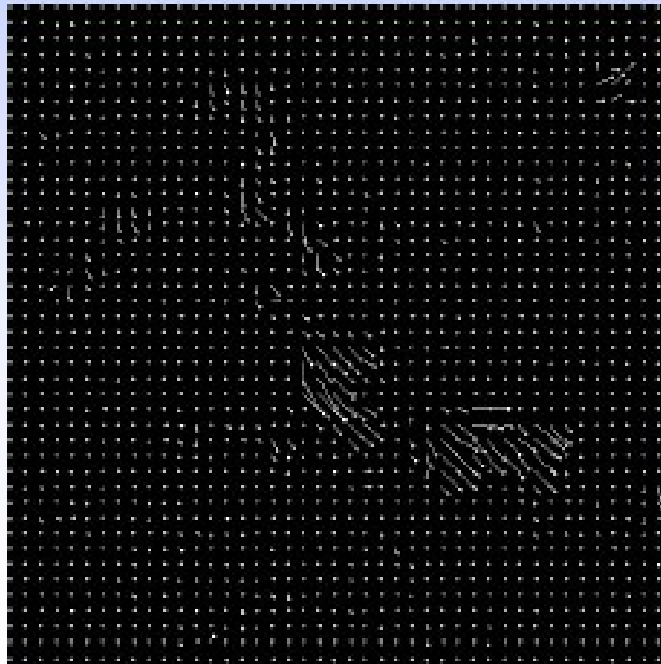
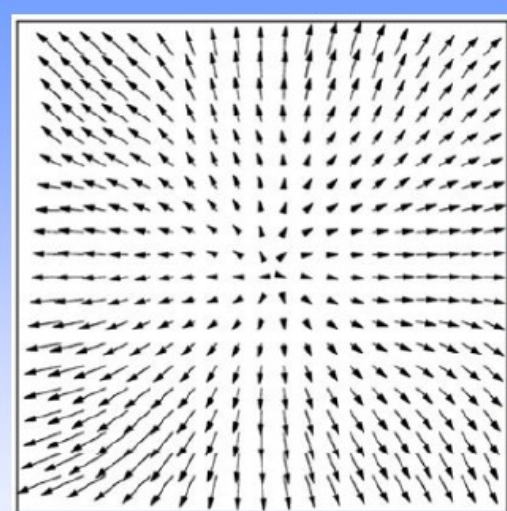
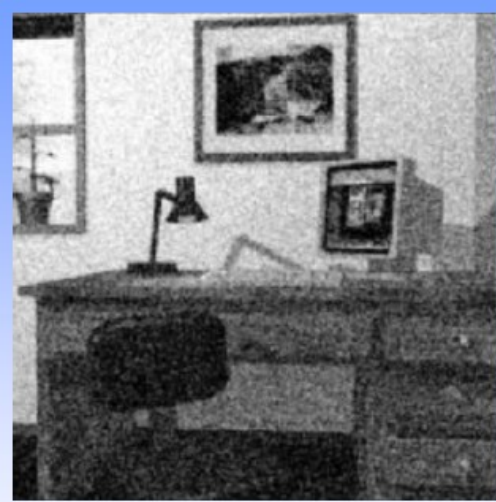


