Department of Computer Engineering & Informatics

Laboratory for Signal Processing and Communications

Wireless and Mobile Communications

Key Technologies:

Multicarrier Transmission and OFDM
(Orthogonal Frequency Division Multiplexing)

Outline

- Single-carrier and multicarrier modulation systems
- Frequency multiplexing
- Quadrature multiplexing in frequency
- Basic characteristics of quadrature multiplexing in frequency
- Applications and related systems

Single-Carrier Modulation

- We have seen that real channels (especially mobile) introduce **intersymbol interference**.
- Generally, ISI is introduced:
  - When the channel is frequency selective (even worse when we have sharp and large dips in the spectrum).
  - When the symbol period is less than the coherence time of the channel's impulse response.
- ISI can be reduced by using a suitable **equalizer**, but it is often the case that the remaining ISI is non-negligible thus causing performance degradation (unless we drastically increase the complexity at the receiver).
- We have so far considered the use of a single frequency carrier, namely:
  - If the channel is bandlimited with a certain bandwidth,
  - then the information signal (baseband) is modulated and shifted to the frequencies of the channel transition band.

Multicarrier Modulation

■ Multicarrier Transmission is a very successful alternative to Single Carrier Transmission

■ Let us divide the available bandwidth W (baseband or passband):

- in K sub-channels of equal bandwidth

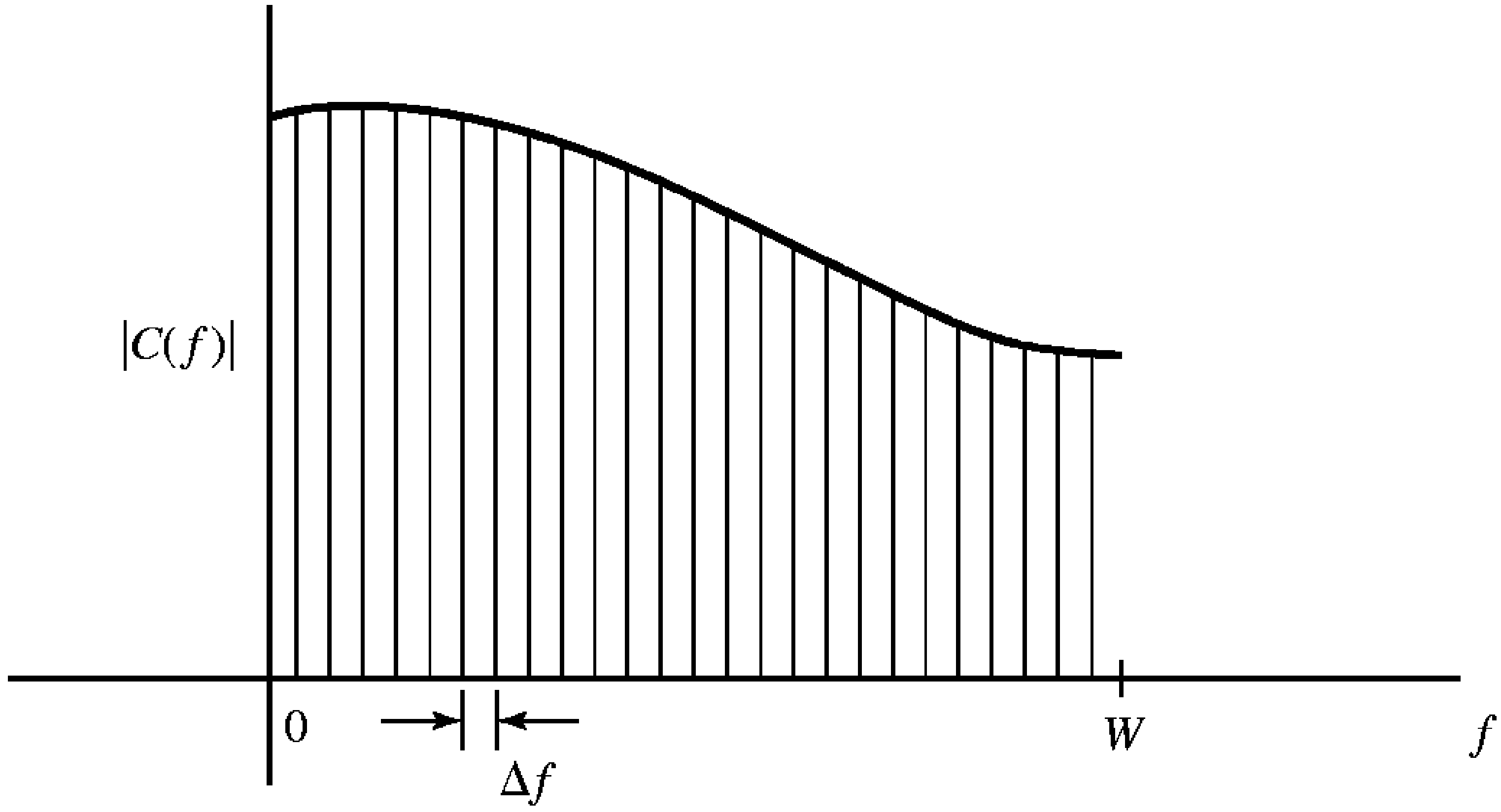
$$\Delta f = W/K$$

■ If the bandwidth of each sub-channel is narrow enough:

