \le \varepsilon_m \le 0.0035 \end{cases} \tag{9.9}$$

where

$$\varepsilon_m = \varepsilon_{f,lim} \frac{\frac{x}{t}}{1-\frac{x}{t}} \leq 0.0035 \quad (9.10)$$

The normalized moment resistance is plotted in Fig. 9.3 in terms of the mechanical reinforcing ratio ω_f, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$ assumed equal to 0.002 and 0.004.



(a) (b)

Fig. 9.3 Normalized design resistance in out-of-plane flexure in terms of the mechanical reinforcing ratio, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$.

9.3 In-plane flexure combined with axial force

Let us now consider a masonry element of length ℓ and thickness t at the ultimate limit state of in-plane flexure, with a moment resistance $M_{Rd,i}$ corresponding to an axial force N_{Rd}. The FRP reinforcement is assumed uniformly distributed in the tension side, with a total cross sectional area A_f within the length ℓ. As before, failure will be governed either by masonry compressive crushing or by failure of the most highly stressed FRP (most likely debonding, but tensile fracture is a possibility). From stress and strain analysis of the cross section (Fig. 9.5), neglecting the contribution of FRP in carrying compression, the following expressions for the neutral axis depth and the design moment resistance corresponding to a specific axial load are derived:

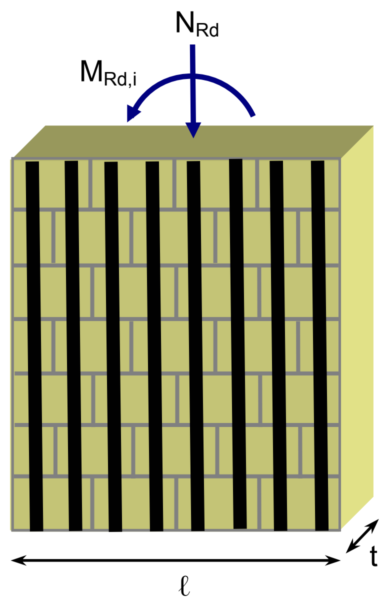


Fig. 9.4 Masonry element subjected to in-plane flexure combined with axial force.

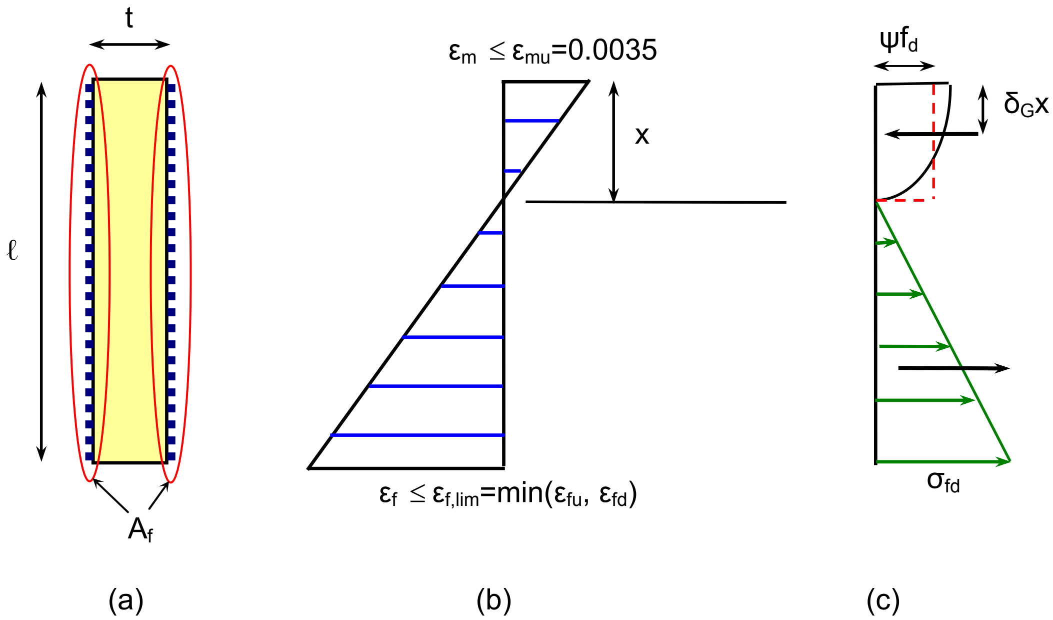


Fig. 9.5 Cross section analysis at the ultimate limit state in in-plane flexure: (a) geometry, (b) strain distribution, (c) internal force distribution.

