# Representation Learning for Text and Applications 

"a word is defined by the company it keeps" (Firth, 1957)

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## Language model

- Goal: determine $\mathrm{P}\left(\mathrm{s}=\mathrm{w}_{1} \ldots \mathrm{w}_{\mathrm{k}}\right)$ in some domain of interest

$$
P(s)=\prod_{i=1}^{k} P\left(w_{i} \mid w_{1} \ldots w_{i-1}\right)
$$

e.g., $P\left(w_{1} w_{2} w_{3}\right)=P\left(w_{1}\right) P\left(w_{2} \mid w_{1}\right) P\left(w_{3} \mid w_{1} w_{2}\right)$

- Traditional n-gram language model assumption:
"the probability of a word depends only on context of $n-1$ previous words"

$$
\Rightarrow \widehat{\mathrm{P}}(\mathrm{~s})=\prod_{\mathrm{i}=1}^{\mathrm{k}} \mathrm{P}\left(\mathrm{w}_{\mathrm{i}} \mid \mathrm{w}_{\mathrm{i}-\mathrm{n}+1} \ldots \mathrm{w}_{\mathrm{i}-1}\right)
$$

- Typical ML-smoothing learning process (e.g., Katz 1987):

1. compute $\widehat{P}\left(w_{i} \mid w_{i-n+1} \ldots w_{i-1}\right)=\frac{\# w_{i-n+1} \ldots w_{i-1} w_{i}}{\# w_{i-n+1} \cdots w_{i-1}}$ on training corpus
2. smooth to avoid zero probabilities

## Representing Words

> One-hot vector

- high dimensionality
- sparse vectors
- dimensions=|V| (10^6<|V|)
- unable to capture semantic similarity between words


## > Distributional vector

- words that occur in similar contexts, tend to have similar meanings
- each word vector contains the frequencies of all its neighbors
- dimensions=|V|
- computational complexity for ML algorithms



## Representing Words

$>$ Word embeddings

- store the same contextual information in a lowdimensional vector
- densification (sparse to dense)
- compression
- dimensionality reduction
- dimensions=m
$100<m<500$

| eat |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

- able to capture semantic similarity between words
- learned vectors (unsupervised)
- Learning methods
- SVD
- word2vec
- GloVe


## Example

- We should assign similar probabilities (discover similarity) to Obama speaks to the media in Illinois and the President addresses the press in Chicago
- This does not happen because of the "one-hot" vector space representation

One hot
$\left.\begin{array}{rl}\text { obama } & =\left[\begin{array}{lllllll}0 & 0 & 0 & 0 & \ldots & 0 & 1\end{array}\right)\end{array}\right]$

Word embeddings


## 

- Dimensionality reduction on co-occurrence matrix
- Create a $|\mathrm{V}| \mathrm{x}|\mathrm{V}|$ word co-occurrence matrix X
- Apply SVD $X=U S V^{T}$
- Take first k columns of U
- Use the $k$-dimensional vectors as representations for each word
- Able to capture semantic and syntactic similarity


## SVD application - Latent Structure in documents

-Documents are represented based on the Vector Space Model
-Vector space model consists of the keywords contained in a document.
-In many cases baseline keyword based performs poorly - not able to detect synonyms.
-Therefore document clustering is problematic
-Example where of keyword matching with the query: "IDF in computerbased information look-up"

|  | access | document | retrieval | information | theory | database | indexing | computer |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Doc1 | x | x | x |  |  | x | x |  |
| Doc2 |  |  |  | x | x |  |  |  |
| Doc3 |  |  |  | x |  |  |  |  |

## Latent Semantic Indexing (LSI) -I

- Finding similarity with exact keyword matching is problematic.
- Using SVD we process the initial document-term document.
- Then we choose the k larger singular values. The resulting matrix is of order k and is the most similar to the original one based on the Frobenius norm than any other k-order matrix.


## Latent Semantic Indexing (LSI) - II

- The initial matrix is SVD decomposed as: $\mathrm{A}=\mathrm{ULV}^{\top}$
- Choosing the top-k singular values from $L$ we have:
$A_{k}=U_{k} L_{k} V_{k}^{\top}$,
- $L_{k}$ square $k x k$ - top-k singular values of the diagonal in matrix $L$,
- $\mathrm{U}_{\mathrm{k},}$ mxk matrix - first k columns in U (left singular vectors)
- $\mathrm{V}_{\mathrm{k}}{ }^{\top}$, kxn matrix - first k lines of $\mathrm{V}^{\top}$ (right singular vectors)

Typical values for $\kappa \sim 200-300$ (empirically chosen based on experiments appearing in the bibliography)