- the **frequency response** within each sub-channel can be considered constant
- which, in the time domain, means that the **impulse response** of the subchannel has shot time duration (tending to delta)
- therefore, the introduced ISI may be very small to insignificant

■ **Frequency Division Multiplexing (FDM)**: a different information symbol can be transmitted to each subchannel

Channel bandwidth division

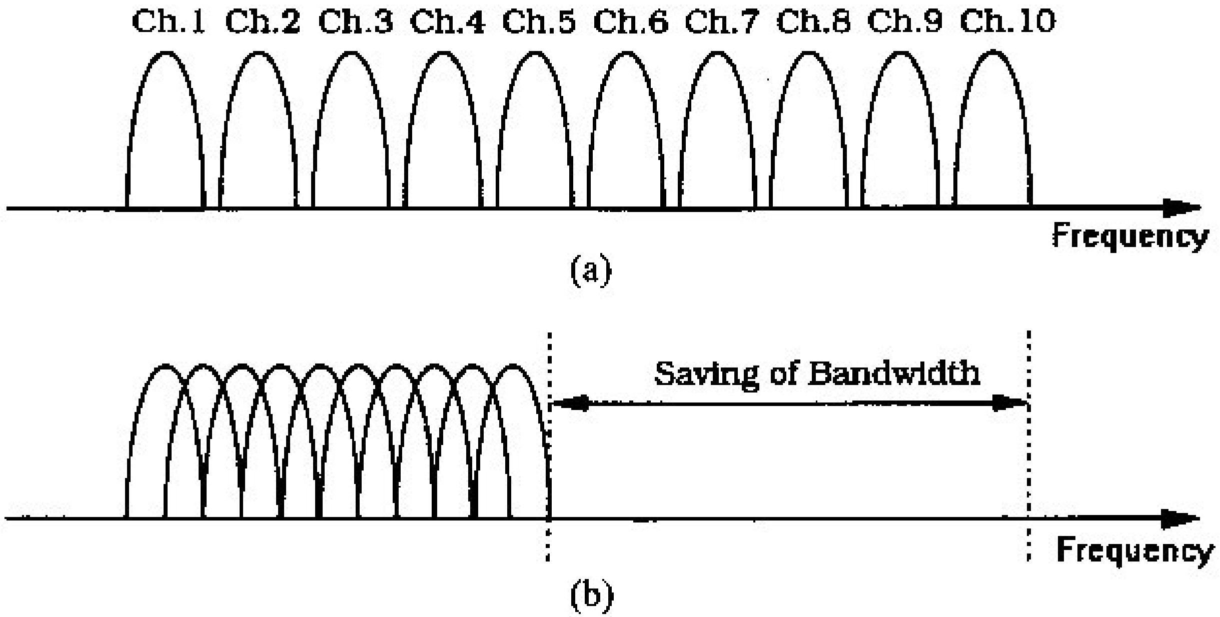


- Bandwidth W is divided into K subchannels.
- Each subchannel has quite narrow bandwidth.

Orthogonal FDM: General concept

■ In OFDM system the spectra of the sub-carriers overlap but this does not cause the phenomenon of inter-carrier interference.

■ To achieve this, the sub-carriers must be mathematically orthogonal (as long as they are at a distance equal to k/T, where T is the duration of the OFDM symbol).



Orthogonal FDM (1/5)

- A different carrier is used in each subchannel k,

$$y_k(t) = cos2\pi f_k t, \qquad k = 0,1,\dots,K-1$$

  - Where f_k the central frequency of the subchannel.