(1) Compression failure of masonry prior to failure of the most highly stressed FRP

Force equilibrium:

$$\psi f_d t x - \frac{1}{2}\sigma_{fd} A_f \frac{\ell - x}{\ell} = N_{Rd} \tag{9.11}$$

Strain compatibility:

$$\frac{\sigma_{fd}}{E_f} = \varepsilon_{mu} \frac{\ell - x}{x} \le \varepsilon_{f,lim} \tag{9.12}$$

From the above equations we determine the neutral axis position:

$$\frac{x}{\ell} = \frac{1}{2\left(\psi - \frac{\omega_f}{2}\right)}\left[-\omega_f + \frac{N_{Rd}}{\ell t f_d} + \sqrt{\left(\omega_f - \frac{N_{Rd}}{\ell t f_d}\right)^2 + 2\left(\psi - \frac{\omega_f}{2}\right)\omega_f}\right] \tag{9.13}$$

Hence, the design moment resistance is:

$$\frac{M_{Rd,i}}{t\ell^2 f_d} = \frac{1}{12}\omega_f \frac{\left(1-\frac{x}{\ell}\right)^2\left(1+2\frac{x}{\ell}\right)}{\frac{x}{\ell}} + \frac{1}{2}\psi\frac{x}{\ell}\left(1-2\delta_G\frac{x}{\ell}\right) \tag{9.14}$$

In the above expressions ψ=0.8, δ_G=0.4 and ω_f is the mechanical reinforcing ratio, defined in eq. (9.5).

(2) Failure (fracture or debonding) of the most highly stressed FRP prior to compression failure of masonry

From eq. (9.11) and given that $\varepsilon_f = \varepsilon_{f,lim}$ we calculate the depth of the neutral axis:

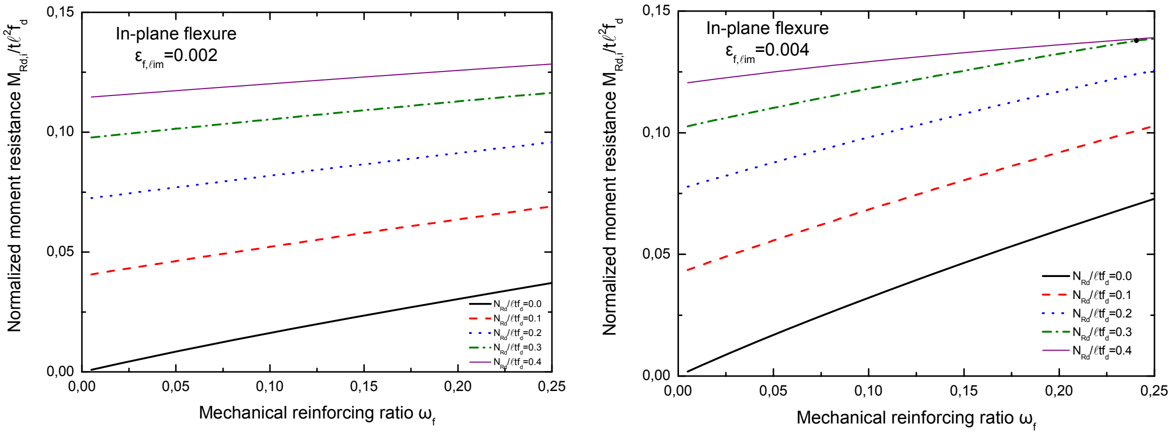
$$\frac{x}{\ell} = \frac{\frac{1}{2}\omega_f \frac{\varepsilon_{f,lim}}{\varepsilon_{mu}} + \frac{N_{Rd}}{\ell t f_d}}{\psi + \frac{1}{2}\omega_f \frac{\varepsilon_{f,lim}}{\varepsilon_{mu}}} \tag{9.15}$$