## LSI capabilities

-     - Term to term similarity: $A_{k} A_{k}{ }^{\top}=U_{k} L_{k}{ }^{2} U_{k}{ }^{\top}$
- Where $A k=U k L$ LVt
-     - Document-document similarity: $\mathrm{A}_{\mathrm{k}}{ }^{\top} \mathrm{A}_{\mathrm{k}}=\mathrm{V}_{\mathrm{k}} \mathrm{L}_{\mathrm{k}}{ }^{2} \mathrm{~V}_{\mathrm{k}}{ }^{\top}$
-     - Term document similarity (as an element of the transformed - document matrix)
-     - Extended query capabilities transforming initial query q to $q_{n} \quad q_{n}=q^{\top} U_{k} L_{k}{ }^{-1}$
-     - Thus $q_{n}$ can be regarded a line in matrix $V_{k}$


## LSI - an example

## LSI application on a term - document matrix

C1: Human machine Interface for Lab ABC computer application
C2: A survey of user opinion of computer system response time
C3: The EPS user interface management system
C4: System and human system engineering testing of EPS
C5: Relation of user-perceived response time to error measurements
M1: The generation of random, binary unordered trees
M2: The intersection graph of path in trees
M3: Graph minors IV: Widths of trees and well-quasi-ordering
M4: Graph minors: A survey

- The dataset consists of 2 classes, 1st: "human - computer interaction" (c1-c5) 2nd: related to graph (m1-m4). After feature extraction the titles are represented as follows.


## LSI - an example

|  | C 1 | C 2 | C 3 | C 4 | C 5 | M 1 | M 2 | M 3 | M 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| human | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Interface | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| computer | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| User | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| System | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| Response | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Time | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| EPS | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Survey | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Trees | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| Graph | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| Minors | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

## LSI - an example

$\mathrm{A}=\mathrm{ULV}{ }^{\top}$

$A=$| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

## LSI - an example

## $A=U L V^{\top}$

$U=$| 0.22 | -0.11 | 0.29 | -0.41 | -0.11 | -0.34 | 0.52 | -0.06 | -0.41 |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.20 | -0.07 | 0.14 | -0.55 | 0.28 | 0.50 | -0.07 | -0.01 | -0.11 |  |  |  |
| 0.24 | 0.04 | -0.16 | -0.59 | -0.11 | -0.25 | -0.30 | 0.06 | 0.49 | 0 | 0 |  |
| 0.40 | 0.06 | -0.34 | 0.10 | 0.33 | 0.38 | 0.00 | 0.00 | 0.01 | 0 | 0 | 0 |
| 0.64 | -0.17 | 0.36 | 0.33 | -0.16 | -0.21 | -0.17 | 0.03 | 0.27 | 0 | 0 | 0 |
| 0.27 | 0.11 | -0.43 | 0.07 | 0.08 | -0.17 | 0.28 | -0.02 | -0.05 |  |  |  |
| 0.27 | 0.11 | -0.43 | 0.07 | 0.08 | -0.17 | 0.28 | -0.02 | -0.05 |  |  |  |
| 0.30 | -0.14 | 0.33 | 0.19 | 0.11 | 0.27 | 0.03 | -0.02 | -0.17 |  |  |  |
| 0.21 | 0.27 | -0.18 | -0.03 | -0.54 | 0.08 | -0.47 | -0.04 | -0.58 |  |  |  |
| 0.01 | 0.49 | 0.23 | 0.03 | 0.59 | -0.39 | -0.29 | 0.25 | -0.23 | 0 |  |  |
| 0.04 | 0.62 | 0.22 | 0.00 | -0.07 | 0.11 | 0.16 | -0.68 | 0.23 | 0 | 0 |  |
| 0.03 | 0.45 | 0.14 | -0.01 | -0.30 | 0.28 | 0.34 | 0.68 | 0.18 |  |  |  |
| 0 | 0 | 0 | 0 |  |  |  |  |  |  |  |  |
| 0 | 0 | 0 | 0 |  |  |  |  |  |  |  |  |
| 0 | 0 | 0 |  |  |  |  |  |  |  |  |  |

## LSI - an example

## $\mathrm{A}=\mathrm{ULV}{ }^{\top}$

$L=$| 3.3 <br> 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 2.54 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2.35 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1.64 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1.50 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1.31 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.85 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.56 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.36 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## LSI - an example

