Enhanced optimal portfolios – A controlled integration of quantitative predictors

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Abstract

Bayesian portfolio construction has received large attention since it was first advocated in form of the Black-Litterman approach. This Bayesian model aims at investors holding an arbitrary, well balanced and – according to their preferences – quasi optimal portfolio, but are keen to *enhance* the respective by considering quantitative return predictions. We show that weighting factors required for the mixed estimation can be directly derived from predictive regressions in form of goodness-of-fit measures, which enables an unambiguous determination of certainty levels. The model can self-adjust rapidly to changing market conditions and is potentially able to generate and preserve excess return on the long-run over an initial portfolio whilst restraining additional downside risk. Supporting results based on a robustly constructed simulation framework also reveal that the increasing number of correlation breakdowns and the variance of correlation between assets and predictor drives average excess returns towards, but never below zero. An empirical setting initialized with a global portfolio and the Baltic Dry Index as a single, universal predictor confirms the findings.

Keywords: Baltic Dry Index, Bayesian portfolio construction, Black-Litterman, downside risk, goodness-of-fit, random sampling
JEL classification: C11; C22; C53; C61; D24; G11; G12

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1 Introduction

Active management of portfolios and its possible benefits over passive strategies has been extensively discussed for years and it is obvious that two camps have emerged scrutinizing this matter. A vast number of asset managers believe in their abilities to identify mispriced assets and, thereby, aim to generate an excess return over a passive benchmark. However, in reality sooner or later most market participants are faced with the fact that long-term outperformance over the aggregated index is only reserved for a handful of managers and the majority cannot persistently generate alpha (Jensen, 1968; Carhart, 1997; Kosowski, Timmermann, Wermers, & White, 2006). The actual skill a manager has in picking the right assets is most relevant during market downturns. 'Riding the wave' in market boom phases can hardly be perceived as skill, therefore, the containment of risk in recessions has to be the main factor for identifying a managers ability. *Enhanced portfolio management* or *enhanced indexing* is a mix of both active- and passive management, where the investors accept a certain tracking error aiming to generate a small degree of alpha whilst not encountering additional downside risk relative to the passive benchmark (Fabozzi, 1999).

Bayesian portfolio construction is a natural choice for the constitution of enhanced optimal portfolios as it is a mixed estimation procedure allowing for two sources of information being combined. The Black-Litterman (BL) approach is arguably the most prominent example of Bayesian portfolio selection and resolves the issue of corner solutions commonly experienced under short-selling constraints with Markowitz optimization (Fabozzi, Focardi, & Kolm, 2006). Although this model can be interpreted as an extension to the BL model we intentionally stand back from closely referencing the BL approach as the intuition behind both models is contrary. Crucially, the central feature of incorporating subjective views and confidences is removed and return estimates and respective certainty levels are added from a linear regression. By deriving certainty levels towards predictions directly from a linear regression in form of the goodness-of-fit measure, we implicitly account for estimation errors in the optimization procedure. This is a key feature disregarded by standard optimizers (Fabozzi et al., 2006). Furthermore, we do not restrict application to implied equilibrium returns but allow the model to be adopted to an arbitrary portfolio and predictor. Thereby, we provide an add-on to an investors existing portfolio – including, but not limited to, a passive benchmark – in form of enhancing the respective by means of quantitative predictions.

In an empirical setting we apply the Baltic Dry Index (BDI) as a universal predictor and test its predictive power. Furthermore, a simulation is conducted to test for robustness of the introduced methodology and show that enhanced optimal portfolios can conserve generated excess return over the long-run and restrain downside risk measured by second order lower partial moments.

2 The Model

We consider an investor holding an arbitrary and – according to their preferences – optimal portfolio, but are keen to *enhance* the respective by considering quantitative return predictions. We propose an approach on which additional quantitatively estimates can be incorporated, thereby, offering upside return potential whilst controlling for risk in form of lower partial moments. Fundaments of the model rely on the procedure of Bayesian portfolio construction. First the implied returns from the underlying portfolio are backed out via a mean-reversion procedure. Next our return estimates and certainty levels are generated based on an arbitrary quantitative predictor. The revised return vector (posterior) is derived via the mixed estimation procedure according to the weighting factors Ω and τ. In the following lower-case letters refer to scalars, bold-face letters denote vectors and upper-case symbols stand for matrices.