Next we calculate the design moment resistance:

$$\frac{M_{Rd,i}}{t\ell^2 f_d} = \frac{1}{2}\omega_f \frac{\varepsilon_{f,lim}}{\varepsilon_{mu}} \frac{\left(1-\frac{x}{\ell}\right)\left(1+2\frac{x}{\ell}\right)}{6} + \frac{1}{2}\psi\frac{x}{\ell}\left(1-2\delta_G\frac{x}{\ell}\right) \tag{9.16}$$

ψ and δ_G in eqs. (9.15) - (9.16) are given by eqs. (9.8) – (9.9) and the maximum compression strain ε_m is:

$$\varepsilon_m = \varepsilon_{f,lim} \frac{\frac{x}{\ell}}{\left(1-\frac{x}{\ell}\right)} \leq 0.0035 \qquad (9.17)$$



(a) (b)

Fig. 9.6 Normalized design resistance in in-plane flexure in terms of the mechanical reinforcing ratio, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$.

The normalized moment resistance is plotted in Fig. 9.6 in terms of the mechanical reinforcing ratio ω_f, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$ assumed equal to 0.002 and 0.004.

9.4 Design examples on flexural strengthening

Consider a masonry wall element with length ℓ = 1 m and width t = 0.30 m in **out-of-plane flexure**. The wall carries an axial load of 150 kN and has compressive strength (in the loading direction) equal to f_d = 5 N/mm^2. We assume that strengthening is provided using unidirectional carbon fiber fabrics with width 100 mm, thickness 0.2 mm and elastic modulus 235 kN/mm^2. Determine the design moment resistance of the FRP-strengthened masonry assuming the use of three fabrics per meter.

$$A_f = 3 \times 100 \times 0.2 = 60 \text{ mm}^2, \quad \frac{N_{Rd}}{\ell t f_d} = \frac{150000}{1000 \times 300 \times 5} = 0.1, \quad \omega_f = \frac{60 \times 0.0035 \times 235000}{1000 \times 300 \times 5} = 0.033.$$

For the case of masonry compressive crushing, eq. (9.3) gives x/t = 0.249 and from eq. (9.4) we obtain $M_{Rd,o}$ = 58.22 kNm. For the case of FRP debonding (at an assumed strain $\varepsilon_{f,lim}$=0.003), eq. (9.6) gives x/t = 0.276 and from eq. (9.7) we obtain $M_{Rd,o}$ =

29.56 kNm. Note that the maximum masonry compressive strain corresponding to the latter value for $M_{Rd,o}$ equals [from eq. (9.10)] ε_m = 0.00115, that is much smaller than the ultimate value ε_{mu}.

Next we consider the same masonry wall subjected to **in-plane flexure.** We assume that strengthening is provided using unidirectional carbon fiber fabrics with width 50 mm, thickness 0.12 mm and elastic modulus 235 kN/mm^2. Determine the design moment resistance of the FRP-strengthened masonry assuming the use of five fabrics on each side of the wall.

$$A_f = 10 \times 50 \times 0.12 = 60 \text{ mm}^2, \ \frac{N_{Rd}}{\ell t f_d} = 0.1, \ \omega_f = \frac{60 \times 0.0035 \times 235000}{1000 \times 300 \times 5} = 0.033 .$$

For the case of masonry compressive crushing, eq. (9.13) gives x/ℓ = 0.194 and from eq. (9.14) we obtain $M_{Rd,i}$ = 117.50 kNm. For the case of FRP debonding (at an assumed strain $\varepsilon_{f,lim}$=0.003), eq. (9.15) gives x/ℓ = 0.257 and from eq. (9.16) we obtain $M_{Rd,i}$ = 71.86 kNm. Note here too that the maximum masonry compressive strain corresponding to the latter value for $M_{Rd,i}$ equals [from eq. (9.17)] ε_m = 0.00104, that is much smaller than the ultimate value ε_{mu}.

9.5 In-plane shear combined with axial force

Considering that composites play the role of shear reinforcement, the design of strengthened masonry may be carried out along the lines of Eurocode 6 for reinforced masonry. The relevant methodology differs slightly depending on whether the design refers to shear walls or beam-type elements.

9.5.1 Shear walls

Let us consider a masonry element of length ℓ and thickness t at the ultimate limit state of in-plane shear (Fig. 9.7), with a shear resistance V_{Rd} corresponding to an axial force N_{Rd}. The FRP reinforcement is assumed uniformly distributed at an equal spacing s_f in both sides, with a cross sectional area A_f at each level.