$\mathrm{A}=\mathrm{ULV}{ }^{\top}$

$\mathbf{V}=$| 0.20 | -0.06 | 0.11 | -0.95 | 0.05 | -0.08 | 0.18 | -0.01 | -0.06 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.61 | 0.17 | -0.50 | -0.03 | -0.21 | -0.26 | -0.43 | 0.05 | 0.24 |
| 0.46 | -0.13 | 0.21 | 0.04 | 0.38 | 0.72 | -0.24 | 0.01 | 0.02 |
| 0.54 | -0.23 | 0.57 | 0.27 | -0.21 | -0.37 | 0.26 | -0.02 | -0.08 |
| 0.28 | 0.11 | -0.51 | 0.15 | 0.33 | 0.03 | 0.67 | -0.06 | -0.26 |
| 0.00 | 0.19 | 0.10 | 0.02 | 0.39 | -0.30 | -0.34 | 0.45 | -0.62 |
| 0.01 | 0.44 | 0.19 | 0.02 | 0.35 | -0.21 | -0.15 | -0.76 | 0.02 |
| 0.02 | 0.62 | 0.25 | 0.01 | 0.15 | 0.00 | 0.25 | 0.45 | 0.52 |
| 0.08 | 0.53 | 0.08 | -0.03 | -0.60 | 0.36 | 0.04 | -0.07 | -0.45 |

## LSI - an example

Choosing the 2 largest singular values we have


## LSI (2 singular values)

$\mathrm{A}_{\mathrm{k}}=$|  | C 1 | C 2 | C 3 | C 4 | C 5 | M 1 | M 2 | M 3 | M 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| human | 0.16 | 0.40 | 0.38 | 0.47 | 0.18 | -0.05 | -0.12 | -0.16 | -0.09 |
| Interface | 0.14 | 0.37 | 0.33 | 0.40 | 0.16 | -0.03 | -0.07 | -0.10 | -0.04 |
| Computer | 0.15 | 0.51 | 0.36 | 0.41 | 0.24 | 0.02 | 0.06 | 0.09 | 0.12 |
| User | 0.26 | 0.84 | 0.61 | 0.70 | 0.39 | 0.03 | 0.08 | 0.12 | 0.19 |
| System | 0.45 | 1.23 | 1.05 | 1.27 | 0.56 | -0.07 | -0.15 | -0.21 | -0.05 |
| Response | 0.16 | 0.58 | 0.38 | 0.42 | 0.28 | 0.06 | 0.13 | 0.19 | 0.22 |
| Time | 0.16 | 0.58 | 0.38 | 0.42 | 0.28 | 0.06 | 0.13 | 0.19 | 0.22 |
| EPS | 0.22 | 0.55 | 0.51 | 0.63 | 0.24 | -0.07 | -0.14 | -0.20 | -0.11 |
| Survey | 0.10 | 0.53 | 0.23 | 0.21 | 0.27 | 0.14 | 0.31 | 0.44 | 0.42 |
| Trees | -0.06 | 0.23 | -0.14 | -0.27 | 0.14 | 0.24 | 0.55 | 0.77 | 0.66 |
| Graph | -0.06 | 0.34 | -0.15 | -0.30 | 0.20 | 0.31 | 0.69 | 0.98 | 0.85 |
| Minors | -0.04 | 0.25 | -0.10 | -0.21 | 0.15 | 0.22 | 0.50 | 0.71 | 0.62 |

## LSI Example

- Query: "human computer interaction" retrieves documents: $\mathrm{c}_{1}, \mathrm{c}_{2}, \mathrm{c}_{4}$ but not $\mathrm{c}_{3}$ and $\mathrm{c}_{5}$.
- If we submit the same query (based on the transformation shown before) to the transformed matrix we retrieve (using cosine similarity) all $c_{1}-c_{5}$ even if $c_{3}$ and $\mathrm{c}_{5}$ have no common keyword to the query.
- According to the transformation for the queries we have:


## Query transformation

|  | query |
| :--- | :--- |
| human | 1 |
| Interface | 0 |
| computer | 1 |
| User | 0 |
| System | 0 |
| Response | 0 |
| Time | 0 |
| EPS | 0 |
| Survey | 0 |
| Trees | 0 |
| Graph | 0 |
| Minors | 0 |


$q=$| 1 |
| :--- |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

## Query transformation



$U_{\mathrm{k}}=$| 0.22 | -0.11 |
| :--- | :--- |
| 0.20 | -0.07 |
| 0.24 | 0.04 |
| 0.40 | 0.06 |
| 0.64 | -0.17 |
| 0.27 | 0.11 |
| 0.27 | 0.11 |
| 0.30 | -0.14 |
| 0.21 | 0.27 |
| 0.01 | 0.49 |
| 0.04 | 0.62 |
| 0.03 | 0.45 |

$$
\begin{gathered}
L_{k}=\begin{array}{|l|l|}
\hline 0.334 & 0 \\
\hline 0 & 0.254 \\
\hline
\end{array} \\
\mathrm{q}_{\mathrm{n}}=\mathrm{q}^{\top} \mathrm{U}_{\mathrm{k}} \mathrm{~L}_{\mathrm{k}}=\begin{array}{|l|l|}
\hline 0.138 & -0.0273 \\
\hline
\end{array}
\end{gathered}
$$