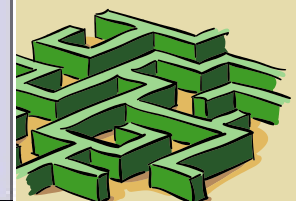
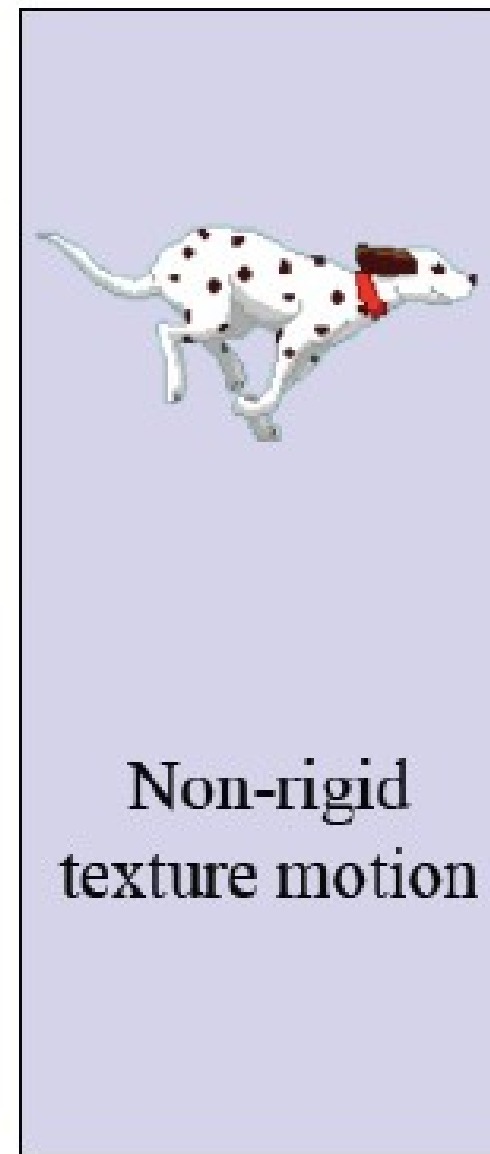
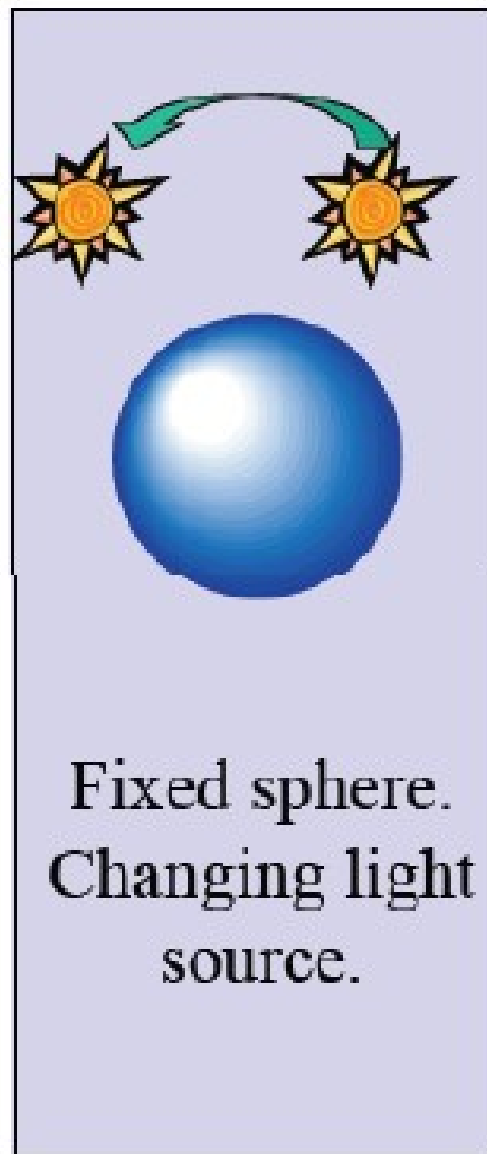
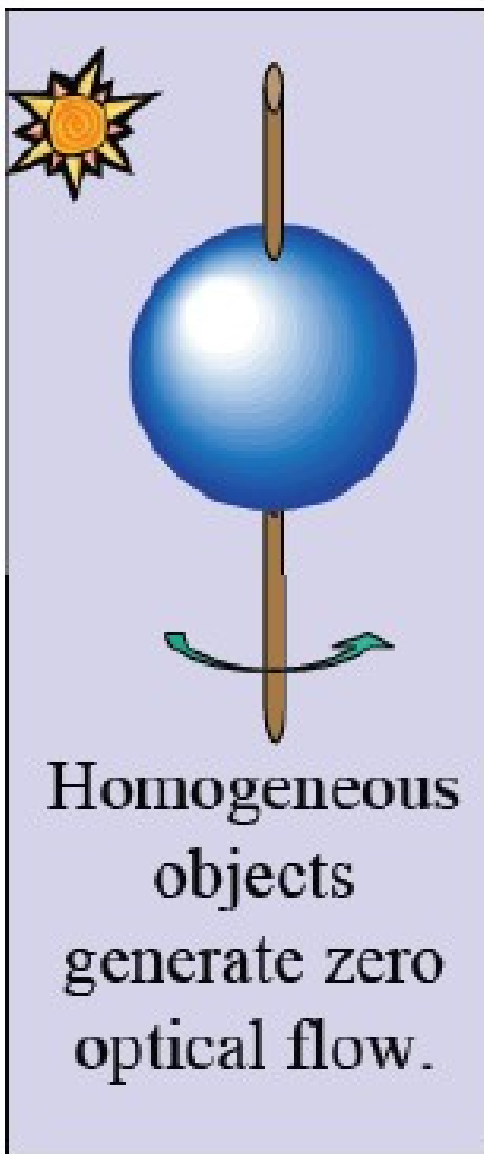
Κατάτμηση εικόνας-video – οπτική ροή