Prior

As for this model the prior can be any arbitrary portfolio defined as optimal, given investors specific preferences. We make use of the reverse optimization procedure to back out expected asset returns implied by the weights of the investors current optimal portfolio given by $\mathbf{z} = \gamma\Sigma\mathbf{w}$ (Black & Litterman, 1992). We denote the vector of implied optimal portfolio returns ($\mathbf{z}$) as a function of investors risk aversion (γ), covariance matrix (Σ) and the vector of portfolio weights ($\mathbf{w}$).

We specify the covariance matrix based on a rolling window of 120 months of historical stock returns. The risk aversion factor γ is set to one.[1] The resulting distribution of the prior is $N \sim (\mathbf{z}, \tau\Sigma)$. Where the scalar τ is scaling factor towards the covariance matrix, which reflects the uncertainty in our return estimates and serves as a weighting factor for the mixed estimation procedure.[2] The scaling factor τ enables the investor to specify the acceptable degree of deviation of the posterior from the prior. A small value implies a posterior closely tracking the prior and vice versa. We specify τ as being close to 0 given the optimality of the existing portfolio.[3]

Quantitative Predictions

Performance of optimized portfolios rely upon generating reasonably good estimates of future asset returns and their covariance. Hereby we focus on the former aspect and attempt to estimate monthly returns for each asset i of the investment set defined by the investors existing portfolio. Methodological framework of generating quantitative predictions ($\hat{\mathbf{r}}$) relies on a classical ordinary least squares approach of the form:[4] $r_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1}$. This model allows for any arbitrary factor (f) to be applied as a source of return forecasts.[5] In order to update the vector of prior return expectations according to quantitative predictions, one requires a quantification of the accuracy of predictions. We propose a method to derive confidences towards quantitative predictions directly from the linear regression and thereby provide an intuitive relation between the certainty levels and expected returns.

Hereto, we apply the goodness-of-fit measures to specify accuracy of predictions and plug

[1] We set risk aversion to 1 as the reverse optimization procedure is applied to an already optimal portfolio, which implies that investors risk aversion is already accounted for. Respectively, expected returns derived from the optimal portfolio are not pre-scaled when entering the Bayesian framework and, therefore, have equal weight relative to quantitative predictions.

[2] Although, the scaling factor τ of this study enters the procedure the same way as in the original Black-Litterman model, the intuition behind it is different. In contrast to standard versions of the BL model who commonly struggle with the specification of τ and its intuition, we explicitly incorporate τ as tool for investors to specify the acceptable degree of deviation from their existing *optimal portfolio*.

[3] Alternatively, a specification of $\tau = 1$ implies an investor being indifferent between prior and quantitative predictor and, therefore, both enter the updating procedure with equal weights. A value specification of $\tau \to \infty$ implies an investor being more confident about the quantitative predictions relative to the implied return estimates of his portfolio. Such an investor should reconsider his 'optimal' portfolio. Concluding, a specification of $\tau \to 0$ is most reasonable in this setting, given the optimality of the underlying portfolio.

[4] Although we make use of the regular OLS approach, instead of classical standard errors we use adjusted measures as proposed by Newey and West (1987) to overcome heteroscedasticity of error terms during time series regressions.

[5] The only constrained regarding the predictor in the linear- or multiple regression is that the Gauss-Markov assumption have to be fulfilled.

them into a certainty matrix Ω, entailing a specific level of certainty for each asset. We utilize the adjusted R^2 as a measure unrelated to the number of independent variables in the equation and point out that it comes along with a virtually standardized 0-1 scale.[6] This enables us to unambiguously determine the quality of estimates and as such offers an intuitive solution for specifying elements of the Ω. Fabozzi et al. (2006) also provide a brief theoretical introduction on incorporating factors models in a Bayesian setting but specify elements of Ω according to the variance of residuals. However, their methodology does not provide an intuitive scale compared to adjusted R^2 and applicability has not been tested. A mathematical depiction of the connection between their approach and the one applied in this study is provided in appendices. Furthermore, Connor (1997) applies a shrinkage approach to the predictive regression in a Bayesian portfolio setting by recalibrating the regression coefficient according to R^2. Thereby he shrinks estimated coefficients towards zero, which is intuitive given the application to the market portfolio and respective market efficiency. However, both approaches deviate from the one at hand in their implementation and intuition, respectively.