## Query transformation



$$
\mathrm{q}_{\mathrm{n}} \mathrm{~L}_{\mathrm{k}}=\begin{array}{|l|l|}
\hline 0.138 & -0.0273 \\
\hline
\end{array} \quad \begin{array}{|l|l|}
\hline 3.34 & 0 \\
\hline 0 & 2.54 \\
\hline
\end{array}=\begin{array}{|l|l|}
\hline 0.46 & -0.069 \\
\hline
\end{array}
$$

## Query transformation

- The cosine similarity matrix of query vector to the documents is:



## SVD problems

- The dimensions of the matrix change when dictionary changes
- The whole decomposition must be re-calculated when we add a word
- Sensitive to the imbalance in word frequency
- Very high dimensional matrix
- Not suitable for millions of words and documents
- Quadratic cost to perform SVD
- Solution: Directly calculate a low-dimensional representation


## Word analogy

- Words with similar meaning end up laying close to each other
- Words that share similar contexts may be analogous
- Synonyms
- Antonyms
- Names
- Colors
- Places
- Interchangeable words
- Vector arithmetics to work with analogies
- i.e. king - man + woman = queen

https://Iamyiowce.github.io/word2viz/


## But why?

- what's an analogy?

$$
\frac{p\left(w^{\prime} \mid \text { man }\right)}{p\left(w^{\prime} \mid \text { woman }\right)} \approx \frac{p\left(w^{\prime} \mid \text { king }\right)}{p\left(w^{\prime} \mid \text { queen }\right)}
$$

Assume PMI is approximated by a low rank approximation of the co-occurrence matrix.

1. $\operatorname{PMI}\left(w^{\prime}, w\right) \approx v_{w} v_{w^{\prime}}{ }^{*}$ inner product*
2. Isotropic: $E_{w^{\prime}}\left[\left(v_{w^{\prime}} v_{u}\right)\right]^{2}=\left\|v_{u}\right\|^{2}$

Then
3. $\operatorname{argmin}_{w} E_{w^{\prime}}\left[\ln \frac{p\left(w^{\prime} \mid w\right)}{p\left(w^{\prime} \mid \text { queen }\right)}-\ln \frac{p\left(w^{\prime} \mid \text { man }\right)}{p\left(w^{\prime} \mid \text { woman }\right)}\right]^{2}$
4. $\operatorname{argmin}_{w} E_{w^{\prime}}\left[\left(P M I\left(w^{\prime} \mid w\right)-P M I\left(w^{\prime} \mid q u e e n\right)\right)-\left(P M I\left(w^{\prime} \mid \text { man }\right)-P M I\left(w^{\prime} \mid \text { woman }\right)\right)\right]^{2}$
5. $\operatorname{argmin}_{w}| |\left(v_{w}-v_{\text {queen }}\right)-\left(v_{\text {man }}-v_{\text {woman }}\right) \|^{2}$
6. $v_{w} \approx v_{\text {queen }}-v_{\text {woman }}+v_{\text {man }}$ which is an analogy!

- Arora et al (ACL 2016) shows that if (2) holds then (1) holds as well
- So we need to construct vectors from co-occurrence that satisfy (2)
- $\mathrm{d} \ll|\mathrm{V}|$ in order to have isotropic vectors


## Learning Word Vectors

Corpus containing a sequence of T training words
$>$ Objective: $\mathrm{f}\left(w_{t}, \ldots, w_{t-n+1}\right)=$ $\widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \ldots \mathrm{w}_{\mathrm{t}-1}\right)$
$>$ Decomposed in two parts:

$>$ Mapping C (1-hotv => lower dimensions)
$>$ Mapping any $\mathbf{g}$ s.t. (estimate prob $\mathrm{t}+1 \mid \mathrm{t}$ previous)
$\mathrm{f}\left(w_{t-1}, \cdots, w_{t-n+1}\right)=\mathrm{g}\left(\mathrm{C}\left(w_{t-1}\right), \cdots, \mathrm{C}\left(w_{t-n+1}\right)\right)$

- $\mathrm{C}(\mathrm{i})$ is the i -th word feature vector
(Word embedding)
$>$ Objective function: $J=\frac{1}{T} \sum \mathrm{f}\left(w_{t}, \ldots, w_{t-n+1}\right)$


Bengio, Yoshua, et al. "A neural probabilistic language model." The Journal of Machine Learning Research 3 (2003): 1137-1155.