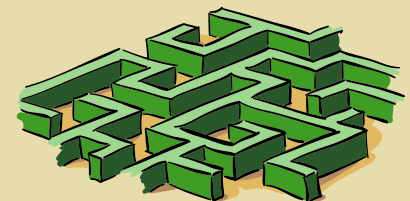
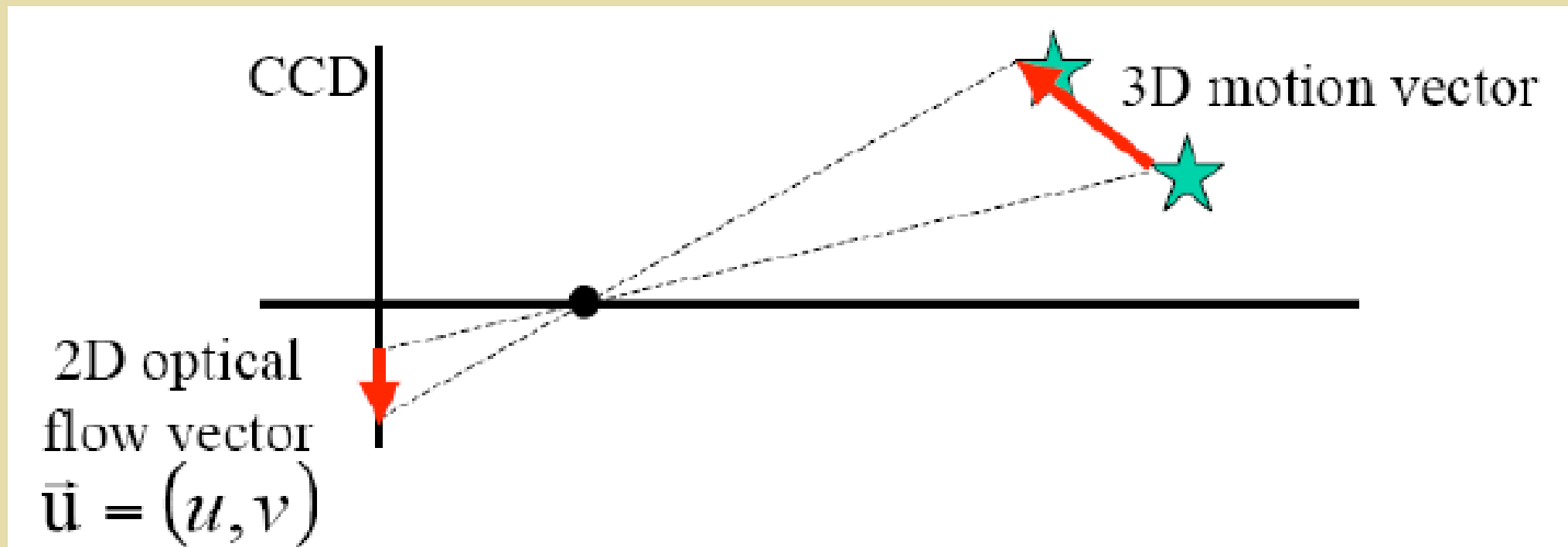
Κατάτμηση εικόνας-video – οπτική ροή



