CEID

MSc on DATA DRIVEN COMPUTING AND DECISION MAKING (DDCDM)

Description Logics-DLs)

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Introductory (1)

- Knowledge Representation Methods
  - Logic based (variations of FOL)
    - Performance problems
    - Clear semantics
  - Structured (Semantic Nets, Frames)
    - Non-clear semantics
    - Good performance

Introductory (2)

- How can we assign explicit semantics to non-logical representations?

↓

- Expressing structured representations through logical representations.

↓ Consequences?

- Loss of some features of structured representations.
- Gain the clear semantics.

} Description Logics-DLs

DL-Basic Elements (1)

- concepts
  - Represented by unary predicates
  - They represent subsets of the domain D (e.g. female, person) and correspond to classes of the structured representations.
  - Distinguished in primitive (e.g. male, person) and defined (e.g. man is defined as person with gender male).
- roles
  - Represented by binary predicates
  - Represent relations between concepts (e.g. hasChild)

DL-Basic Elements (2)

- constructors or operators
  - Used for creating concept or term descriptions
  - To produce new (defined) concepts.
  - Basic constructors:
    - $\cup$ (union)
    - $\cap$ (intersection)
    - $\subset$, $\subseteq$ (subset)
    - $\neg$ (negation)
  - Examples: Person$\cap$Female, Male$\cup$Female, Person$\cap\neg$Female
- quantifiers
  - $\forall$ (universal), $\exists$ (existential)

DL-Basic Elements (3)

- Value restrictions
  - They are used to assign constraints on role values.
  - Format: ∀R.C, ∃R.C (R: role, C: concept)

    C is a universal or existential constraint on the values of R.
  - Examples:

    ∃hasChild.Female (entities that have at least one daughter)

    ∀hasChild.Female (entities that have only daughters)
- Individuals or nominals
  - They correspond to logic constants or class instances of structured representations (e.g. JOHN, MARIA).

Equivalences DL-FOL (1)

Constructor	DL Syntax	FOL Syntax	DL example	FOL example
intersectionOf	$C1 \cap C2$	$C1(x) \wedge C2(x)$	$Human \cap Male$	$Human(x) \wedge Male(x)$
unionOf	$C1 \cup C2$	$C1(x) \vee C2(x)$	$Doctor \cup Lawyer$	$Doctor(x) \vee Lawyer(x)$
complementOf	$\neg C$	$\neg C(x)$	$\neg Male$	$\neg Male(x)$
one of	$\{a1\} \cup \{a2\}$	$x=a1 \vee x=a2$	$\{John\} \cup \{Mary\}$	$John \vee Mary$
allValuesFrom	$\forall P.C$	$\forall y P(x,y) \Rightarrow C(y)$	$\forall hasChild.Doctor$	$\forall y\ hasChild(x,y) \Rightarrow Doctor(y)$
someValuesFrom	$\exists P.C$	$\exists y P(x,y) \wedge C(y)$	$\exists hasChild.Doctor$	$\exists y\ hasChild(x,y) \wedge Doctor(y)$
hasValue	$\exists P.\{a\}$	$P(x,a)$	$\exists hasChild.\{Mary\}$	$hasChild(x,Mary)$
subClassOf	$C1 \subseteq C2$	$\forall x\ C1(x) \Rightarrow C2(x)$	$Human \subseteq Animal$	$\forall x\ Human(x) \Rightarrow Animal(x)$
equivalentClass	$C1 \equiv C2$	$\forall x\ C1(x) \Leftrightarrow C2(x)$	$Man \equiv Human \cap Male$	$\forall x\ Man(x) \Leftrightarrow Human(x) \wedge Male(x)$

Equivalences DL-FOL (1)

Constructor	DL Syntax	FOL Syntax	DL example	FOL example
disjointWith	$C1 \subseteq \neg C2$ ή $C2 \subseteq \neg C1$	$\forall x\ C1(x) \Rightarrow \neg C2(x)$	$\text{Female} \subseteq \neg \text{Male}$	$\forall x\ \text{Female}(x) \Rightarrow \neg \text{Male}(x)$
minCardinality	$\geq nP.C$ ή $\geq nP$	$\exists^{\geq n} y\ P(x,y) \wedge C(y)$ ή $\exists^{\geq n} y\ P(x,y)$	$\geq 2\text{hasChild.Doctor}$ ή $\geq 2\text{hasChild}$	$\exists^{\geq n} y\ \text{hasChild}(x,y) \wedge \text{Doctor}(y)$ ή $\exists^{\geq n} y\ \text{hasChild}(x,y)$
maxCardinality	$\leq nP.C$ ή $\leq nP$	$\exists^{\leq n} y\ P(x,y) \wedge C(y)$ ή $\exists^{\leq n} y\ P(x,y)$	$\leq 2\text{hasChild.Doctor}$ ή $\leq 2\text{hasChild}$	$\exists^{\leq n} y\ \text{hasChild}(x,y) \wedge \text{Doctor}(y)$ ή $\exists^{\leq n} y\ \text{hasChild}(x,y)$

DL Knowledge Base

- Tbox (Terminological Box)
  - It represents general knowledge related to the domain of the problem (intensional knowledge).
  - This knowledge concerns definitions and taxonomic-hierarchical relationships of concepts.
- ABox (Assertional Box)
  - It represents special knowledge related to the specific problem (extensional knowledge).
  - This knowledge is about specific facts related to the problem.

