

Theorem 3.1.1 For all formulas $\phi, \psi \in L_n$, structures $M \in M_n$, and agents $i = 1, \dots, n$,

- (a) if ϕ is an instance of a propositional tautology, then $M \models \phi$,
- (b) $M \models (Ki\phi \wedge Ki(\phi \Rightarrow \psi)) \Rightarrow Ki\psi$,
- (c) if $M \models \phi$ and $M \models \phi \Rightarrow \psi$ then $M \models \psi$,
- (d) if $M \models \phi$ then $M \models Ki\phi$.

Axiom system K_n :

- A1. All tautologies of propositional calculus
- A2. $(Ki\phi \wedge Ki(\phi \Rightarrow \psi)) \Rightarrow Ki\psi$, $i = 1, \dots, n$ (Distribution Axiom)
- R1. From ϕ and $\phi \Rightarrow \psi$ infer ψ (modus ponens)
- R2. From ϕ infer $Ki\phi$, $i = 1, \dots, n$ (Knowledge Generalization)

Proposition For all formulas $\phi, \psi \in L_n$ where $\phi \Rightarrow \psi$ is a propositional tautology, if $K_n \vdash \phi$ then $K_n \vdash \psi$.

Lemma For all formulas $\phi, \psi \in L_n$, structures $M \in M_n$, and agents $i = 1, \dots, n$,

- A** if $M \models \phi \Rightarrow \psi$ then $M \models Ki\phi \Rightarrow Ki\psi$
- B** if $K_n \vdash \phi \Rightarrow \psi$ then $K_n \vdash Ki\phi \Rightarrow Ki\psi$

Prove B:

$$\phi \Rightarrow \psi$$

$$Ki(\phi \Rightarrow \psi) \quad \mathbf{R2}$$

$$Ki(\phi \Rightarrow \psi) \Rightarrow (Ki\phi \Rightarrow Ki\psi)$$

A2 και Proposition

$$Ki\phi \Rightarrow Ki\psi \quad \mathbf{R1}$$

Σχετικό εκπαιδευτικό υλικό

Huth – Ryan Logic in Computer Science 2nd Ed.

Ενότητες: 1.1 , 1.3 , 1.4 , 1.5.1 , 1.5.2

Fagin – Halpern – Moses – Vardi Reasoning About Knowledge

Ενότητα 2.1. The Possible-Worlds Model

Ενότητα 2.4. The Properties of Knowledge , μέχρι και το Theorem 2.4.1.