

**CHAPTER 3**

**BASIS OF DESIGN**

### 3.1 General

The design of RC and masonry structures strengthened with composites follows the philosophy of the relevant design codes (e.g. Eurocodes 2, 6, 8) and involves the verification for the ultimate and serviceability limit states, with proper modifications to account for the contribution of FRP.

### 3.2 Material constitutive laws

This section describes briefly the material constitutive laws in uniaxial loading and gives data on FRP material safety factors.

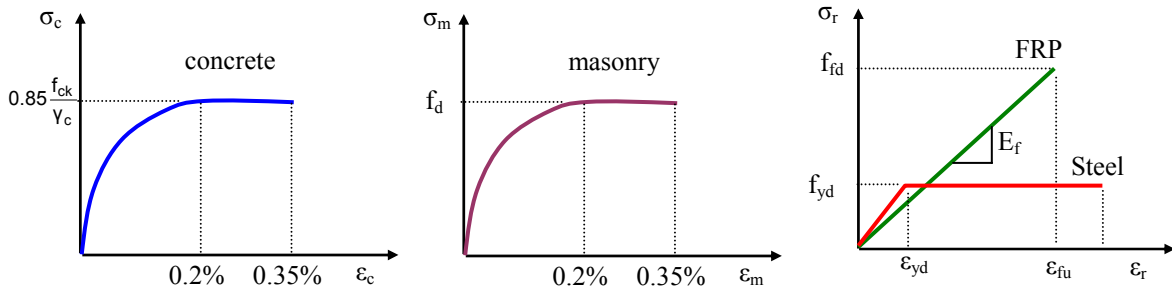
#### 3.2.1 Calculation of resistance – full composite action

For concrete, masonry and steel (“existing” materials) the design values for strength are calculated by dividing the **representative** value of strength  $X_k$  with the material safety factor  $\gamma_m$ . If the limit state verification is performed in terms of strength (“forces”), as representative value is taken the mean value divided by a reliability coefficient (1.0, 1.2, 1.35), which depends on the quantity and reliability of available material data. If the verification is performed in terms of deformations (e.g. displacements, rotations), the representative value is taken as the mean value. In each of the above cases the safety factor  $\gamma_m$  ( $\gamma_c$ ,  $\gamma_M$  and  $\gamma_s$  for concrete, masonry and steel, respectively) depends on the level of reliability for material strength data. For the concrete compressive strength  $f_{cd} = f_{ck} / \gamma_c$ , where  $f_{ck}$  = representative strength and  $\gamma_c$  = safety factor for concrete. For masonry  $f_d = f_k / \gamma_M$ , where  $f_k$  = representative strength and  $\gamma_M$  = safety factor for masonry. Finally, for steel reinforcement  $f_{yd} = f_{yk} / \gamma_s$ , where  $f_{yk}$  = representative value of yield stress and  $\gamma_s$  = safety factor for steel.

The strength of composite materials (“added” materials) is represented by the characteristic value if the safety verification is performed in terms of strength, or by the mean value if the safety verification is performed in terms of deformations. Their behavior in uniaxial tension is assumed linear elastic to failure, according to eq. (3.1); failure is defined at a (design) stress  $f_{fd} = f_{fk} / \gamma_f$ :

$$\sigma_f = E_f \varepsilon_f \leq f_{fd} \quad (3.1)$$

The elastic modulus of FRP is determined by dividing the representative values of strength to ultimate strain,  $E_f = f_{fk} / \varepsilon_{fuk}$ . The design stress-strain curves for concrete, masonry and steel and FRP are summarized in Fig. 3.1.



**Fig. 3.1** Design stress – strain curves.

At this point we should point out that the in-situ tensile strength of FRP is lower than that measured in a uniaxial tension test, due to stress concentrations, complex multiaxial states of stress, several layers, environmental degradation effects etc. All these reduction factors may be taken into account by assuming that FRP reaches failure at an **effective strain**  $\varepsilon_{fue}$ , which is less than the mean ultimate strain  $\varepsilon_{fum}$  determined through testing. On the basis of the above, the design value of the effective strength for FRP,  $f_{fde}$ , is given as follows:

$$f_{fde} = \frac{\varepsilon_{fue}}{\varepsilon_{fum}} \frac{f_{fk}}{\gamma_f} = \eta_e f_{fd} \quad (3.2)$$

More details on the effective strain  $\varepsilon_{fue}$  will be given in the sections where this strain plays an important role (e.g. shear strengthening, confinement).