The design shear resistance is calculated as follows:

$$\frac{V_{Rd}}{t\ell f_d} = \frac{V_{Rd,M}}{t\ell f_d} + \frac{V_{Rd,f}}{t\ell f_d} = \frac{f_{vd}}{f_d} + \frac{A_f}{ts_f}\frac{\sigma_{fed}}{f_d} \leq \frac{2}{f_d} \qquad (9.18)$$

where f_{vd} = design shear strength of masonry, σ_{fed} = effective design stress in FRP at the ultimate limit state in shear and f_d in N/mm^2. f_{vd} is given by the following expression:

$$f_{vd} = \frac{1}{\gamma_M}\min\left(f_{vko} + 0.4\sigma_d, f_{vlt}\right) \qquad (9.19)$$

where f_{vko} = characteristic shear strength under zero compressive stress, σ_d = design value of compressive stress ($\sigma_d = N_{Rd} / \ell t$) and f_{vlt} = maximum allowable value for f_{vko}, which depends mainly on how much the joints are filled with mortar. For fully filled vertical joints $f_{vlt} = 0.065 f_b$, where f_b is the compressive strength of masonry blocks in the direction of loading. As a rather conservative approximation, the stress σ_{fed} may be taken equal to $0.003 E_f$.

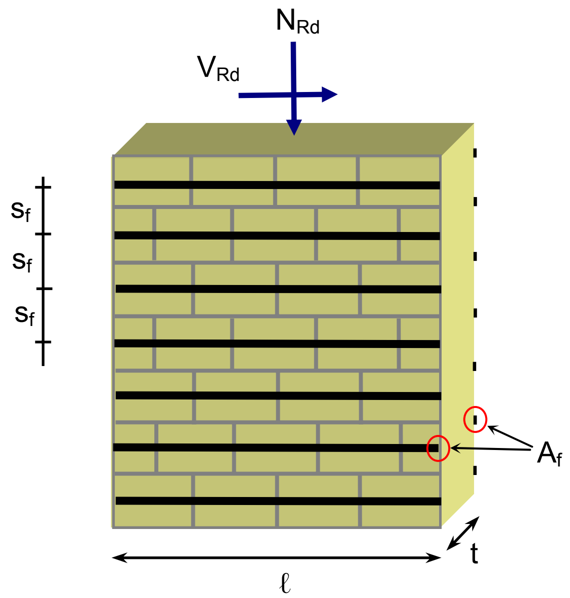


Fig. 9.7 Masonry wall subjected to in-plane shear combined with axial compression.

The normalized shear resistance is plotted in Fig. 9.8 in terms of the mechanical reinforcing ratio ω_f, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$ taken equal to 0.002 and 0.004.

At this point it is important to emphasize that the horizontal configuration of composites is effective towards resisting shear only if failure is governed by diagonal tension (Fig. 9.9a). Other mechanisms, such as *sliding shear* (Fig. 9.9b) and *rocking* (Fig. 9.9c) would necessitate the use of composites vertically and properly anchored in the foundation.

