Randomized Algorithm

Lemma: A running node will stop in the current round with probability $\geq 0.25 = 1/4$

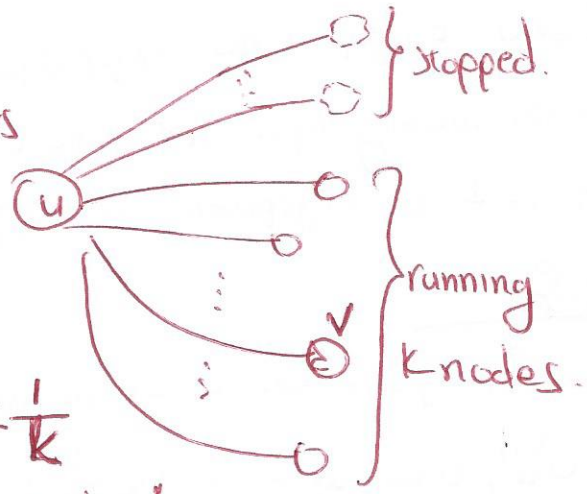
Proof:

Assume we look at the running node u:

u has access to at least $k+1$ colors.

We look at the probability that u conflicts with some node in its running neighbor.

Let v be a node in u's running neighbor.



u conflicts with v with probability $< \frac{1}{k}$

For v to be active it can happen with prob $\frac{1}{2}$.

Thus conflict between active u & v with prob $\leq \frac{1}{2k}$.

For the k nodes we use the union bound.

Thus the prob of conflicting with anyone is $< k \cdot \frac{1}{2k} = \frac{1}{2}$.

Thus the prob that no conflict happens is $\geq \frac{1}{2}$.

The total probability is dictated by u being active $(\frac{1}{2})$. Thus the total probability of stopping is $\boxed{\geq \frac{1}{4}}$ ∎

Corollary: The node is still running after T rounds with prob. $\leq 0.75^T$

Proof: The prob. of failing is $\leq 1 - 1/4 = 3/4 = 0.75$

Thus the prob. of failing T rounds (independent) is $\leq 0.75^T$ ∎

Corollary: All nodes will stop after $O(\log n)$ rounds w.h.p.

Proof: Assume that $T = c \log_{4/3} n$. Then, the prob to fail one node in T rounds is $\left(\frac{3}{4}\right)^{c \log_{4/3} n} = \frac{1}{\left(\frac{4}{3}\right)^{c \log_{4/3} n}} = \frac{1}{n^c}$.

The probability that some node fails in T rounds by the union bound is