**Table 3.1** FRP material safety factors,  $\gamma_f$ .

FRP type	Application type A <sup>(1)</sup>	Application type B <sup>(2)</sup>
CFRP	1.20	1.35
AFRP	1.25	1.45
GFRP	1.30	1.50

<sup>(1)</sup> Application of prefab FRP systems under normal quality control conditions. Application of wet lay-up systems if all necessary provisions are taken to obtain a high degree of quality control on both the application conditions and the application process.

<sup>(2)</sup> Application of wet lay-up systems under normal quality control conditions. Application of any system under difficult on-site working conditions.

Values for the FRP material safety factor are suggested in Table 3.1 (*fib* 2001). Note that these values are still a topic of current research and are subject to further refinements. Note that Eurocode 8 suggests, for simplicity, the use of a single value safety factor,  $\gamma_f = \boxed{1.50}$ .

### 3.2.2 Calculation of resistance - debonding

In many cases fracture of the FRP is not reached due to premature bond failure at the FRP-substrate interface (see next chapter for details). Debonding is mainly caused due to high interfacial shear stresses and is observed as shearing through the substrate (concrete or masonry), due to the lower strength of the latter compared to that of adhesives. When debonding controls failure, the material safety factor concerns the substrate and should be taken as  $\gamma_{fb} = 1.5$ .

### 3.2.3 Serviceability limit state

The elastic modulus of FRP for the serviceability limit state should be taken equal to that for the ultimate limit state.

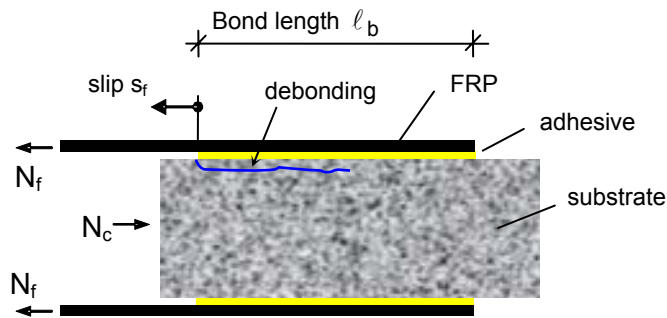
## 3.3 Bond at the FRP – substrate interface

The full composite action between FRP and concrete or masonry can only be achieved through high quality epoxy adhesives. Bond failure is a critical phenomenon, which should be accounted for carefully in the safety verifications. This requires a good understanding of bond mechanics and the development of appropriate bond modeling, as described in the following.

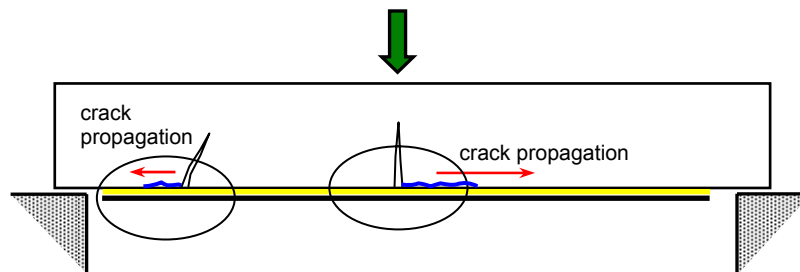
### 3.3.1 General, behavior

The behavior of the bond between externally bonded FRP and concrete or masonry can be analyzed in bond tests, such as the one illustrated in Fig. 3.2, which represents, in a simplified manner, the state of stress and strain near cracks (see Fig. 3.3). In the vicinity of cracks (e.g. Fig. 3.3), the FRP carries a tension force  $N_f$  (Fig. 3.2), which is transferred through shearing in the substrate. Of particular practical interest is the relationship between the mean shear stress  $\tau_b$  at the FRP-substrate interface (equal to  $N_f / \ell_b b_f$  in Fig. 3.2, where  $b_f$  the width of FRP) and the slip  $s_f$ . This relationship depends on many factors, including the substrate strength, the type of adhesive, the FRP characteristics (e.g. thickness, elastic modulus) and the bond length. A typical shear