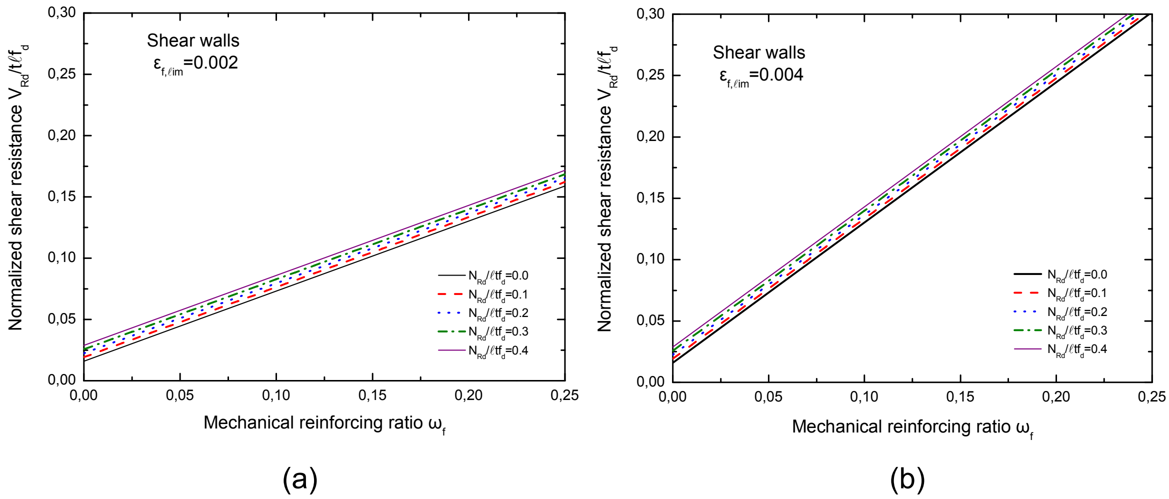


Fig. 9.8 Normalized design resistance in in-plane shear in terms of the mechanical reinforcing ratio, for different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$.

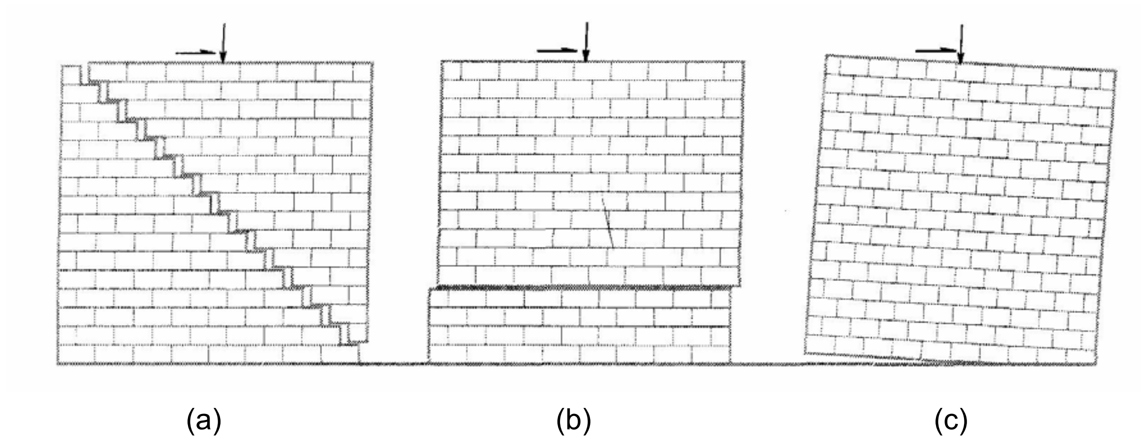


Fig. 9.9 Typical failure mechanisms of unreinforced masonry subjected to in-plane shear: (a) diagonal cracking, (b) shear sliding, (c) rocking.

9.5.2 Beam-type elements

For beam-type elements subjected to in-plane shear (e.g. lintels) V_{Rd} is determined as follows:

$$\frac{V_{Rd}}{t\ell f_d} = \frac{V_{Rd,M}}{t\ell f_d} + \frac{V_{Rd,f}}{t\ell f_d} = \frac{f_{vd}}{f_d}\frac{d}{\ell} + 0.9\frac{d}{\ell}\frac{A_f}{ts_f}\frac{\sigma_{fd}}{f_d}(1+\cot\alpha)\sin\alpha \le 0.25\frac{d}{\ell} \qquad (9.20)$$

where d = static depth, which in the case of deep beams may be taken approximately equal to $1.3z$, with $z = \min(0.7\ell_{ef}, 0.4\ell + 0.2\ell_{ef})$, ℓ_{ef} = effective span length and ℓ = beam height, as shown in Fig. 9.10. In eq. (9.20) α = angle between fiber direction and beam axis (typically $\alpha = 90^o$).