Κίνηση αντικειμένων 3D – Περιοχές ομοιάζουσας κατανομής χρώματος

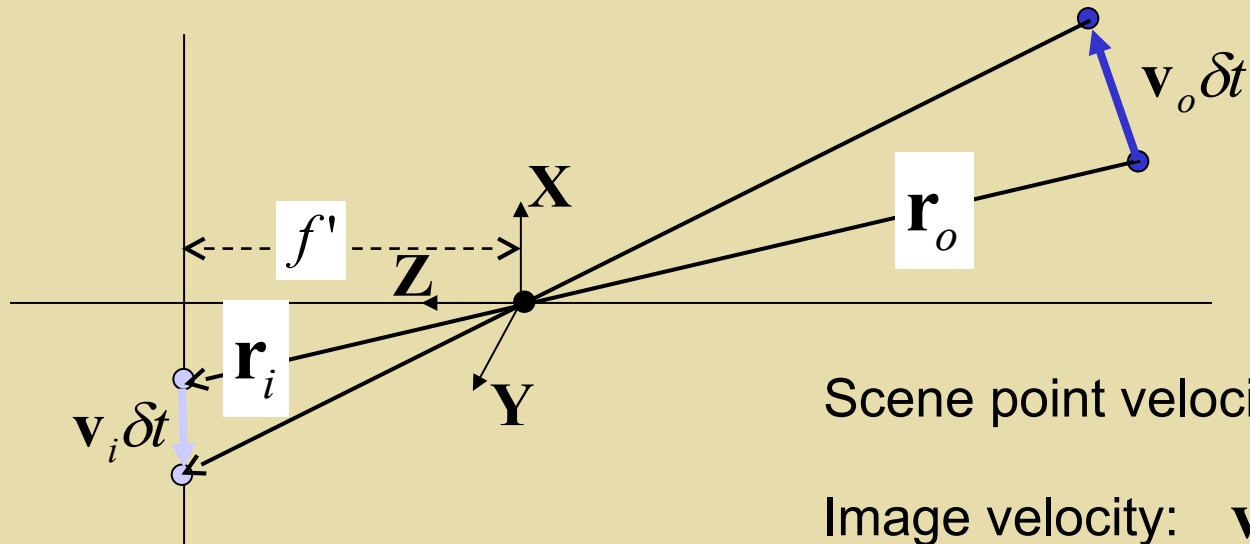


3D -> 2D οπτική ροή



3D -> 2D οπτική ροή

Image velocity of a point moving in the scene



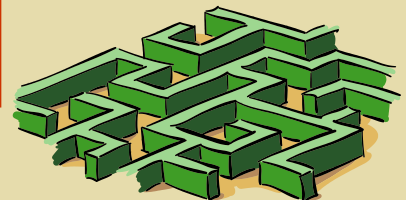
Scene point velocity: $\mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$

Image velocity: $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$

Perspective projection: $\frac{1}{f'} \mathbf{r}_i = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{Z}}$

Motion field

$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f' \frac{(\mathbf{r}_o \cdot \mathbf{Z})\mathbf{v}_o - (\mathbf{v}_o \cdot \mathbf{Z})\mathbf{r}_o}{(\mathbf{r}_o \cdot \mathbf{Z})^2} = f' \frac{(\mathbf{r}_o \times \mathbf{v}_o) \times \mathbf{Z}}{(\mathbf{r}_o \cdot \mathbf{Z})^2}$$



