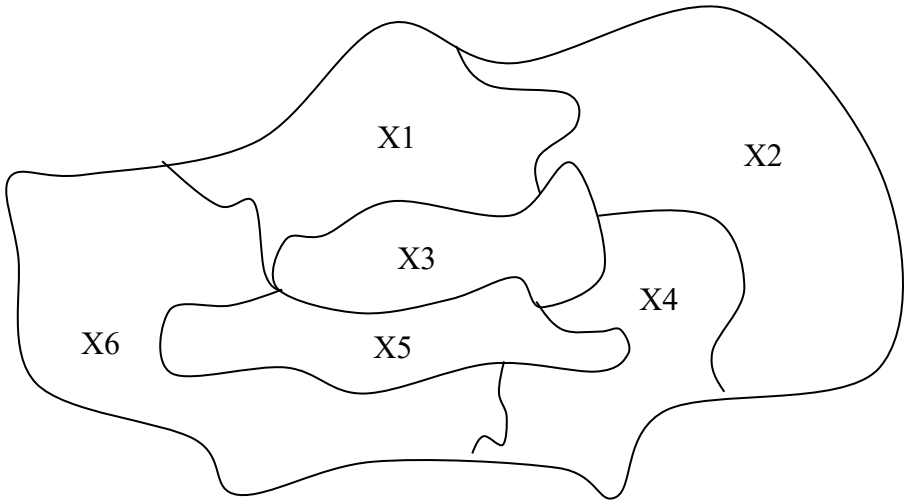
UNIVERSITY OF PATRAS
DEPT. OF COMPUTER ENGINEERING & INFORMATICS
ARTIFICIAL INTELLIGENCE

SOLUTIONS of 2nd Assignment

CONSTRAINT SATISFACTION

You are given the following problem: "There is a plane map (the one in the figure below) that contains six (6) different regions and you are asked to color the regions using only four (4) colors (red, yellow, green, blue) in a way that no two adjacent regions have the same color".



To solve it, describe the problem as a constraint satisfaction problem, by specifying the variables, their domains and the constraints between them.

Problem representation:
Variables: V1, V2, V3, V4, V5 ,V6
Domains : D1≡D2≡D3≡D4≡D5≡D6≡={R,Y,G,B}