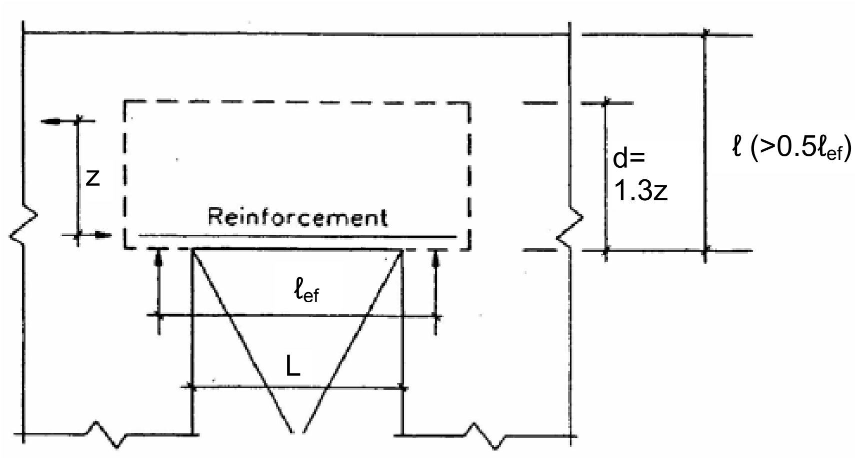
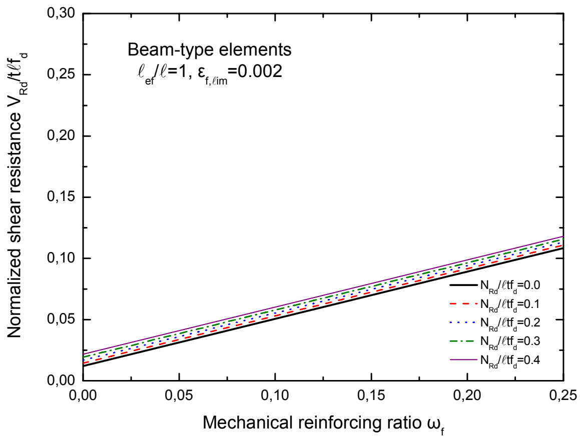
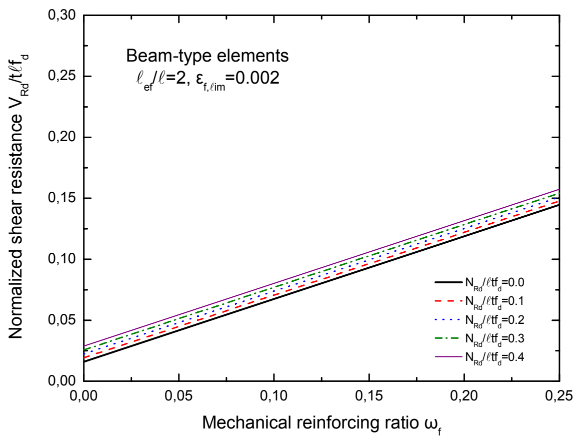


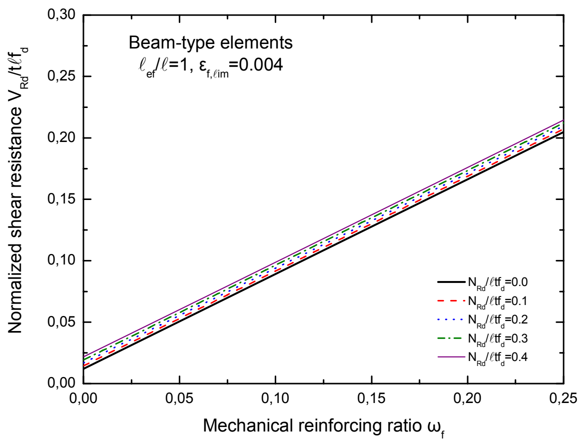
Fig. 9.10 Geometry of beam-type masonry element subjected to in-plane shear.



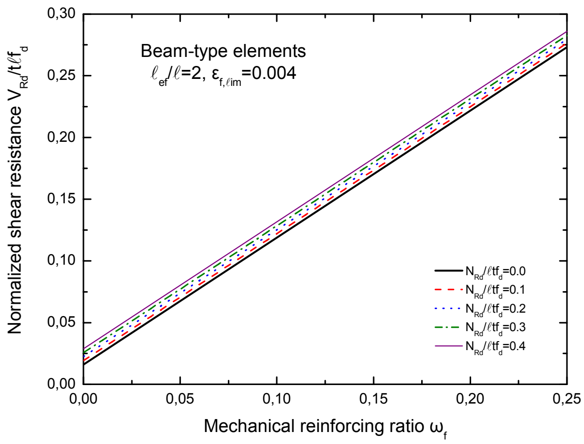
(a)



(b)



(c)



(d)

Fig. 9.11 Normalized design resistance in in-plane shear in terms of the mechanical reinforcing ratio, for two different geometries and different values of the normalized axial force (at failure) and $\varepsilon_{f,lim}$.