$$\Omega_t = \begin{pmatrix} 1 - R^2_{1,t} & 0 & \cdots & 0 \\ 0 & 1 - R^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - R^2_{i,t} \end{pmatrix}$$

We calculate the additive inverse of R^2 and add it to 1 in order to make the values appropriate to the nature of certainty. Elements of Ω take large values for unreliable predictions, while small values correspond to less noisy forecasts. As a consequence elements of Ω indicating noisy predictions exhibit low effect on the *enhanced* optimal portfolio and vice versa. We calibrate Ω for every out-of-sample period.

Posterior

Taking the prior return distribution $N \sim (\mathbf{z}, \tau\Sigma)$ as the basis we can subjoin the expected returns derived from predictions given by $N \sim (\hat{\mathbf{r}}, \Omega)$. Applying a rearranged version of Black and Litterman (1992) according to Da Silva, Lee, and Pornrojnangkool (2009, p.3) and adjusting the respective to meet the properties of this study, we derive the posterior return vector as follows:[7]

$$\mathbf{z}^* = \mathbf{z} + \Sigma \left[\frac{\Omega}{\tau} + \Sigma \right]^{-1} \cdot (\hat{\mathbf{r}} - \mathbf{z})$$

The revised return vector constructed as a weighted average of the prior and quantitative

[6] The model can be extended to a multi-factor form in order to increase predictive power. Therefore, we make use of the *adjusted* R^2 measure as correction for the degrees of freedom is especially necessary when performing this approach with multifactor forecasting. With regards to the standardized scale one has to differentiate between the standard- and adjusted R^2 measure, later accounting for the number of explanatory terms. Where the standard form always stays between 0-1 guaranteeing the standardized scale, the other can go out of bounds, but offers more accuracy on the explanatory power of the regression. Ultimately, this leads to a slight reconsideration of the range of feasible τ values.

[7] In contrast to the standard form of the BL model, we drop identity matrix P which assigns the views to the respective assets. Given that we only have *absolute* return estimates on all assets at all times, we can drop this matrix. In the periods where the predictor yields weak predictions on certain assets, the according element of Ω will get close to 1 (i.e. no influence on the optimal portfolio), hence eliminating the need for the identification matrix.

predictions according to the weighting factors τ and Ω. Recalling, a value specification for τ close to zero refers to a low degree of uncertainty regarding prior return estimates. As we define the prior to be an optimal portfolio, a high certainty in the respective is implied. Consequently, the uncertainty in the prior is smaller compared to the quantitative predictions and, therefore, the enhanced optimal portfolio will track the underlying prior closely.

In a final step the updated return expectations are fed into the initial portfolio optimizer to generate the portfolio weights of the *enhanced* optimal portfolio (EOP). This is achieved by rearranging the formula for deriving the prior in order to back out the revised portfolio weights: $\mathbf{w}^* = (\gamma\Sigma)^{-1}\mathbf{z}^*$. We chose to apply the initial optimizer in order to derive a clear picture of the contribution from our quantitative predictions and certainty levels. Thereon, we analyze whether EOP's are capable of generating sustainable excess return over the initial (prior) portfolio. Furthermore, we evaluate whether this excess return comes at the cost of additional downside risk relative to the underlying portfolio. Due to unequal distribution characteristics we measure risk by second order raw and central lower partial moments (LPM) based on Nantell and Price (1982). We are particularly interested in the differences between the portfolios: $\Delta LPM_{\{h,g\}}(r) = \int_{-\infty}^{h}(r-h)^2 f_{EOP}(r)\,dr - \int_{-\infty}^{g}(r-g)^2 f_{OP}(r)\,dr$. We denote h and g to represent the respective target return, while j denotes the order of LPM.