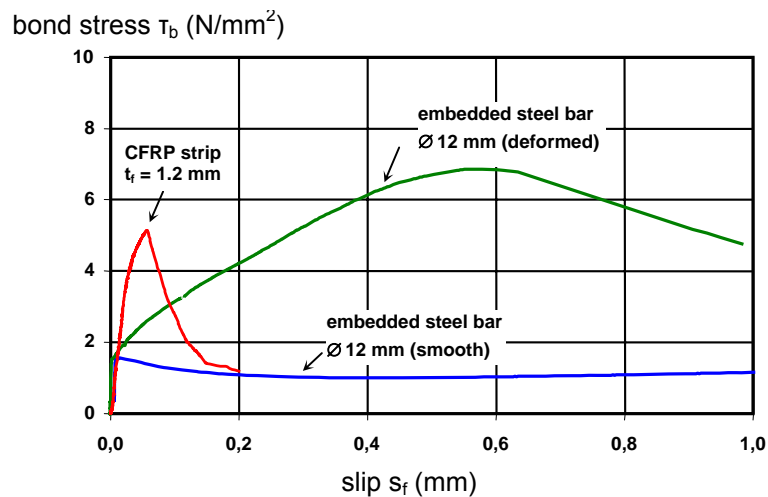
stress – slip curve is plotted in Fig. 3.4, along with others for deformed and smooth steel rebars, which are provided for the sake of comparison.



**Fig. 3.2** Simplified FRP-substrate bond test (e.g. Zilch et al 1998, Bizindavyi and Neale 1999).



**Fig. 3.3** Cracking and possible debonding (the arrows indicate the crack propagation).



**Fig. 3.4** Bond stress – slip relationships (Zilch et al. 1998).

Contrary to the case of embedded steel rebars in concrete, an important characteristic of the FRP-substrate bond is that FRP fracture rarely precedes debonding.

The force in the FRP to cause debonding, that is the maximum anchorable force,  $N_{fa}$ , increases with the bond length  $\ell_b$ , until this length reaches a limiting value, beyond which the maximum anchorable force remains practically unchanged, equal to  $N_{fa,max}$  (Fig. 3.5).

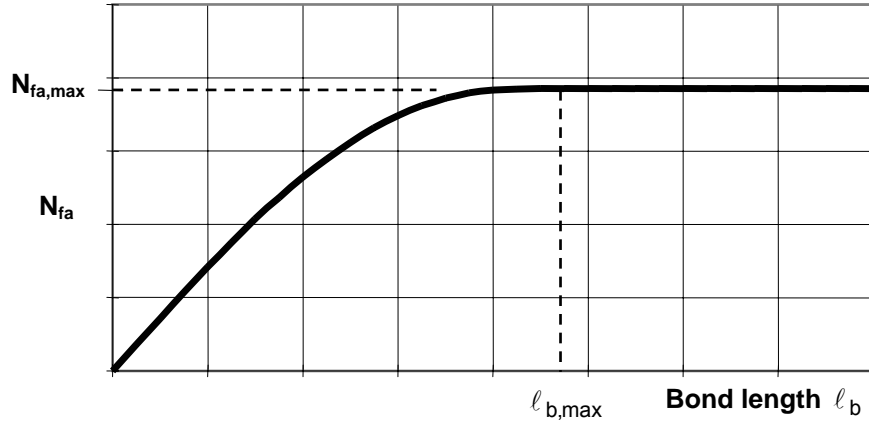


Fig. 3.5 Anchorable force – bond length relationship.