The normalized shear resistance is plotted in Fig. 9.11a-d in terms of the mechanical reinforcing ratio ω_f, for different values of the normalized axial force (at failure) and the ratio ℓ_{ef}/ℓ, with $\varepsilon_{f,lim}$ taken equal to 0.002 and 0.004. In these figures $\alpha = 90^o$.

9.6 Design examples on shear strengthening

Consider a **shear wall** with length ℓ = 1 m and thickness t = 0.30 m in **in-plane shear**. The wall carries an axial load of 150 kN and has compressive strength (in the loading direction) equal to f_d = 5 N/mm^2. The masonry blocks are solid with compressive strength f_b = 7 N/mm^2, the vertical joints are fully filled, f_{vko} = 0.2 N/mm^2 and γ_M = 2.5. We assume that strengthening is provided using unidirectional carbon fiber strips with width 20 mm, thickness 1.5 mm and elastic modulus E_f = 170 kN/mm^2. Determine the required spacing of FRP strips so that V_{Rd} = 100 kN.

$$\sigma_d = \frac{150000}{1000 \times 300} = 0.5 \text{ N/mm}^2, \quad f_{vd} = \frac{1}{2.5}\min(0.2 + 0.4 \times 0.5, 0.065 \times 7) = 0.16 \text{ N/mm}^2$$

From eq. (9.18) with σ_{fed} =0.003$\times$170000 = 510 N/mm^2 and $A_f = 2 \times 20 \times 1.5$ = 60 mm^2 we calculate s_f = 588 mm. [If we assume that the spacing of bed joints is about 140 mm, we could use near-surface mounted strips every fourth joint].

Next we consider a **beam-type** masonry element with height ℓ = 1 m and static depth d = 0.80 m, subjected to in-plane shear with zero axial load. The material properties are as given above. Determine the required spacing of FRP strips so that V_{Rd} = 100 kN.

$$\sigma_d = 0, \; f_{vd} = \frac{1}{2.5}\min(0.2, 0.065 \times 7) = 0.08 \text{ MPa}.$$

From eq. (9.20) with σ_{fed} =0.003$\times$170000 = 510 N/mm^2 and A_f = 60 mm^2 we calculate s_f = 273 mm.

9.7 Confinement

The effect of composite material confinement on masonry is similar to that on concrete. Based on the simple analytical model of Krevaikas and Triantafillou (2005), which has been calibrated using test results from short columns of rectangular cross sections (Fig. 9.12), the compressive strength f_{dc} and the ultimate strain ε_{mcu} of confined masonry may be obtained from the following expressions:

$$f_{dc} = f_d \quad \text{if} \quad \frac{\sigma_{\ell ud}}{f_d} \le 0.24 \qquad (9.21a)$$

$$f_{dc} = f_d\left(0.6 + 1.65\frac{\sigma_{\ell ud}}{f_d}\right) \quad \text{if} \quad \frac{\sigma_{\ell ud}}{f_d} \ge 0.24 \qquad (9.21b)$$

$$\varepsilon_{mcu} = \varepsilon_{mu} + 0.034\frac{\sigma_{\ell ud}}{f_d} \qquad (9.22)$$


(a)


(b)

Fig. 9.12 (a) Compressive failure of unreinforced masonry, (b) failure of confining jacket.

In the equations given above $\sigma_{\ell ud}$ is the confining stress, which, for circular cross sections with diameter D, equals $(2t_f/D)f_{fde}$. For rectangular cross sections with dimensions b and d, the confining stress may be taken approximately equal to the average of the confining stresses in each direction, as explained in Ch. 6 for the confinement of concrete:

$$\sigma_{\ell ud} = \frac{\sigma_{\ell ud,d} + \sigma_{\ell ud,b}}{2} = \frac{1}{2}\alpha_f\left(\frac{2t_f}{d}f_{fde} + \frac{2t_f}{b}f_{fde}\right) = \alpha_f\frac{(b+d)}{bd}t_f f_{fde} \qquad (9.23)$$

where α_f is the confinement effectiveness coefficient according to eq. (6.10).