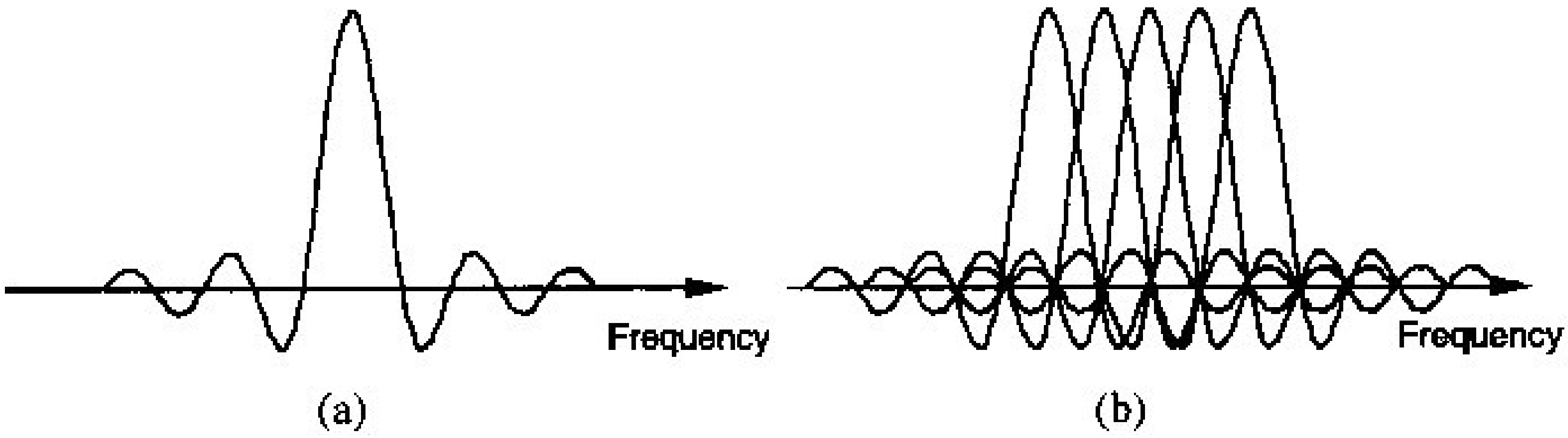
- If the frequency difference between successive sub-channels is (at least) $\Delta f = 1/T$, where T s the symbol rate in each sub-channel,

  - then the sub-carriers are **orthogonal** to each other regardless of their phases

$$\int_0^T \cos(2\pi f_k t + \phi_k)\cos\left(2\pi f_j t + \phi_j\right) dt = 0, k \neq j$$

- Then we get, **Orthogonal Frequency Division Multiplexing - OFDM**

Orthogonal FDM (2/5)



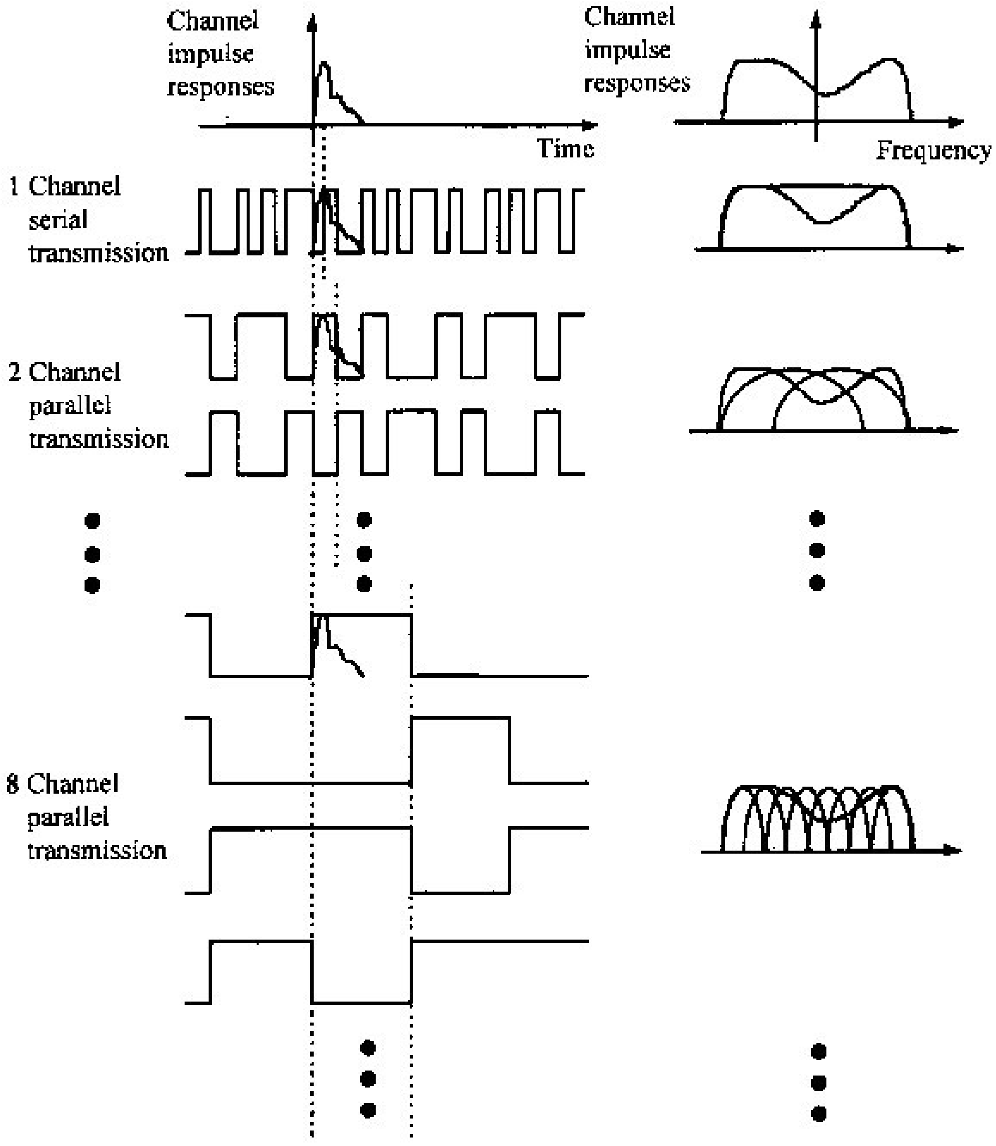
- In figure (a) we see the spectrum of an OFDM sub-channel
- In figure (b) we see the spectrum of the total OFDM signal.