## Neural Net Language Model

For each training sequence: input $=($ context, target $)$ pair: $\left(w_{t-n+1} \ldots w_{t-1}, w_{t}\right)$ objective: minimize $\mathrm{E}=-\log \widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \ldots \mathrm{w}_{\mathrm{t}-1}\right)$

OUTPUT
LAYER

HIDDEN
LAYER
nonlinear

PROJECTION LAYER
linear

## INPUT LAYER

input context:

$$
\text { softmax. } \quad i^{\text {th }} \text { output }=\widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{i}}=\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \cdots \mathrm{w}_{\mathrm{t}-1}\right)
$$


$\mathrm{w}_{\mathrm{t}-\mathrm{n}+1}$
$\stackrel{\uparrow}{\mathrm{w}_{\mathrm{t}-2}}$

## Objective function

- $\mathrm{E}=-\log \widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \cdots \mathrm{w}_{\mathrm{t}-1}\right)$
- a probability between 0 and 1.
- On this support, the log is negative =>-log term positive.
- makes sense to try to minimize it.
- Probability of word given the context be as high as possible (1 for a perfect prediction).
- case the error is equal to 0 (global minimum).

| $\mathbf{p}$ | $\log (\mathbf{p})$ | $-\log (\mathbf{p})$ |
| :---: | :---: | :---: |
| 0,7 | $-0,15490196$ | 0,15490196 |
| 0,2 |  |  |

## NNLM Projection layer

> Performs a simple table lookup in $\mathrm{C}_{|\mathrm{VV}| \mathrm{m}}$ : concatenate the rows of the shared mapping matrix $\mathrm{C}_{|\mathrm{V}| \mathrm{m}}$ corresponding to the context words

Example for a two-word context $\mathrm{w}_{\mathrm{t}-2} \mathrm{w}_{\mathrm{t}-1}$ :


Concatenate (1) and (2) $\rightarrow$ C $\left(\mathrm{w}_{\mathrm{t}-2}\right) \quad \mathrm{C}\left(\mathrm{w}_{\mathrm{t}-1}\right)$
$>\mathrm{C}_{\mid \mathrm{VV}, \mathrm{m}}$ is critical: it contains the weights that are tuned at each step. After training, it contains what we're interested in: the word vectors

## NNLM hidden/output layers and training

> Softmax (log-linear classification model) is used to output positive numbers that sum to one (a multinomial probability distribution):
for the $i^{\text {th }}$ unit in the output layer: $\widehat{P}\left(w_{i}=w_{t} \mid w_{t-n+1} \ldots w_{t-1}\right)=\frac{e^{y w_{i}}}{\sum_{i^{\prime}=1}^{\mid V_{1}} e^{y w_{i^{\prime}}}}$
Where:
$-y=b+U \cdot \tanh (d+H . x)$

- tanh : nonlinear squashing (link) function
-x : concatenation C(w) of the context weight vectors seen previously
- b : output layer biases (|V| elements)
- d : hidden layer biases (h elements). Typically $500<\mathrm{h}<1000$
$-\mathrm{U}:|\mathrm{V}|$ * h matrix storing the hidden-to-output weights
$-\mathrm{H}:(\mathrm{h} *(\mathrm{n}-1) \mathrm{m})$ matrix storing the projection-to-hidden weights
$\rightarrow \boldsymbol{\theta}=(\boldsymbol{b}, \boldsymbol{d}, \boldsymbol{U}, \boldsymbol{H}, \boldsymbol{C})$
- Complexity per training sequence: $\mathrm{n} * \mathrm{~m}+\mathrm{n} * \mathrm{~m} * \mathrm{~h}+\mathbf{h} *|\mathbf{V}|$ computational bottleneck: nonlinear hidden layer ( $\mathrm{h} *|\mathrm{~V}|$ term)
> Training is performed via stochastic gradient descent (learning rate $\varepsilon$ ):


$$
\theta \leftarrow \theta+\varepsilon \cdot \frac{\partial \mathrm{E}}{\partial \theta}=\theta+\varepsilon \cdot \frac{\partial \log \widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \cdots \mathrm{w}_{\mathrm{t}-1}\right)}{\partial \theta}
$$

(weights are initialized randomly, then updated via backpropagation)

## NNLM facts

- tested on Brown (1.2M words, $|\mathrm{V}| \cong 16 \mathrm{~K}$ ) and AP News (14M words, $|\mathrm{V}| \cong 150 \mathrm{~K}$ reduced to 18K) corpuses
- Brown: $\mathrm{h}=100, \mathrm{n}=5, \mathrm{~m}=30$
- AP News: $\mathrm{h}=60, \mathrm{n}=6, \mathrm{~m}=100,3$ week training using 40 cores
- $24 \%$ and $8 \%$ relative improvement (resp.) over traditional smoothed n -gram LMs
- in terms of test set perplexity: geometric average of $1 / \widehat{P}\left(w_{t} \mid w_{t-n+1} \ldots w_{t-1}\right)$
- Due to complexity, NNLM can't be applied to large data sets $\rightarrow$ poor performance on rare words
- Bengio et al. (2003) initially thought their main contribution was a more accurate LM. They let the interpretation and use of the word vectors as future work
- On the opposite, Mikolov et al. (2013) focus on the word vectors