3D -> 2D οπτική ροή

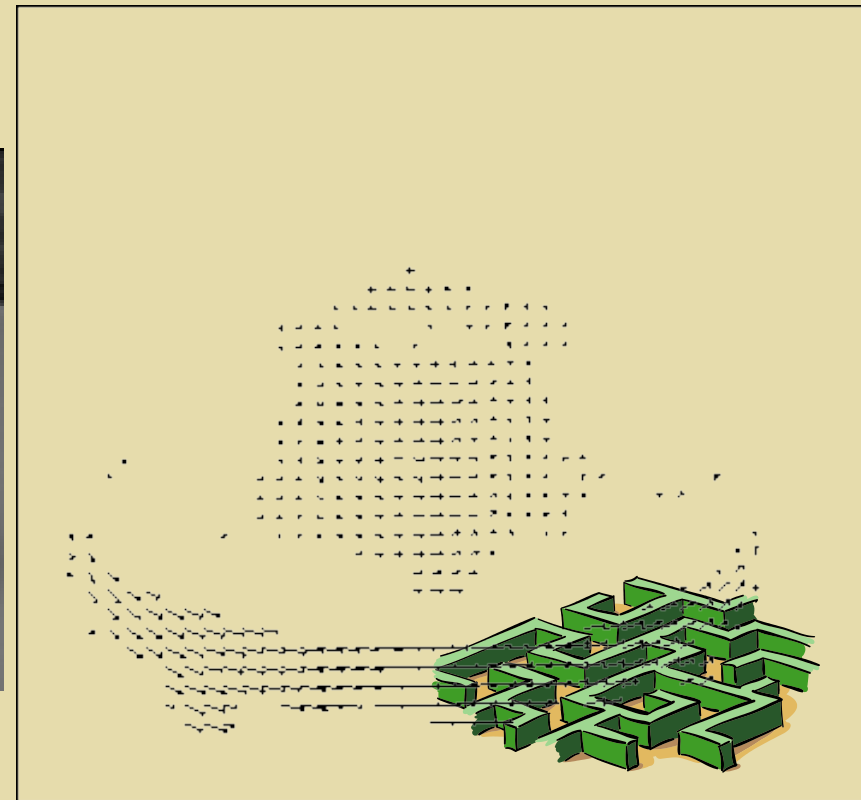
Ideally Optical flow = Motion field

Αλλαγή φωτισμού -> Κίνηση αντικειμένων;

Κίνηση φωτιστικού σημείου -> Κίνηση αντικειμένων;

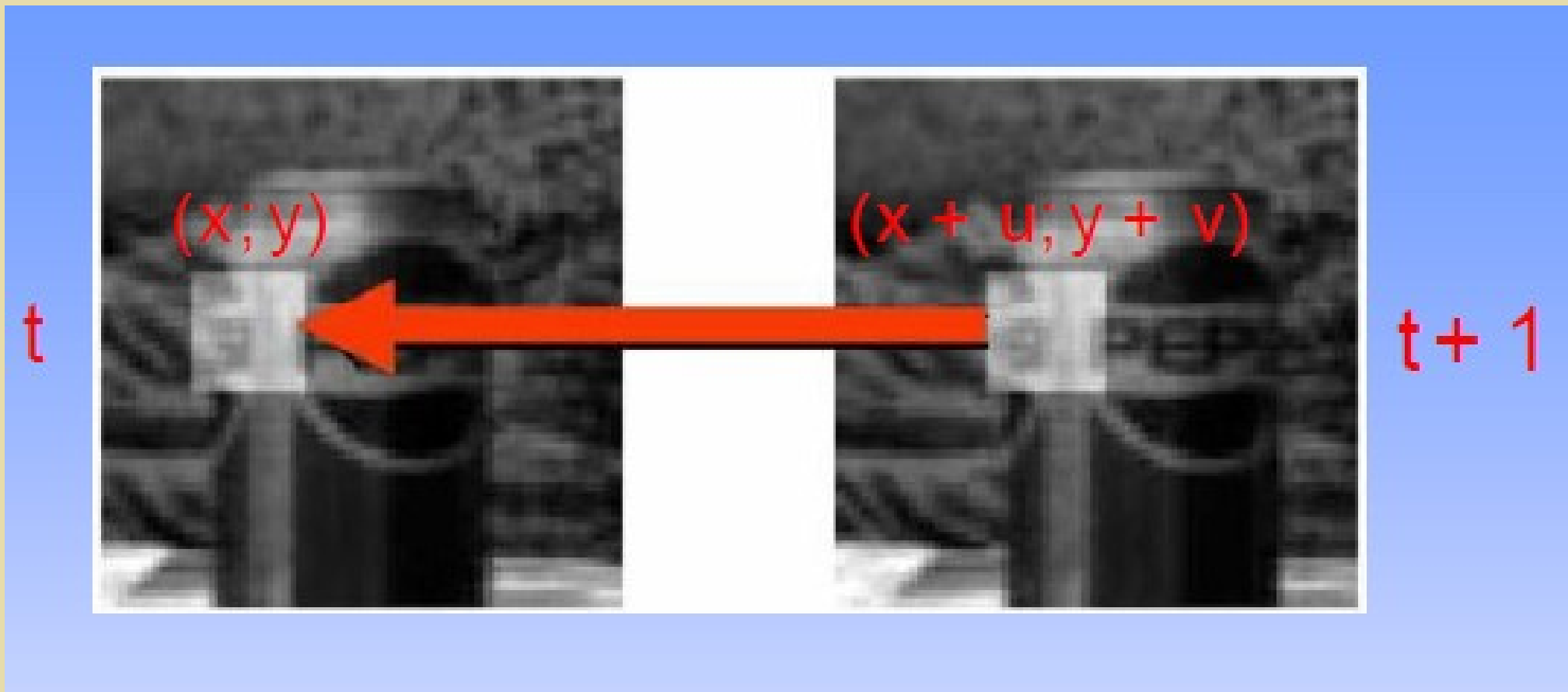
Κίνηση κάμερας -> Κίνηση αντικειμένων;