Orthogonal FDM (3/5)

- In such a system,
  - the rate of symbols in each subchannel is reduced K times with respect to a single carrier system
  - therefore, the symbol period in OFDM becomes $T = KT_s$, where T_s is the period of the initial information symbols (e.g., QAM symbols)
- If K is large enough,
  - the symbol interval can be significantly longer than the subchannel duration
  - and ISI becomes negligible
- Thus, if K is large enough, then each subchannel appears to have a constant frequency response

$$C(f_k) = C_k$$

Orthogonal FDM (4/5)



(a) Time domain (b) Frequency domain

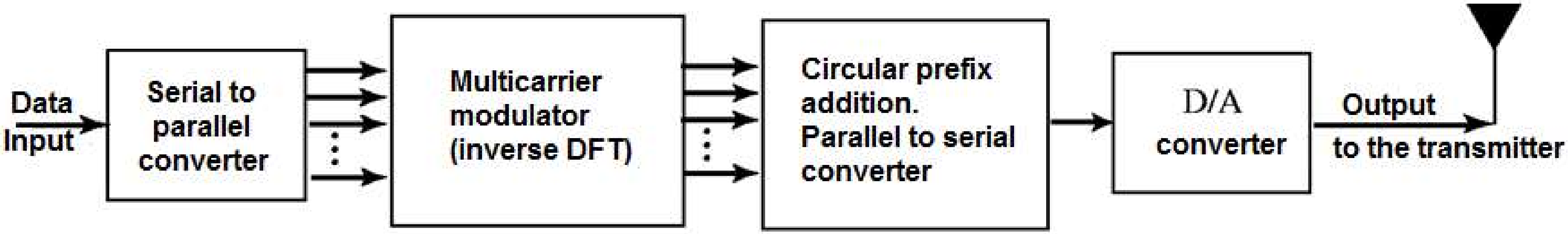
Orthogonal FDM (5/5)

- Initially, implementing OFDM was considered as an extremely complicated process involving *K* transmitters and K receivers.

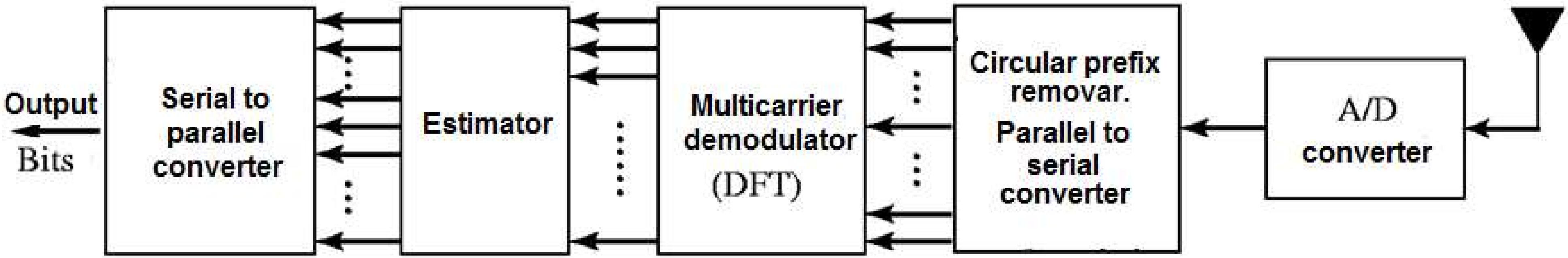
- However, it was later shown that, the OFDM modulator and demodulator can be implemented as a filter bank using the **discrete Fourier transform (DFT)**

- If K is large enough, then the above is implemented efficiently using **Fast Fourier Transform (FFT)**