Tbox (1)

- Concept definitions

  Woman $\equiv$ Person$\cap$Female

  Man $\equiv$ Person$\cap\neg$Female

  Hypotheses:

  – Only one definition for each concept is allowed.

  – Definitions are acyclic (a concept is not defined by itself or through other concepts that indirectly refer to it).

Tbox (2)

- Taxonomic relations-General axioms

  Man $\subset$ Human

  $\exists$hasChild.Person $\subseteq$ Person

  (Only persons can have children who are persons)

  More analytically

  (Entities that have at least one child that is a person are persons)

Abox

- Specific knowledge (facts)

  Man(BOB) ή BOB: Man

  hasChild(BOB, MARY) ή (BOB, MARY): hasChild

  $\neg$Doctor(MARY) ή MARY: $\neg$Doctor

Types of Description Logics (DL)

- Basic DL: $\mathcal{AL}$ (Atrributive Language)

  Offers:
  - concepts: C, D
  - Atomic concepts: A
  - Most general concept (Top): T
  - Bottom concept (Bottom): $\perp$
  - Concepts intersection: $C \cap D$
  - Value restrictions: $\forall R.C$
  - Restricted existential quantification: $\exists R.\perp$
  - Negation only to atomic concepts
  - Equivalence: $C \equiv D$
  - Subsumption: $C \subseteq D$

Types of Description Logics (DL)

- Basic DL: $\mathcal{AL}$ (Atrributive Language)

Examples:

Person, Female : atomic/primitive concepts

Person $\cap$ Female
Person $\cap$ $\neg$Female
Person $\cap$ $\exists$hasChild.T
Person $\cap$ $\forall$hasChild.Female
Person $\cap$ $\forall$hasChild.T

} (defined) concepts

$\mathcal{AL}$ Extensions

- Addition of «union»: $C \cup D$ (symbol $\mathcal{U}$)
- Addition of complete $\exists$: $\exists R.C$ (symbol $\mathcal{E}$)
- Addition of numerical restrictions: $\geq \mathrm{n}\ R$, $\leq \mathrm{n}\ R$ (symbol $\mathcal{N}$)
- Addition of negation to any concept: $\neg C$, $\neg(C \cap D)$ (symbol $\mathcal{C}$)

Depending on the extensions we have different variations ΠΛ: $\mathcal{ALC}$, $\mathcal{ALEN}$, $\mathcal{ALUN}$ κλπ.

When a DL has the $\mathcal{C}$ extension, then it can simulate $\mathcal{U}$ και $\mathcal{E}$. So, we have e.g. $\mathcal{ALUE} \equiv \mathcal{ALC}$ and $\mathcal{ALUEN} \equiv \mathcal{ALCN}$.

More Extensions

- Roles hierarchy (symbol: $\mathcal{H}$)
- Limited complex roles inclusion, reflexivity and non-reflexivity, role incompatibility: (symbol: $\mathcal{R}$)
- Nominals (Enumerate classes of constraints on object values): (symbol: $\mathcal{O}$)
- Reverse properties: (symbol: $\mathcal{I}$)
- Functional properties: (symbol: $\mathcal{F}$)
- Qualified cardinality restrictions: (symbol: $\mathcal{Q}$)
- Using properties of data types, data values, or data types: (symbol : ($\mathcal{D}$))

DL Examples

- $\mathcal{S}$: abbreviation of $\mathcal{ALC}$ with transitional roles
- $\mathcal{SHIQ}$
- $\mathcal{SHOIN}^{(\mathcal{D})}$ (OWL-DL)
- $\mathcal{SROIQ}^{(\mathcal{D})}$ (OWL 2)
- $\mathcal{SHIF}^{(\mathcal{D})}$ (OWL-Lite)

Examples

1. $\exists hasChild.Person \cap \forall hasChild.Male$

Entities of which at least one child is a person.

Entities whose children are all boys.

Entities whose at least one child is a person and a boy.

Human, Doctor, Professor, Female, hasChild, married

Examples

Human, Doctor, Professor, Female, hasChild, married

2. Represent/Define in DL the following concept/class:

«Individuals whose children are all either doctors or professors»

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$\forall$hasChild.(Doctor $\cup$ Professor)

Examples

Human, Doctor, Professor, Female, hasChild, married

3. Represent/Define in DL the following concept/class :

«Men who are married to a doctor and their children are either doctors or professors»

Man

$$\underbrace{Human \sqcap \neg Female}_{Man} \sqcap \exists married.Doctor \sqcap \forall hasChild.(Doctor \sqcup Professor)$$

Examples

4. $(\geq 3\ \text{hasChild}) \cap (\leq 2\ \text{hasFemaleRelative})$

   What does it mean in natural language?

   Individuals who have at least 3 children and at most 2 female relatives

5. $\text{Woman} \cap \leq 2\ (\text{hasChild} \cap \text{hasFemaleRelative})$

   What does it mean in natural language?

   A woman who has at most 2 daughters