Αλλαγή εστίασης -> Κίνηση αντικειμένων;

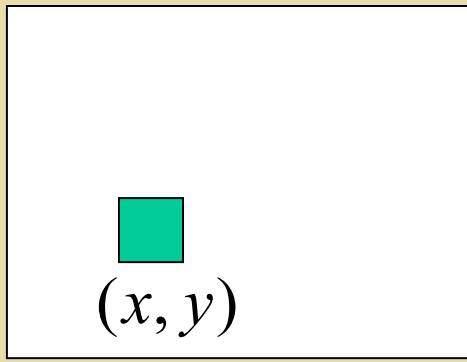


Οπτική ροή – Υποθέσεις

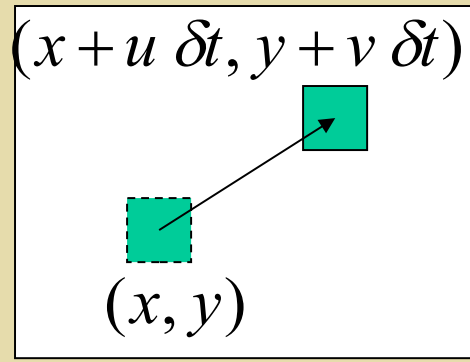
$$I(x, y, t) = (I(x + dx, y + dy, t + 1))$$



Οπτική ροή – Επίλυση



time t



time t + delta t

Optical Flow: Velocities (u, v)

Displacement:

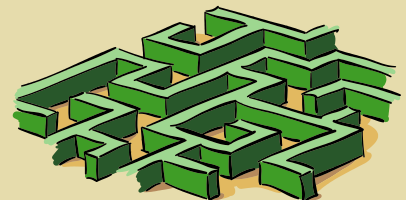
$$(\delta x, \delta y) = (u \delta t, v \delta t)$$

- Assume brightness of patch remains same in both images:

$$E(x + u \delta t, y + v \delta t, t + \delta t) = E(x, y, t)$$

- Taylor:

$$E(x, y, t) + \delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} = E(x, y, t)$$



Οπτική ροή – Επίλυση

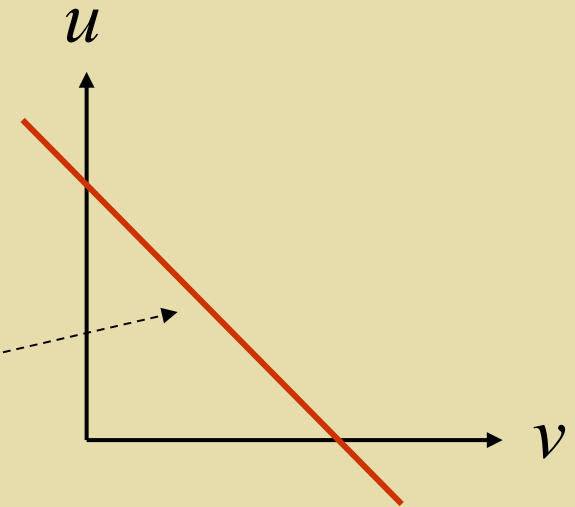
$$\delta x \frac{\partial E}{\partial x} + \delta y \frac{\partial E}{\partial y} + \delta t \frac{\partial E}{\partial t} = 0$$