OFDM System



(a) Transmitter



(b) Receiver

Using IFFT to implemend OFDM

$$x(n) = \sum_{k=0}^{K-1} X(k) e^{j\frac{2\pi}{K}k\cdot n}$$

$$x(0) = X(0)e^{j\frac{2\pi}{4}0\cdot 0} + X(1)e^{j\frac{2\pi}{4}1\cdot 0} + X(2)e^{j\frac{2\pi}{4}2\cdot 0} + X(3)e^{j\frac{2\pi}{4}3\cdot 0}$$

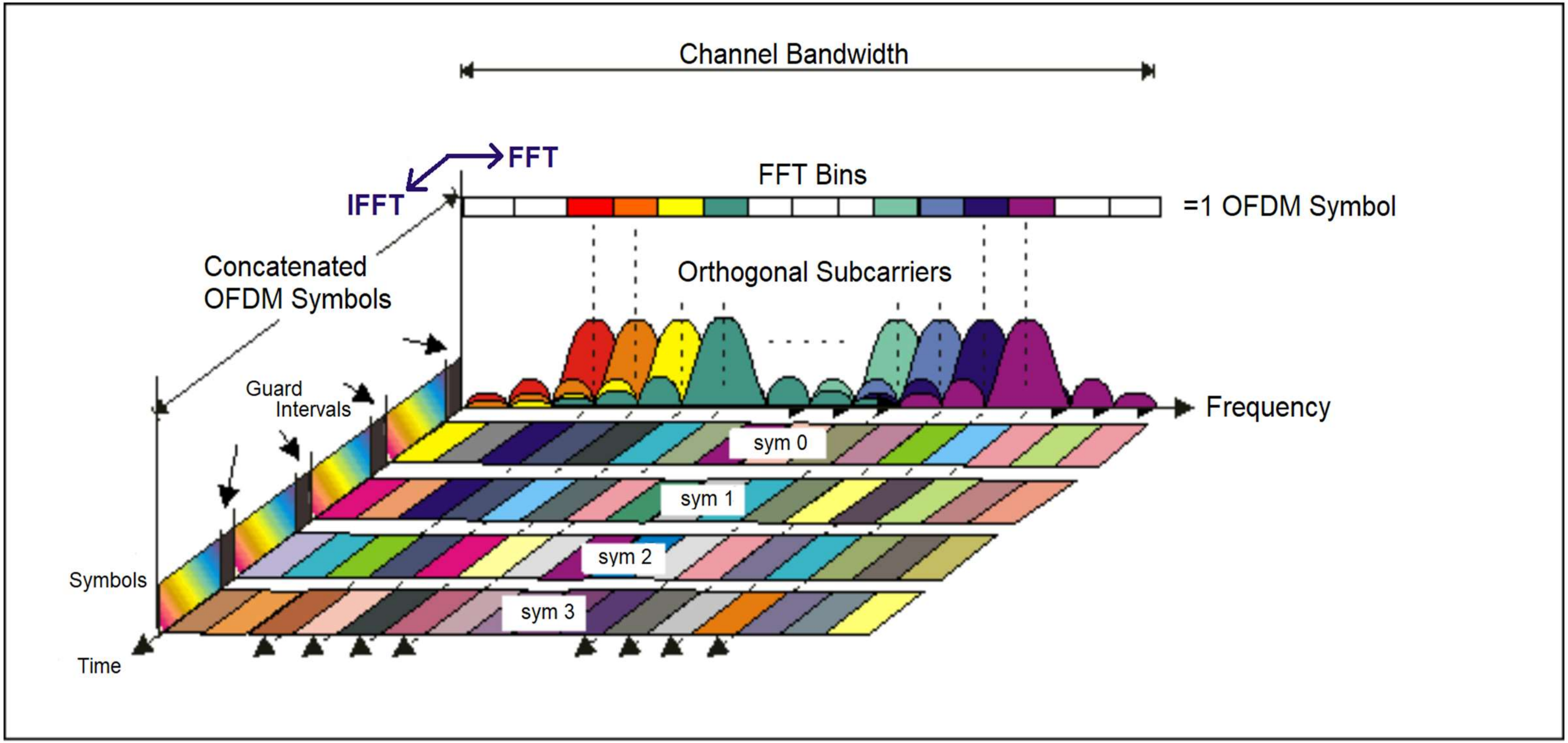
$$x(1) = X(0)e^{j\frac{2\pi}{4}0\cdot 1} + X(1)e^{j\frac{2\pi}{4}1\cdot 1} + X(2)e^{j\frac{2\pi}{4}2\cdot 1} + X(3)e^{j\frac{2\pi}{4}3\cdot 1}$$

$$x(2) = X(0)e^{j\frac{2\pi}{4}0\cdot 2} + X(1)e^{j\frac{2\pi}{4}1\cdot 2} + X(2)e^{j\frac{2\pi}{4}2\cdot 2} + X(3)e^{j\frac{2\pi}{4}3\cdot 2}$$

$$x(3) = X(0)e^{j\frac{2\pi}{4}0\cdot 3} + X(1)e^{j\frac{2\pi}{4}1\cdot 3} + X(2)e^{j\frac{2\pi}{4}2\cdot 3} + X(3)e^{j\frac{2\pi}{4}3\cdot 3}$$