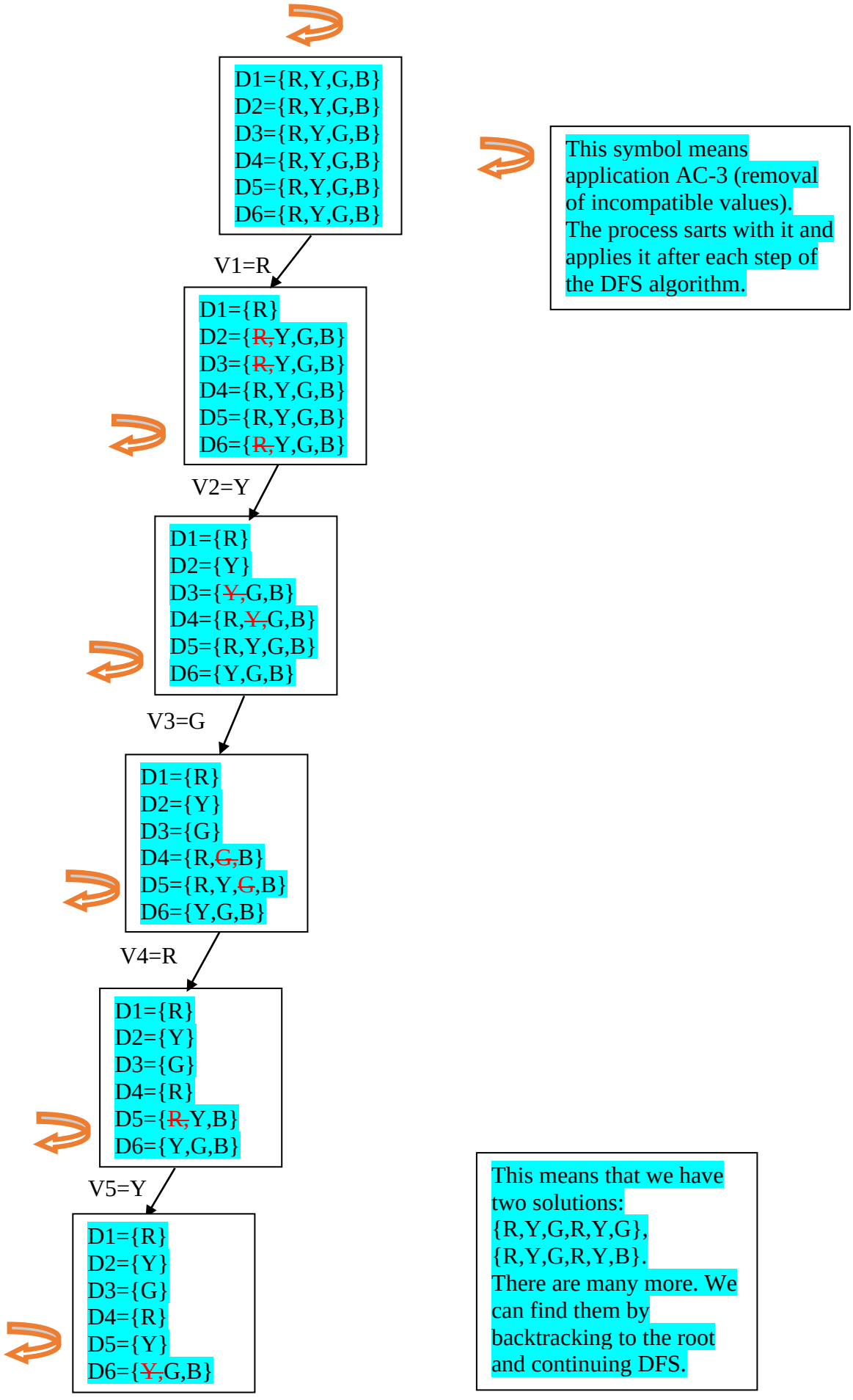
The only general constraint given to us by the problem is that two neighboring variables (regions on the map) cannot have the same value (color) at the same time.

Therefore, the following specific constraints should be satisfied:

V1 ≠ V2 (C1)	V1 ≠ V6 (C3)	V3 ≠ V5 (C7)	V5 ≠ V4 (C10)
V1 ≠ V3 (C2)	V6 ≠ V3 (C4)	V3 ≠ V2 (C8)	V2 ≠ V4 (C11)
V1 ≠ V6 (C3)	V6 ≠ V5 (C5)	V3 ≠ V4 (C9)	

(1) Apply an arc consistency algorithm combining a classical depth-first search approach with the AC-3 consistency algorithm. Draw the first three levels of the search tree.