## Word2Vec

$>$ Mikolov et al. in 2013
$>$ Key idea of word2vec: achieve better performance not by using a more complex model (i.e., with more layers), but by allowing a simpler (shallower) model to be trained on much larger amounts of data
$>$ no hidden layer (leads to 1000X speedup)
$>$ projection layer is shared (not just the weight matrix) - C
$>$ context: words from both history \& future:

- Two algorithms for learning words vectors:
- CBOW: from context predict target
- Skip-gram: from target predict context


## Continuous Bag-of-Words (CBOW)

$>$ continuous bag-of-words
$>$ continuous representations whose order is of no importance
$>$ uses the surrounding words to predict the center word
$>$ n-words before and after the target word


## Continuous Bag-of-Words (CBOW)

For each training sequence: input $=($ context, target $)$ pair: $\left(w_{t-\frac{n}{2}} \ldots W_{t-1} W_{t+1} \ldots w_{t+\frac{n}{2}}, W_{t}\right)$
objective: minimize $-\log \widehat{\mathrm{P}}\left(\mathrm{w}_{\mathrm{t}} \mid \mathrm{w}_{\mathrm{t}-\mathrm{n}+1} \ldots \mathrm{w}_{\mathrm{t}-1}\right)$


## Weight updating

$>$ For each (context, target $=w_{\mathrm{t}}$ ) pair, only the word vectors from matrix C corresponding to the context words are updated
$>$ Recall that we compute $\mathrm{P}\left(\mathrm{w}_{\mathrm{i}}=\mathrm{w}_{\mathrm{t}} \mid\right.$ context $) \forall \mathrm{w}_{\mathrm{i}} \in \mathrm{V}$. We compare this distribution to the true probability distribution ( 1 for $w_{t}, 0$ elsewhere)

## Back propagation

If $P\left(w_{i}=w_{t} \mid\right.$ context $)$ is overestimated (i.e., $>0$, happens in potentially $|V|-1$ cases), some portion of $\mathrm{C}^{\prime}\left(\mathrm{w}_{\mathrm{i}}\right)$ is subtracted from the context word vectors in C , proportionally to the magnitude of the error

Reversely, if $P\left(w_{i}=w_{t}\right.$ I context) is underestimated (<1, happens in potentially 1 case), some portion of $C^{\prime}\left(w_{i}\right)$ is added to the context word vectors in $C$
$\rightarrow$ at each step the words move away or get closer to each other in the feature space $\rightarrow$ clustering

input $\rightarrow$ projection weight matrix
constant adjustments


$$
\mathrm{C}^{\prime}\left(\mathrm{w}_{1}\right)
$$

$$
\text { projection } \rightarrow \text { output }
$$

1
weight matrix


## Skip-gram

$>$ skip-gram uses the center word to predict the surrounding words
$>$ instead of computing the probability of the target word $w_{t}$ given its previous words, we calculate the probability of the surrounding word $w_{t+j}$ given $w_{t}$
$>\mathrm{p}\left(w_{t+j} \mid w_{t}\right)=\frac{\exp \left(v_{w_{t}}^{T} v_{w_{t+j}}^{\prime}\right)}{\sum_{w \in V} \exp \left(v_{w_{t}}^{T} v_{w_{t+j}}^{\prime}\right)}$
$>\boldsymbol{v}^{T}{ }_{w t}$ is a column of $\boldsymbol{W}_{V x N}$ and $\boldsymbol{v}_{\boldsymbol{w}_{t+j}}^{\prime}$ is a column of

$W^{\prime}{ }_{N x V}$

$$
J=\frac{1}{T} \sum_{t=1}^{T} \sum_{-n \leq j \leq n} \log \mathrm{p}\left(w_{t+j} \mid w_{t}\right)
$$

## Word2vec facts

$>$ Complexity is $\mathbf{n} * \mathbf{m}+\mathbf{m} * \log |\mathbf{V}|$ (Mikolov et al. 2013a)
$>\mathrm{n}$ :size of the context window ( $\sim 10$ ) $\mathbf{n x m}$ : dimensions of the projection layer, $|\mathrm{V}|$ size of the vocabulary
$>$ On Google news 6B words training corpus, with $|\mathbf{V}| \sim 10^{6}$ :