3 Simulation

First we show robustness of the model by applying the model to a simulated data set in order to overcome the potential of experiencing empirical results by chance due to the application of a single predictor to a limited observation period of selected indices. Consequently, the setup of this simulation study is as arbitrary as possible – in terms of generating asset returns and the predicting factor – in order to identify an devaluate the deterministic factors of the model.

Setup and Sampling Properties

We calculate enhanced optimal portfolios based on two *equally-weighted* assets (with returns r_A and r_B). Quantitative forecasts for both assets are generated according to a standard OLS prediction model. Independent variable f is a random series with arbitrary first and second order moments. Both series r_A and r_B, as well as r_A and f are simulated based on bivariate normal distributions. We consider correlation breakdowns, leading to a shift in moments as for the representation of genuine stock market crashes. Notation for variables is given in subscripts – possibly in braces when multiple series involved –, while parentheses in superscripts state breakdowns as total number of unique periods.

First we define correlation of assets A and B to follow a random walk process with mean 0.75 and upper and lower boundaries of 0.5 and 1, respectively. Furthermore, correlation of asset A and predictor f is also characterized by a stochastic process with an arbitrary mean and standard deviation that changes $k-1$ times after each breakdown and due to the nature of correlation is bounded between -1 and 1. Random shocks of both models follow normal distributions: $\eta \sim \mathcal{N}(0, 0.01)$ and $\upsilon \sim \mathcal{N}(0, \sigma_\upsilon)$.

$$
\begin{aligned}
\rho_{\{r_A,r_B\},t} = &\quad \rho_{\{r_A,r_B\},t-1} + \eta_t \qquad \text{s.t. } \rho_{\{r_A,r_B\},1} = 0.75 \text{ and } 0.5 \le \rho_{\{r_A,r_B\}} \le 1 \\
\rho^{(k)}_{\{r_A,f\},t} = &\quad \rho^{(k)}_{\{r_A,f\},t-1} + \upsilon_t \qquad \text{s.t. } \rho^{(k)}_{\{r_A,f\},1} \sim \mathcal{U}(-1,1) \text{ and } |\rho^{(k)}_{\{r_A,f\}}| \le 1
\end{aligned}
$$

Now we are able to simulate observations r_A and r_B with covariance conditional on $\rho_{\{r_A,r_B\}}$. We set one asset to be riskier than the other with the following parameters: $\mu_{r_A}, \mu_{r_B} = 0$ and $\sigma_A = 0.35, \sigma_B = 0.15$.

$$\langle r_A, r_B \rangle \sim \mathcal{N}(\kappa, \Sigma_t), \text{ with } \kappa = \begin{pmatrix} \mu_{r_A} \\ \mu_{r_B} \end{pmatrix} \text{ and } \Sigma_t = \begin{pmatrix} \sigma^2_{r_A} & \sigma_{\{r_A,r_B\},t} \\ \sigma_{\{r_A,r_B\},t} & \sigma^2_{r_B} \end{pmatrix}$$

Finally, we generate individual moments for subperiods of predictor f in order to simulate correlation breakdowns. Means are based on a normal distribution around 0, while standard deviation – being strictly positive – follow a uniform distribution between 0 and 1. As such we not only have a variance-covariance matrix (Θ) that is time-varying, but also first and second moments are unique for k periods.

$$\mu_f^{(k)} \sim \mathcal{N}(0,1) \text{ and } \sigma_f^{(k)} \sim \mathcal{U}(0,1)$$

$$\left\langle r_A, f^{(k)} \right\rangle \sim \mathcal{N}\left(\xi^{(k)}, \Theta_t^{(k)}\right), \text{ with } \xi^{(k)} = \begin{pmatrix} \mu_{r_A} \\ \mu_f^{(k)} \end{pmatrix} \text{ and } \Theta_t^{(k)} = \begin{pmatrix} \sigma^2_{r_A} & \sigma^{(k)}_{\{r_A,f\},t} \\ \sigma^{(k)}_{\{r_A,f\},t} & \sigma_f^{(k)2} \end{pmatrix}$$