### 3.3.2 Analytical model

For FRP-concrete interfaces, the anchorable force – bond length relationship shown in Fig. 3.5 can be described analytically as follows (Holzenkämpfer 1994, Brosens and Van Gemert 1999):

$$\text{if } \ell_b \geq \ell_{b,max} : \quad N_{fa} = N_{fa,max} = b_f \sqrt{0.6k_b E_f f_{ctm} t_f} \quad (\text{N}) \quad (3.3a)$$

$$\text{if } \ell_b < \ell_{b,max} : \quad N_{fa} = N_{fa,max} \frac{\ell_b}{\ell_{b,max}} \left( 2 - \frac{\ell_b}{\ell_{b,max}} \right) \quad (\text{N}) \quad (3.3b)$$

$$\ell_{b,max} = 0.6 \sqrt{\frac{E_f t_f}{\sqrt{f_{ctm} k_b}}} \quad (\text{mm}) \quad (3.4)$$

with

$$k_b = \sqrt{\frac{1.5 \left( 2 - \frac{b_f}{b} \right)}{1 + \frac{b_f}{100}}} \geq 1 \quad (3.5)$$

where  $b_f$  = width of FRP (mm),  $b$  = width of RC member cross section (mm),  $f_{ctm}$  = mean tensile strength of concrete (N/mm<sup>2</sup>),  $E_f$  = elastic modulus of FRP (N/mm<sup>2</sup>) and  $t_f$  = thickness of FRP (mm).

In terms of **stresses**, the above model results in the following equations for the FRP design stress ( $f_{fbd} = N_{fad} / b_f t_f$ ) corresponding to debonding:

$$\text{if } \ell_b \geq \ell_{b,max} : \quad f_{fbd} = \frac{1}{\gamma_{fb}} \sqrt{\frac{0.6E_f f_{ctm} k_b}{t_f}} \quad (\text{N, mm}) \quad (3.6a)$$

$$\text{if } \ell_b < \ell_{b,max} : \quad f_{fbd} = \frac{1}{\gamma_{fd}} \sqrt{\frac{0.6E_f f_{ctm} k_b}{t_f}} \frac{\ell_b}{\ell_{b,max}} \left( 2 - \frac{\ell_b}{\ell_{b,max}} \right) \quad (\text{N, mm}) \quad (3.6b)$$

### Example 3.1

Consider an FRP strip with width  $b_f = 50$  mm, thickness  $t_f = 1.2$  mm, elastic modulus  $E_f = 180$  kN/mm<sup>2</sup> and tensile strength  $f_f = 3000$  kN/mm<sup>2</sup>, epoxy-bonded on a concrete member with a width  $b = 100$  mm (Fig. 3.6). The mean tensile strength of concrete is assumed  $f_{ctm} = 1.9$  kN/mm<sup>2</sup>.

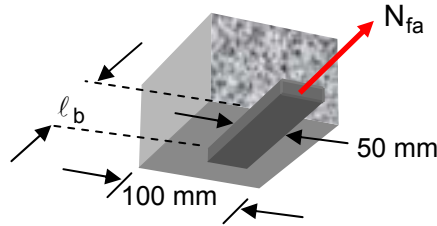


Fig. 3.6

Eq. (3.5) gives

$$k_b = \sqrt{\frac{1.5 \left( 2 - \frac{50}{100} \right)}{1 + \frac{50}{100}}} = 1.22 \geq 1$$

Eq. (3.4) gives  $\ell_{b,max} = 0.6 \sqrt{(180000 \times 1.2) / \sqrt{1.9 \times 1.22}} = 226$  mm and from eq. (3.3a) we calculate  $N_{fa,max} = 50 \sqrt{0.6 \times 1.22 \times 180000 \times 1.9 \times 1.2} = 27405$  N  $\approx$  27.4 kN, corresponding to a stress in the FRP equal to  $27405 / (50 \times 1.2) = 457$  N/mm<sup>2</sup> [it is worth noting here that if the strip reached its tensile capacity the respective force would be  $N_f = 3000 \times (50 \times 1.2) / 1000 = 180$  kN, that is about 6.5 times higher than that causing debonding].

In terms of stresses, the **design** stress in the FRP at debonding (assuming a bond length at least equal to 226 mm) is given by eq. (3.6) (with material safety factor  $\gamma_{fb} = 1.5$ ) as  $f_{fbd} = 305 \text{ N/mm}^2$ .