Divide by δt and take the limit $\delta t \rightarrow 0$

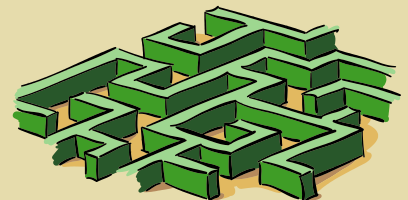
$$\frac{dx}{dt} \frac{\partial E}{\partial x} + \frac{dy}{dt} \frac{\partial E}{\partial y} + \frac{\partial E}{\partial t} = 0$$

Constraint Equation

$$E_x u + E_y v + E_t = 0$$



Η εξίσωση δεν λύνεται διότι έχουμε δύο αγνώστους

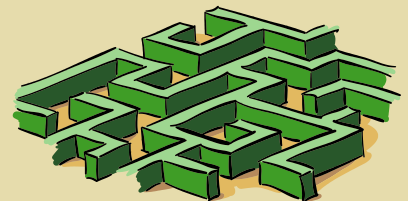


Επίλυση: Lucas Kanade

Υπόθεση: Σε γειτονικά εικονοστοιχεία τα u, v
παραμένουν σταθερά $E_1 = \sum$

Δουλεύω σε μικρά παράθυρα:

$$E_1 = \sum (I_x u + I_y v + E_t)^2$$



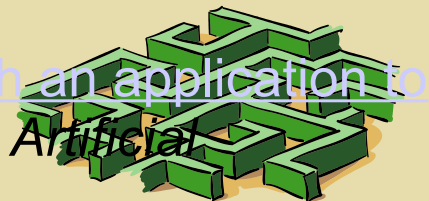
Solving the aperture problem

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \quad \begin{matrix} A & d = b \\ 25 \times 2 & 2 \times 1 & 25 \times 1 \end{matrix}$$

- When is this system solvable?

B. Lucas and T. Kanade. [An iterative image registration technique with an application to stereo vision](#). In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

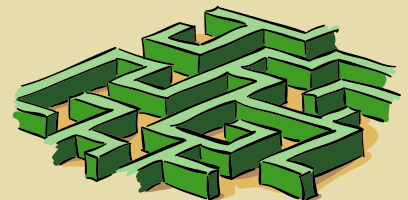


Lucas-Kanade flow

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

$A^T A$ $A^T b$

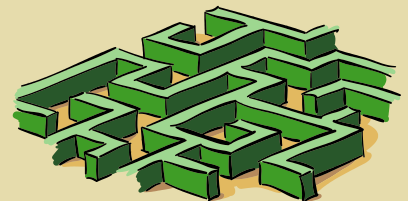
- $M = A^T A$ is the *second moment matrix*
- The system is solvable by looking at the eigenvalues of the second moment matrix
 - The eigenvectors and eigenvalues of M relate to edge direction and magnitude
 - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change, and the other eigenvector is orthogonal to it



Uniform region



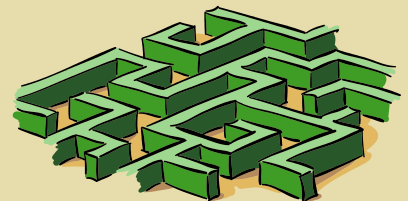
- gradients have small magnitude
- small λ_1 , small λ_2
- system is ill-conditioned