- IDFT outputs are sent one-by-one (P/S) and the resulting signal is an OFDM (frame) symbol.
- The OFDM symbol is essentially the superimposition of the sub-carriers modulated by the QAM symbols.
- We observe that each QAM symbol is transmitted via a specific subcarrier (product of the corresponding IDFT base function with a common carrier) for a duration which is K-times the original QAM symbol period.

Time and Frequency Domain view of OFDM



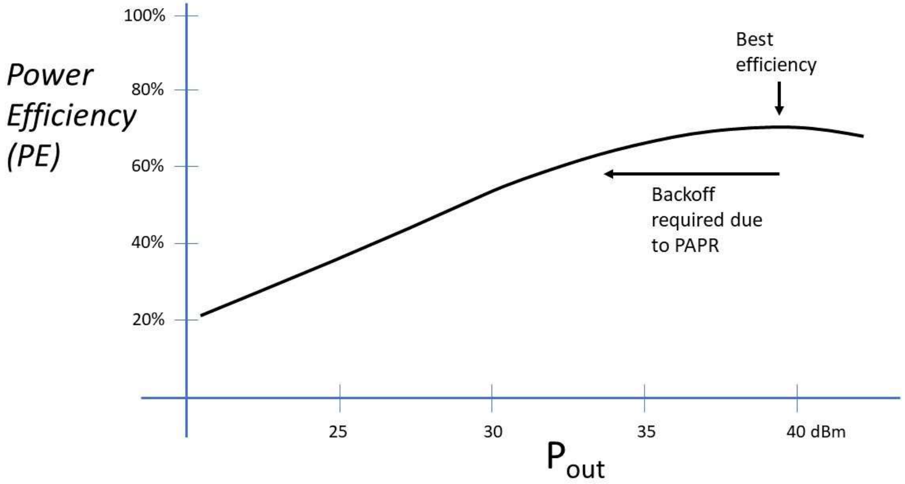
Time-Frequency Representation of OFDM signal

PAR: A major drawback of OFDM

- **Peak-to-Average Ratio (PAR)** of the transmitted signal.

$$PAPR_{dB} = 10\log_{10}\left(\frac{\max[x(n)x^*(n)]}{E[x(n)x^*(n)]}\right)$$

- What is the effect of this high PAR;
  - The instantaneous value of the total transmitted signal can become very large
  - Then the high power amplifier (at the transmitter end) operates in the saturation region, where it exhibits non-linear behavior.



- To deal with this, we need to reduce the transmit power (backoff strategy) which affects the received SNR and the probability of error at the receiver.
- Several other, more power efficient, techniques have been proposed, trading-off power efficiency to implementation complexity (e.g., Predistortion and Adaptive Predistortion, Clipped OFDM, heavier coding)

OFDM System: Adaptive transmission

■ The transmitter collects a frame of B_f bits. These are divided into K groups, and each group is assigned b_i bits,

$$\sum_{i=1}^{K} b_i = B_f$$

■ In the general case, a different number of bits / symbol can be used in each subchannel (i.e., M_i-QAM=2^{b_i} is used in each subchannel). For example,

- if a channel has a low SNR, then 4-QAM may be used
- whereas in a channel with high SNR, 16-QAM could be applied

■ The above process of adaptive power and bit allocation is not simple and requires channel knowledge at the transmitter side

■ IFFT is applied to all sub-channel symbols.

Inter-Frame interference (1/2)

■ There is (almost) flat fading in each subchannel, so practically no inter-symbol interference occurs (at the QAM symbol level).

■ However, there may be interference between two consecutive OFDM symbols. This is called Inter-Frame Interference (IFI).

■ To avoid IFI between consecutive frames, we may,

- either leave an empty space (zero prefix),
- or enter a cyclic prefix.