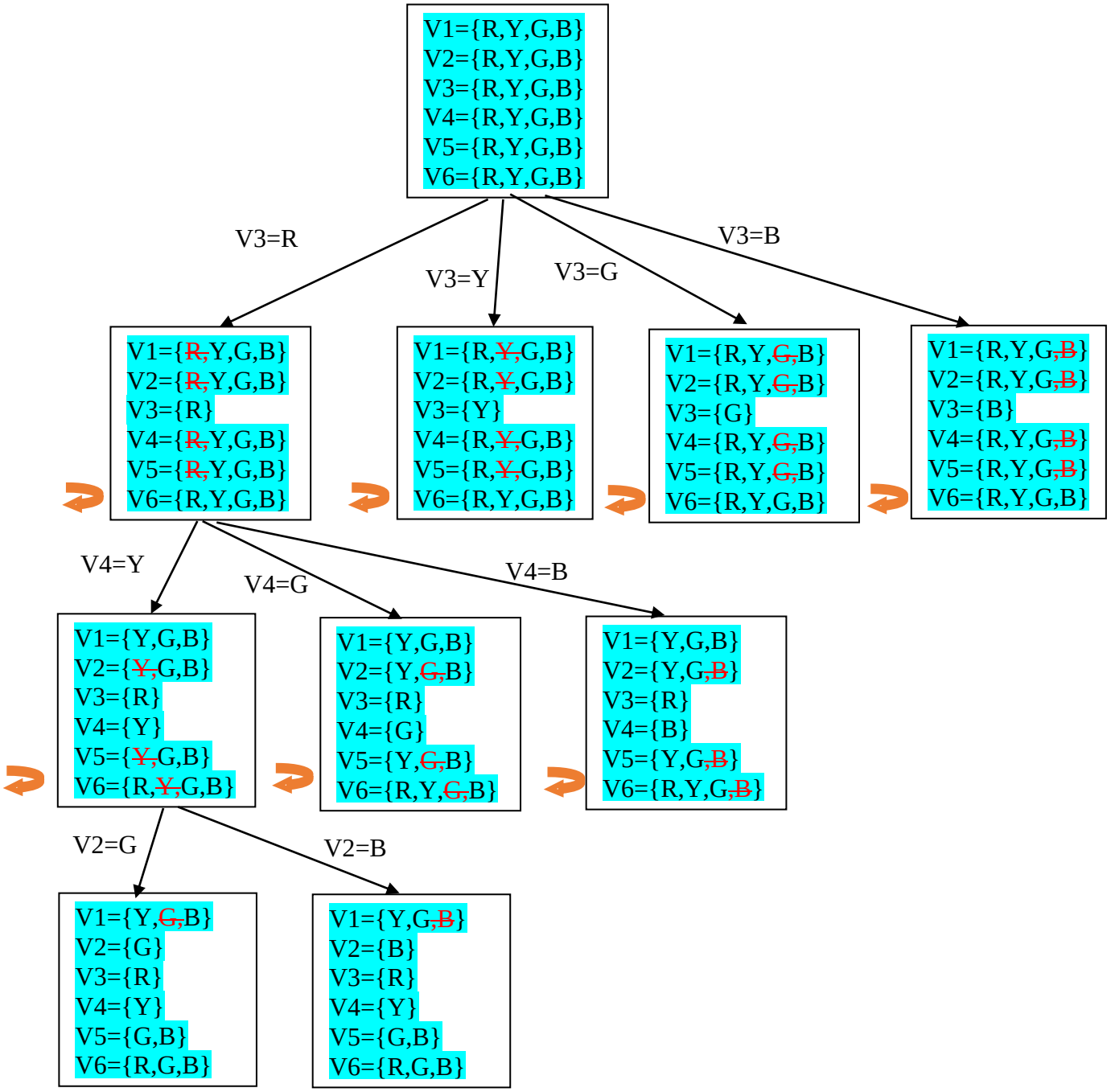
Answer


D1={R,Y,G,B}
D2={R,Y,G,B}
D3={R,Y,G,B}
D4={R,Y,G,B}
D5={R,Y,G,B}
D6={R,Y,G,B}
V1=R
D1={R}
D2={~~R,~~Y,G,B}
D3={~~R,~~Y,G,B}
D4={R,Y,G,B}
D5={R,Y,G,B}
D6={~~R,~~Y,G,B}
V2=Y
D1={R}
D2={Y}
D3={~~Y,~~G,B}
D4={R,~~Y,~~G,B}
D5={R,Y,G,B}
D6={Y,G,B}
V3=G
D1={R}
D2={Y}
D3={G}
D4={R,~~G,~~B}
D5={R,Y,~~G,~~B}
D6={Y,G,B}
V4=R
D1={R}
D2={Y}
D3={G}
D4={R}
D5={~~R,~~Y,B}
D6={Y,G,B}
V5=Y
D1={R}
D2={Y}
D3={G}
D4={R}
D5={Y}
D6={~~Y,~~G,B}
This symbol means application AC-3 (removal of incompatible values). The process sarts with it and applies it after each step of the DFS algorithm.
This means that we have two solutions: {R,Y,G,R,Y,G}, {R,Y,G,R,Y,B}. There are many more. We can find them by backtracking to the root and continuing DFS.

(2) The same as (1), but use a best-first search instead of a depth-first one. Notice that you need a define heuristic metric here.

Answer

Heuristic metric concerns way we choose the current variable to give a value. It is based on (a) the shortest failure principle (choose variable with the smaller domain), (b) most constraint variable principle (choose the variable that participates in most constraints, in case of equivalent domains), and (c) use order priority (in case of equivalent a and b).



We continue in this mode until all variables have a single value. This means a solution is found. To find all solutions, we deploy the whole tree.