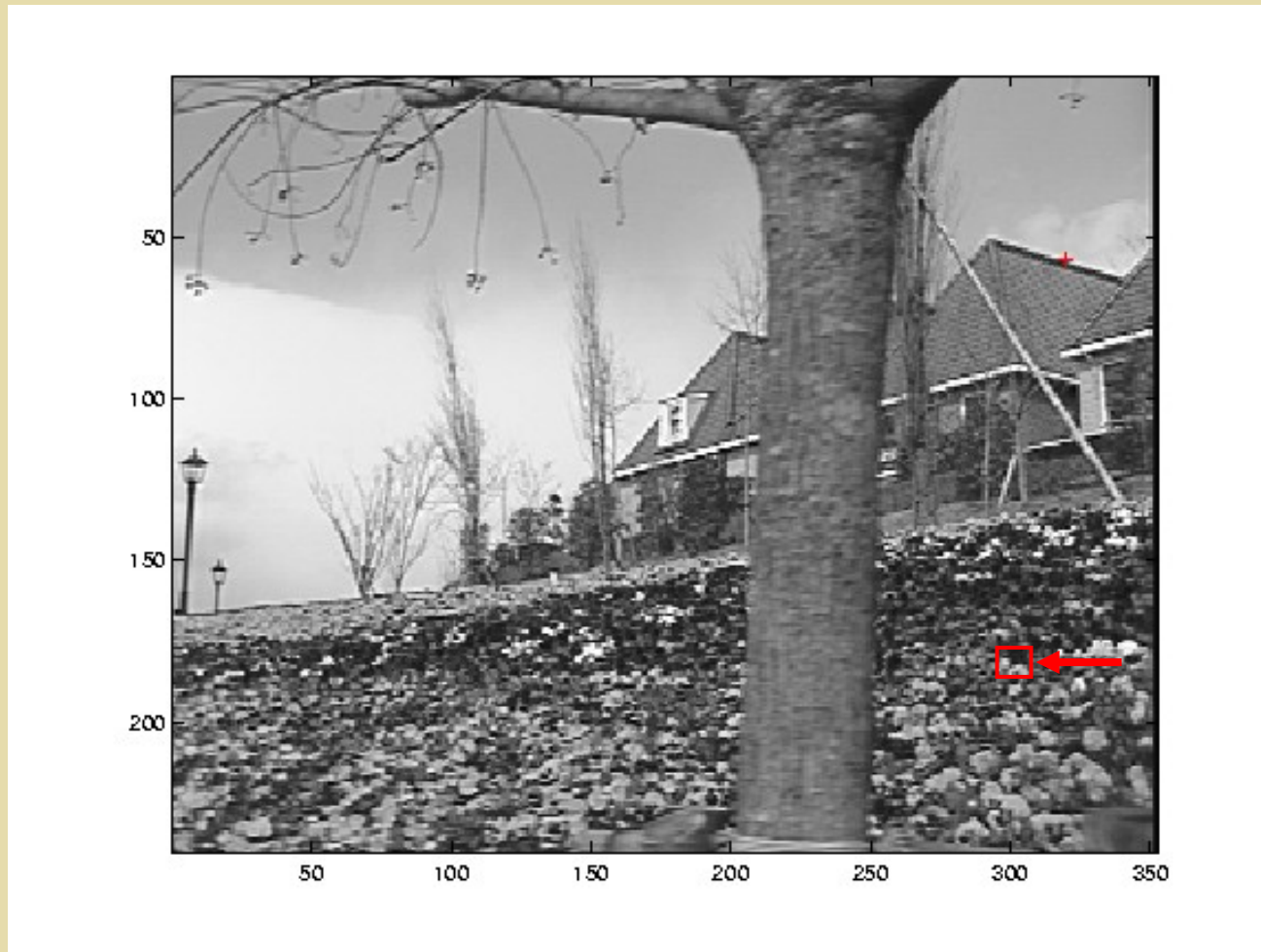
Edge



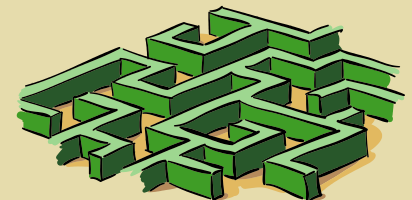
- gradients have one dominant direction
- large λ_1 , small λ_2
- system is ill-conditioned



High-texture or corner region



- gradients have different directions, large magnitudes
- large λ_1 , large λ_2
- system is well-conditioned



Errors in Lucas-Kanade

- The motion is large (larger than a pixel)
 - Iterative refinement
 - Coarse-to-fine estimation
- A point does not move like its neighbors
 - Motion segmentation
- Brightness constancy does not hold
 - Exhaustive neighborhood search with normalized correlation