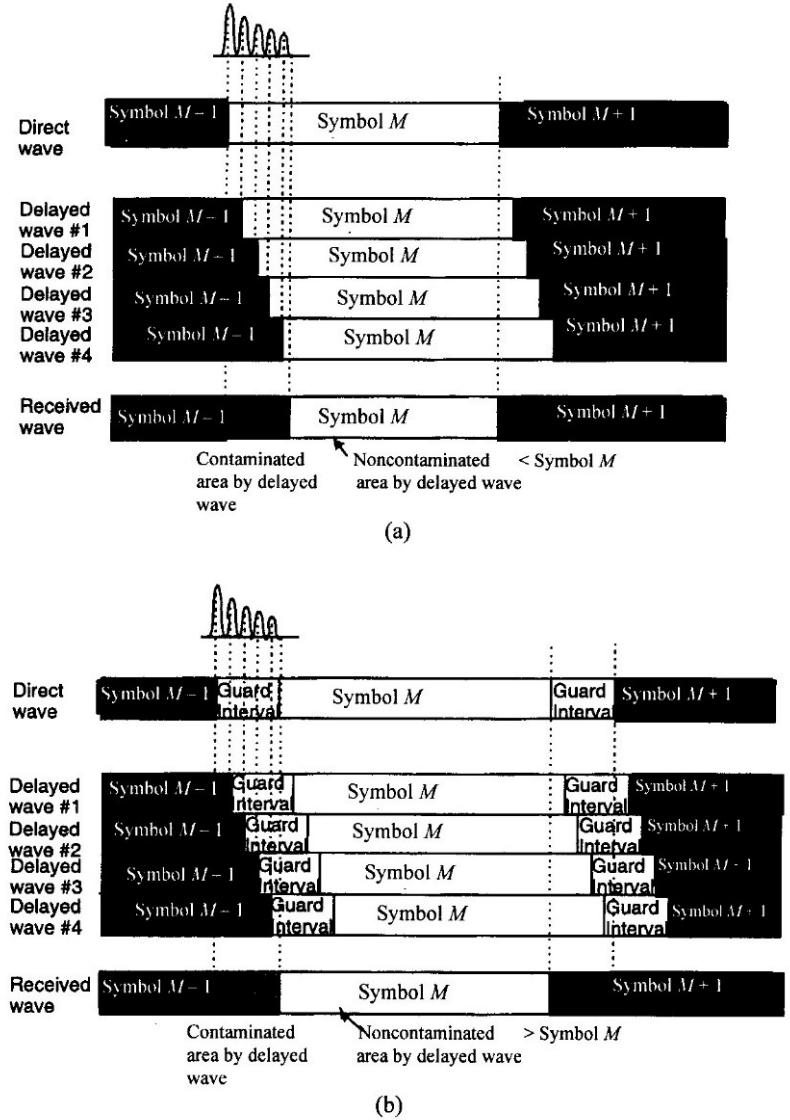
■ After the protection interval (ZF or CP) is inserted, the signal is transmitted in a serial and IFI-free manner.

Frame interference elimination (2/2)

■ The use of a circular prefix is more common in protocols using OFDM.

■ The circular prefix is inserted just before the frame (i.e., at the left end)

■ The circular prefix of length *L* consists of the last *L* samples of the frame.



OFDM: At the receiver side

- The receiver rejects the first L samples corresponding to the CP, where L is the length of the impulse response of the total channel.
- This eliminates any interference between the frames.
- All sub-channels are being demodulated in parallel using FFT.
- The signal of each subchannel is now being received as

$$Y(k) = C_k X(k) + N(k)$$

- The channel effects can now be compensated. To this end,
  - each subchannel needs to be estimated by applying a suitable estimation method (if it is time-variant then some adaptive method may be used).
- A decision is made for each symbol.

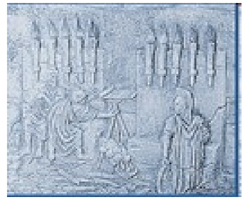
The SNR per subchannel

- The SNR per subchannel is:

$$SNR_k = \frac{P_k |C_k|^2}{\sigma_{nk}^2}$$

  - P_k the average power transmitted to the subchannel k
  - $|C_k|^2$ the frequency response of subchannel k
  - σ_{nk}^2 the noise variance in subchannel k

- As already mentioned, depending on the SNR at each subchannel, we can select different P_k and/or different QAM constellations