- CBOW with $\mathrm{m}=1000$ took 2 days to train on 140 cores
- Skip-gram with m = 1000 took 2.5 days on 125 cores
- NNLM (Bengio et al. 2003) took 14 days on 180 cores, for $m=100$ only!
(note that $\mathrm{m}=1000$ was not reasonably feasible on such a large training set)
> word2vec training speed $\cong 100 \mathrm{~K}-5 \mathrm{M}$ words/s
$>$ Quality of the word vectors:
- 7 significantly with amount of training data and dimension of the word vectors ( m ), with diminishing relative improvements
- measured in terms of accuracy on 20K semantic and syntactic association tasks.
e.g., words in bold have to be returned:

| Capital-Country | Past tense | Superlative | Male-Female | Opposite |
| :--- | :--- | :--- | :--- | :--- |
| Athens: Greece | walking: <br> walked | easy: easiest | brother: sister | ethical: unethical |

$>$ Best NNLM: 12.3\% overall accuracy. Word2vec (with Skip-gram): 53.3\%
> References: http://www.scribd.com/doc/285890694/NIPS-DeepLearning Workshop-NNforText\#scribd https://code.google.com/p/word2vec/

## GloVe

| Probability and Ratio | $k=$ solid | $k=$ gas | $k=$ water | $k=$ fashion |
| :--- | :---: | :---: | :---: | :---: |
| $P(k \mid$ ice $)$ | $1.9 \times 10^{-4}$ | $6.6 \times 10^{-5}$ | $3.0 \times 10^{-3}$ | $1.7 \times 10^{-5}$ |
| $P(k \mid$ steam $)$ | $2.2 \times 10^{-5}$ | $7.8 \times 10^{-4}$ | $2.2 \times 10^{-3}$ | $1.8 \times 10^{-5}$ |
| $P(k \mid$ ice $) / P(k \mid$ steam $)$ | 8.9 | $8.5 \times 10^{-2}$ | 1.36 | 0.96 |

- Ratio of co-occurrence probabilities best distinguishes relevant words
$F\left(w_{i}, w_{j}, \tilde{w}_{k}\right)=\frac{P_{i k}}{P_{j k}} \quad \square \quad w_{i}^{T} \tilde{w}_{k}+b_{i}+\tilde{b}_{k}=\log \left(X_{i k}\right)$
- Cast this into a lease square problem:
- $X$ co-occurrence matrix
- $f$ weighting function,

$$
\begin{aligned}
& J=\sum_{i, j=1}^{V} f\left(X_{i j}\right)\left(w_{i}^{T} \tilde{w}_{j}+b_{i}+\tilde{b}_{j}-\log X_{i j}\right)^{2} \\
& f(x)=\left\{\begin{array}{cc}
\left(x / x_{\max }\right)^{\alpha} & \text { if } x<x_{\max } \\
1 & \text { otherwise } .
\end{array}\right.
\end{aligned}
$$

- b biasterms
- $w_{i}=$ word vector
- $\widetilde{w_{j}}=$ context vector
model that utilizes
- count data
- bilinear prediction-based methods like word2vec


## Which is better?

- Open question
- SVD vs word2vec vs GloVe
- All based on co-occurrence
- Levy, O., Goldberg, Y., \& Dagan, I. (2015)
- SVD performs best on similarity tasks
- Word2vec performs best on analogy tasks
- No single algorithm consistently outperforms the other methods
- Hyperparameter tuning is important
- 3 out of 6 cases, tuning hyperparameters is more beneficial than increasing corpus size
- word2vec outperforms GloVe on all tasks
- CBOW is worse than skip-gram on all tasks


## Applications

- Word analogies
- Find similar words
- Semantic similarity
- Syntactic similarity
- POS tagging
- Similar analogies for different languages
- Document classification

https://lamyiowce.github.io/word2viz/


## Applications

$>$ High quality word vectors boost performance of all NLP tasks, including document classification, machine translation, information retrieval...
> Example for English to Spanish machine translation:


About 90\% reported accuracy (Mikolov et al. 2013c)


Mikolov, T., Le, Q. V., \& Sutskever, I. (2013). Exploiting similarities among languages for machine translation. arXiv preprintarXiv:1309.4168.

## Remarkable properties of word vectors


regularities between words are encoded in the difference vectors e.g.,there is a constant country-capital difference vector

Mikolov et al. (2013b)
Distributed representations of
words and phrases and their

## Remarkable properties of word vectors



## Remarkable properties of word vectors


constant male-female difference vector

constant singular-plural difference vector
> Vector operations are supported and make intuitive sense:

$$
\begin{array}{rr}
w_{\text {king }}-w_{\text {man }}+w_{\text {woman }} \cong w_{\text {queen }} & w_{\text {einstein }}-w_{\text {scientist }}+w_{\text {painter }} \cong w_{\text {picasso }} \\
w_{\text {paris }}-w_{\text {france }}+w_{\text {italy }} \cong w_{\text {rome }} & w_{\text {his }}-w_{\text {he }}+w_{\text {she }} \cong w_{\text {her }} \\
w_{\text {windows }}-w_{\text {microsoft }}+w_{\text {google }} \cong w_{\text {android }} & w_{c u}-w_{\text {copper }}+w_{\text {gold }} \cong w_{\text {au }}
\end{array}
$$