Hereafter, arbitrary asset returns and a random predictor draw on varying forecasting accuracy. Factor f is to deliver 'forecasts' for both series r_A and r_B based on two univariate OLS models. Deterministic parameters of the model are the number of correlation breakdowns and variance of shocks (σ_v) affecting correlation of coefficient between r_A and f. A sensitivity analysis of the two parameters is conducted with regards to the impact on excess return of EOP relative to the initial investors portfolio. Given this set-up, for any possible combination we run 100 simulations of 1'200 months of random asset returns each.

Results

Taking a random sample from the set of simulations Figure 1 indicates a representative case illustrating the models characteristics. From correlation levels we can assert the quality (goodness-of-fit) of our predictor with respect to the two assets. Given a high level of absolute correlation our predictor will have high predictive power and consequently our certainty levels assigned to the derived predictions will be high leading to a stronger tilt in portfolio weights and vice versa. This relation is clearest for the high correlation level in period two. EOP starts generating outperformance – after a calibration period identical to the estimation window – reflecting the increase in predictive quality and rising certainty levels derived from adjusted R^2.

Cumulative return plot reveals a constant widening of the gap until the second correlation breakdown 800 periods into the sample window. At this point the correlation drops to around 0 and consequently our predictor is jimmied for the remaining months. At this point, the significant contribution of our model comes into place and clarifies what makes this portfolio an *enhanced* optimal portfolio. By definition, the EOP cannot be restricted to exhibit temporarily modest negative returns subsequent to the correlation breakdown, during recalibration. Given the significant correlation breakdown from almost 1 to 0, the model adapts fast by means of a drop in certainty levels assigned to the predictions thereby tilting the EOP towards its prior. Hence, the model is able to preserve the outperformance previously generated even during phases where the quantitative predictor is weak. This is clearly reflected by the constant gap over remaining 400 months where weights of the EOP

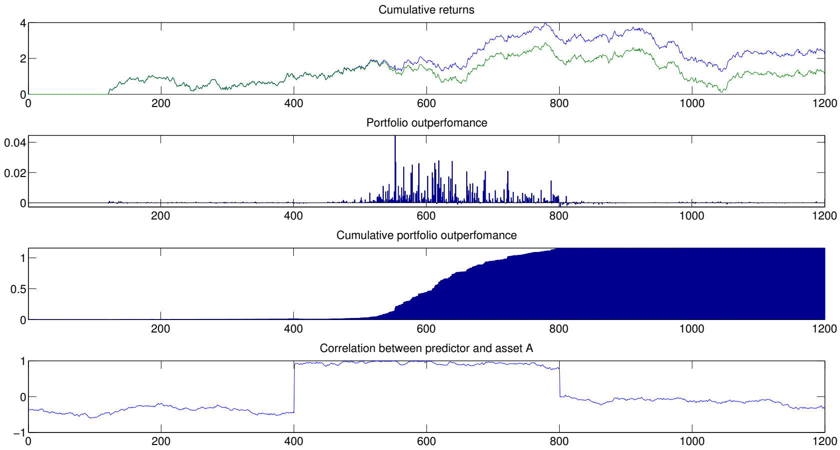

Figure 1: This figure is a composition of: (1) cumulative returns of an equally-weighted portfolio, (2) monthly outperformance of the EOP, (3) cumulative out-performance and (4) correlation and respective breakdowns between the dependently simulated asset A and predictor f. In this setting we enforce $k = 3$ correlation breakdowns, therefore, generating three periods of different correlation means equal to -0.4, +0.9 and 0. We set the variance of correlation to 0.05 for all levels to ensure simulated correlation is close to the set mean for each period.

are approximately identical to the optimal portfolio. This is a major improvement over standard forms of the BL model incorporating subjective certainties subject to investors irrational behavior. Especially overconfidence – commonly observed with financial analysts – and hesitation to revise personal opinions contribute to this pattern (?, ?).