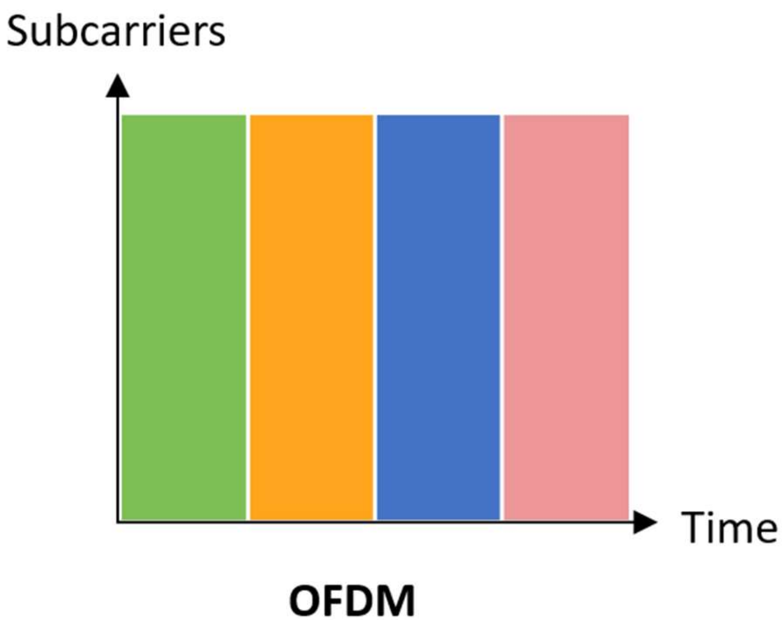
OFDM: General evaluation

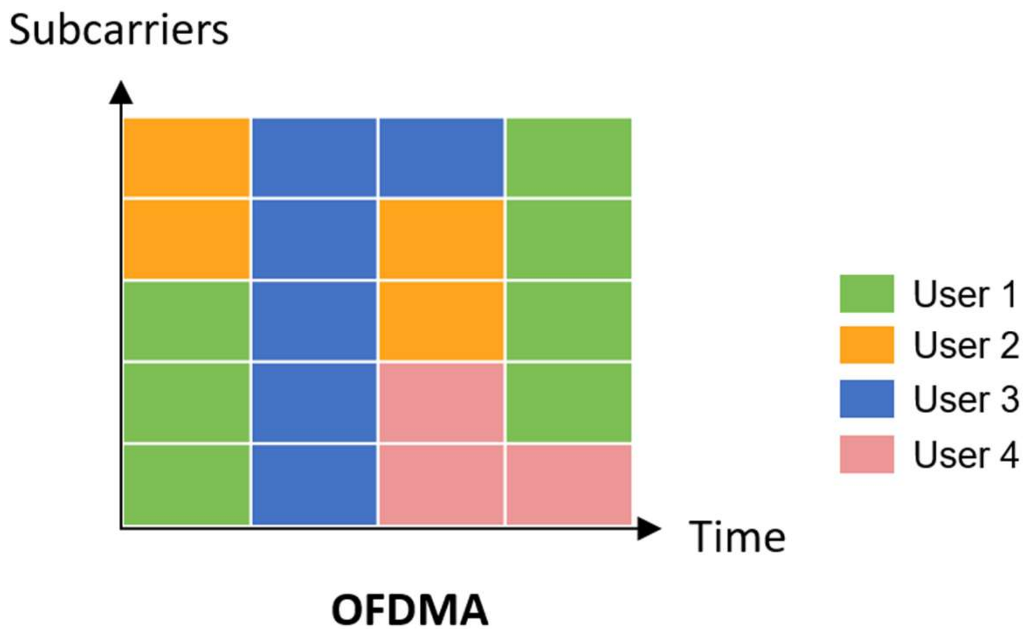
- Key benefits:
  - Better utilization of the available spectrum
  - The problems of frequency selectivity and fast fading are somehow “diffused”. Thus, burst errors are avoided.
- Disadvantages:
  - Sensitivity to carrier frequency offset (due to jitter and Doppler)
  - Constant envelope (curve) is not guaranteed and therefore the technique is sensitive to non-linear channel deformations
  - Strong channel encoding required (COFDM)

OFDMA

■ The OFDM system has been primarily conceived as a spectrally efficient modulation system.

■ However, the basic idea can be used for multiple access, i.e., sharing the same channel to multiple users. This is done by assigning sub-carrier groups to different users (possibly with different QoS requirements).





OFDM applications

- OFDM and OFDMA have been implemented in various systems.
- Discrete MultiTone Modulation - DMT
  - for high-speed transmission over telephone lines, such as digital ADSL, VDSL subscription lines
- Wireless LAN (WLAN) radio interfaces IEEE 802.11a, g, n, ac, ah and HIPERLAN/2
- The mobility mode of the wireless MAN/broadband wireless access (BWA) standard IEEE 802.16e (or Mobile-WiMAX)
- Digital Audio Broadcasting - DAB
  - for digital audio transmission
- Digital Video Broadcasting - DVB (HDTV)
  - for digital video broadcasting
- Cellular communications (4G, 5G)
- Power Line Communications
- Ultra Wide-band (UWB) Communications

Related systems

- SC-FDMA (Single-Carrier FDMA). It employs:
  - Single-Carrier Modulation.
  - DFT-spread orthogonal frequency multiplexing.
  - Frequency-Domain Equalization.
  - Uplink of LTE.

- Filter Bank-based Multi-Carrier (FBMC)
  - Filter-banks are used, instead of DFT, to implement parallel transmission to the sub-carriers.