Γνώση του Α όταν οι Β , C δεν γνωρίζουν το χρώμα τους

Ορισμός Έστω $\mathbf{M_A}$ ένα μοντέλο Kripke που αναπαριστά την **γνώση του Α**:
Ο παίκτης Α *γνωρίζει ότι αληθεύει* θ , άν $M_A \models \theta$.

Take any cluster **S** of *possible worlds that* A *can not distinguish between*.

Partition this cluster into sub-clusters, say T1, T2, etc., that are *indistinguishable for* B. That is, B can not distinguish between members of T1; B can not distinguish between members of T2, and so on.

1) By [B sees A], **A knows** that either B knows AisWh or B knows ¬AisWh.

   This means that in T1 it cannot happen that AisWh is true at some worlds and false at others; similarly for T2, and so on.
   Briefly, for each k, either AisWh is true at all the possible worlds of Tk,
   or else AisWh is false at all the possible worlds of Tk.

2) By [B hears C], **A knows** that B knows that AisWh ∨ BisWh.

   Then AisWh ∨ BisWh must be true throughout each Tk.
   Combining with (1) it follows that, <u>for each k</u> :
   either AisWh is true at every possible world of Tk,
   or BisWh is true at every possible world of Tk.

   Άρα, AisWh ∨ K_B BisWh is true at every possible world of Tk.

Since the Tk are a partition of **S**, AisWh ∨ K_B BisWh must be true at every world of **S**.
Since AisWh ∨ K_B BisWh is true at all the worlds that A thinks are possible,

$\mathbf{M_A} \models$ AisWh ∨ K_B BisWh .

3) By [B doesn't know], **A knows** that B doesn't know BisWh.

   Then $\mathbf{M_A} \models \neg K_B$ BisWh ,

   and $\mathbf{M_A} \models$ AisWh .

ΕΡΩΤΗΜΑ Τι συμβαίνει όταν ο Α είναι *μαύρος*;

Tableau proof for: 1 , 2 |= (AisWh $\vee$ K_B BisWh)

1	(s1)	A knows that (B knows AisWh or B knows ¬AisWh)	
2	(s1)	A knows that B knows that (AisWh $\vee$ BisWh)	
4	(s1)	¬ (AisWh $\vee$ K_B BisWh)	
5	(s1)	¬ AisWh	from 4
6	(s1)	¬ K_B BisWh	from 4
7	(s1 B s3)	¬ BisWh	from 6
8	(s1)	B knows that AisWh $\vee$ BisWh	from 2
9	(s1 B s3)	AisWh $\vee$ BisWh	from 8
10	(s1 B s3)	AisWh	from 9 , 7
11	(s1)	B knows AisWh **or** B knows ¬AisWh	from 1
		BRANCH 1	
11; **1**	(s1)	B knows ¬AisWh	
12	(s1 B s3)	¬ AisWh	from 11; 1
		branch closes with 12, 10	
		BRANCH 2	
11; **2**	(s1)	B knows AisWh	
13	(s1)	AisWh	from 11; 2
		branch closes with 13, 5	

Tableau proof for: 1 , 2 , 3 |= A knows AisWh

1	(s1)	A knows that (B knows AisWh or B knows ¬AisWh)	
2	(s1)	A knows that B knows that AisWh ∨ BisWh	
3	(s1)	A knows that B doesn't know BisWh	
4	(s1)	¬ A knows AisWh	
5	(s1 A s2)	¬ AisWh	from 4
6	(s1 A s2)	B doesn't know BisWh	from 3
7	(s1 A s2)	B knows that AisWh ∨ BisWh	from 2
8	(s1 A s2 B s3)	¬ BisWh	from 6
9	(s1 A s2 B s3)	AisWh ∨ BisWh	from 7
10	(s1 A s2 B s3)	AisWh	from 9 , 8
11	(s1 A s2)	B knows AisWh or B knows ¬AisWh	from 1

BRANCH 1

11;1	(s1 A s2)	B knows ¬AisWh	
12	(s1 A s2 B s3)	¬ AisWh	from 11;1

closes with 10

BRANCH 2

11;2	(s1 A s2)	B knows AisWh	
13	(s1 A s2)	AisWh	from 11;2

closes with 5