Next, we examine the sensitivity of EOP's regarding the previously mentioned two deterministic parameters. All other parameters are held constant and are identical to the deliberations made before. The outcome of the sensitivity analysis is illustrated in Figure 2. We check for robustness of our assertion that an portfolio optimized by means of this method will generate on average excess return without the burden of additional downside risk – in form of raw lower partial moments – relative to the optimal portfolio. We prove that these features prevail even when accounting for correlation breakdowns up to a yearly frequency along with rapidly repeating shifts in correlation between predictor and assets.

On average positive excess return under both parameter variations can be reported. The surface plot is positive for all variations of either parameter and across all simulations, as indicated by the vertical axis. Right-horizontal axis represents a variation in the variance of correlation between predictor and assets and, therefore, reflects the predictive power of the indicator. Along an increase in variance of correlation, a decline in excess return is observable. This is reasonable as an increase in variation of correlation lowers the predictive quality of the factor. Furthermore, sensitivity towards correlation breakdowns is also according to our expectations. As for the variance of correlation, an increase in breakdowns leads to a reduction in excess return. Overall, we show that the model is sensitive to the number of correlation breakdowns and variance of correlation between predictor and assets. However, excess returns are strictly positive on average for all tested combinations with a clear convergence towards 0. In terms of additional returns this means an investor can never be worse off allocating according to the *enhanced* optimal portfolio. This is confirmed when accounting for lower partial moments, where simulation results prove $\forall k, \sigma_\upsilon : max[\Delta LPM_{\{h,g\}}(r)] < 0$ for $\{h, g\} = \{0, 0\}$ and $\{E[\mathbf{r}'\mathbf{w}^*], E[\mathbf{r}'\mathbf{w}]\}$.

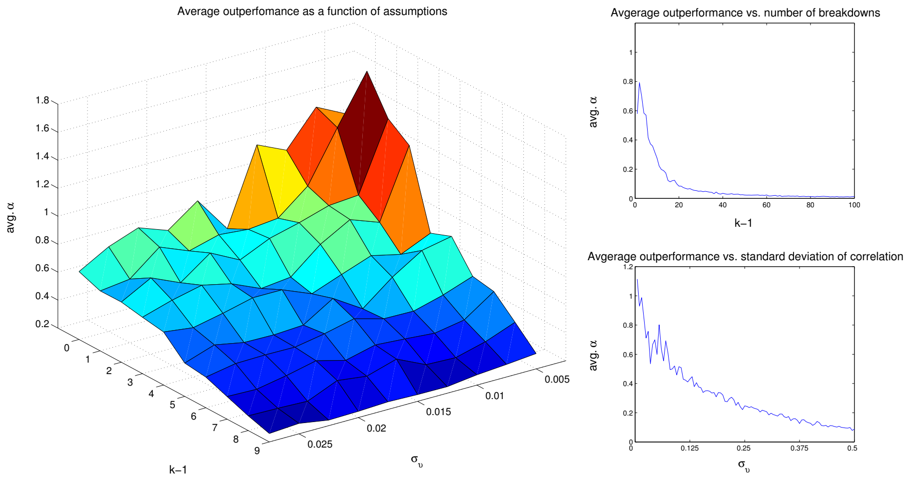

Figure 2: The surface plot shows 10'000 simulation of 100 per parameter combination. Each portfolio simulation is made up of 1'200 months of random asset returns. We depict values for variance of correlation (σ_υ) on the right-horizontal axis, number of correlation breakdown ($k-1$) on the left-horizontal axis and excess return of EOP over the initial (prior) portfolio on the vertical axis. Two graphs to the right are extended sensitivity plots towards the two deterministic parameters.

4 Empirical Evidence

Finally, we test the described approach on historical observations to provide empirical evidence. This is of particular interest as the simulation setting required normal distribution throughout, while stock returns might not exhibit this property at all times.

Data

In the empirical setting we update a naive $1/N$ portfolio according to quantitative prediction derived from a single global predictor, the Baltic Dry Index (BDI). We apply 3-month BDI growth rates to obtain forecasts for various reasons. Firstly, former has been empirically tested and confirmed to be a reliable global equity market predictor, which allows us to implement them it a world equity index portfolio.[8] Furthermore, as a universal predictor it exhibits fairly constant correlations across our indices and, therefore, allows us to limit the number of predictors for simplicity. Finally, it has a uniquely high frequency and also offers decent history for the last 25 years.