$>$ Online demo (scroll down to end of tutorial)

## Distributed Representations of Sentences and Documents

- Doc2vec
- Paragraph or document vectors
- Capable of constructing representations of input sequences of variable length
- Represent each document by a dense vector
- Trained to predict words in the document
- paragraph vector and word vectors are averaged or concatenated to predict the next word in a context
- can be thought of as another word shared across all contexts in document


| Model | Error rate <br> (Positive/ <br> Negative) | Error rate <br> (Fine- <br> grained) |
| :--- | ---: | ---: |
| Naïve Bayes <br> (Socher et al., 2013b) | $18.2 \%$ | $59.0 \%$ |
| SVMs (Socher et al., 2013b) | $20.6 \%$ | $59.3 \%$ |
| Bigram Naïve Bayes <br> (Socher et al., 2013b) | $16.9 \%$ | $58.1 \%$ |
| Word Vector Averaging <br> (Socher et al., 2013b) | $19.9 \%$ | $67.3 \%$ |
| Recursive Neural Network <br> (Socher et al., 2013b) | $17.6 \%$ | $56.8 \%$ |
| Matrix Vector-RNN <br> (Socher et al., 2013b) | $17.1 \%$ | $55.6 \%$ |
| Recursive Neural Tensor Network <br> (Socher et al., 2013b) | $14.6 \%$ | $54.3 \%$ |
| Paragraph Vector | $\mathbf{1 2 . 2 \%}$ | $\mathbf{5 1 . 3 \%}$ |

## Word Mover's distance

- "Edit" distance of 2 documents
- Based on word embedding representations
- Incorporate semantic similarity between individual word pairs into the document distance metric
- Based on "travel cost" between two words
- Calculates the cost of moving d to d'
- hyper-parameterfree
- highly interpretable
- high retrieval accuracy

| document 1 |
| :--- |
| Obama <br> speaks <br> to <br> the <br> media <br> in <br> Illinois |



## Word Mover's distance example

With the BOW
representation $D_{1}$ and $D_{2}$ are at equal distance from $\mathrm{D}_{0}$. Word embeddings allow to capture the fact that $\mathrm{D}_{1}$ is closer.

Kusner, M. J., Sun, E. Y., Kolkin, E. N. I., \& EDU, W. From Word Embeddings To Document Distances. Proceedings of the 32nd International Conference on Machine Learning, Lille, France, 2015. JMLR: W\&CP volume 37.


## Word Mover's distance computation

$d_{i}=\frac{c_{i}}{\sum_{j=1}^{n} c_{j}}:$ Normalized frequency of word $\boldsymbol{i}$
$c(i, j)=\left\|\mathbf{x}_{i}-\mathbf{x}_{j}\right\|_{2}$ the word embed dings distance among words $\boldsymbol{i}, \boldsymbol{j}$

- Assume documents d,d’.
- Assume each word ifrom d can be transformed into any word $j$ in $d^{\prime}$
- $\boldsymbol{T} i j \geq \mathbf{0}$ denotes how much of word $\boldsymbol{i}$ in $\boldsymbol{d}$ travels to word $j$ in $\boldsymbol{d}^{\prime}$.
- To transform $\boldsymbol{d}$ entirely into $\boldsymbol{d}^{\prime}$ : entire outgoing flow from word $\boldsymbol{i}$ equals $\boldsymbol{d}_{\boldsymbol{i}}$ : .
- Transportation problem:

$$
\begin{aligned}
\min _{\mathbf{T} \geq 0} & \sum_{i, j=1}^{n} \mathbf{T}_{i j} c(i, j) \\
\text { subject to: } & \sum_{j=1}^{n} \mathbf{T}_{i j}=d_{i} \quad \forall i \in\{1, \ldots, n\} \\
& \sum_{i=1}^{n} \mathbf{T}_{i j}=d_{j}^{\prime} \quad \forall j \in\{1, \ldots, n\} .
\end{aligned}
$$

- Learn parameters $\boldsymbol{T}_{i j}$ then the distance is: $\sum_{i, j=1}^{n} \mathbf{T}_{i j} c(i, j)$


## Representation Learning for Greek

- Prototype and resources
http://archive.aueb.gr:7000
- Paper: Word Embeddings from Large-Scale Greek Web Content
https://arxiv.org/abs/1810.06694


## EYXAPIITIEL ...!

Google Scholar: https://bit.ly/2rwmvQU
Twitter: @mvazirg

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- nice blog post on neural nets applied to NLP: http://colah.github.io/posts/2014-07-NLP-RNNsRepresentations/
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