As part of this empirical study we apply 37 USD denominated MSCI country indices for an observation period from April 1985 to March 2012 as dependent variables in the predictive regression. Table 1 presents a detailed description of the dataset. Alongside an intercept, we estimate different factor coefficients for every market i at every time t. In order to find the best fitting model across the investigated markets, once again we follow Bakshi, Panayotov, and Skoulakis (2011) and initialize regressions with a 120-month in-sample window. Hence we conduct our calculations recursively until the last observation, which enables us to generate forecasts for 105 and 137 months for emerging and emerged markets, respectively. The following regression is applied: $r_{i,[t\to t+1]} = \alpha_i + \beta_i\, g_{[t-3\to t]} + \varepsilon_{i,t+1}$. BDI growth rates are included with one lag as an independent variable into the model. Its effect on the dependent variable, excess returns (r_i), is positive across all markets. According to common literature we set the lag size at 3 thus p-values are also calculated accordingly.

[8]We form a world portfolio composed of the G7 markets, 14 emerged markets and 11 emerging markets.

	obs	mean	std dev	skew	kurt	max	min	$\rho_{g_t, r_{t+1}}$
				Predictor				
Baltic Dry Index	320	0.018	0.557	-1.792	19.710	0.671	-1.330	-
				Regional Indices				
EAFE	320	0.049	0.159	-0.875	5.271	0.107	-0.211	0.179
Europe	320	0.046	0.183	-0.594	4.184	0.138	-0.226	0.145
Emerging Markets	320	0.065	0.185	-0.844	5.164	0.133	-0.240	0.163
G7	288	0.080	0.247	-1.019	5.926	0.167	-0.346	0.175
World	320	0.046	0.158	-0.761	4.843	0.115	-0.203	0.179
				G7 markets				
Canada	320	0.057	0.201	-1.130	7.555	0.193	-0.315	0.178
France	320	0.067	0.222	-0.530	4.055	0.186	-0.255	0.127
Germany	320	0.059	0.246	-0.709	4.716	0.212	-0.280	0.148
Italy	320	0.008	0.256	-0.100	3.604	0.258	-0.270	0.107
Japan	320	0.007	0.224	0.013	3.485	0.211	-0.219	0.064
UK	320	0.059	0.184	-0.523	4.886	0.138	-0.248	0.158
USA	320	0.057	0.160	-1.028	6.213	0.120	-0.245	0.186
				Emerged markets				
Australia	320	0.078	0.248	-2.166	18.000	0.164	-0.590	0.130
Austria	320	0.035	0.276	-1.055	8.326	0.227	-0.463	0.187
Belgium	320	0.072	0.227	-1.549	12.445	0.228	-0.456	0.136
Denmark	320	0.085	0.206	-0.734	5.454	0.182	-0.297	0.166
Greece	288	0.014	0.368	0.232	6.212	0.443	-0.458	0.115
Hong Kong	320	0.081	0.281	-1.309	11.677	0.284	-0.570	0.077
Ireland	288	-0.007	0.234	-0.982	5.533	0.163	-0.303	0.216
Netherlands	320	0.070	0.200	-1.127	6.203	0.134	-0.290	0.138
Norway	320	0.066	0.281	-1.191	7.027	0.194	-0.407	0.134
Portugal	288	-0.009	0.232	-0.339	4.665	0.246	-0.305	0.092
Singapore	320	0.044	0.277	-1.345	10.470	0.226	-0.532	0.138
Spain	320	0.090	0.249	-0.508	4.752	0.220	-0.292	0.092
Sweden	320	0.100	0.265	-0.587	4.486	0.227	-0.311	0.149
Switzerland	320	0.085	0.182	-0.510	4.028	0.138	-0.200	0.134
				Emerging Markets				
Argentina	288	0.107	0.578	-0.265	13.394	0.924	-1.083	0.060
Brazil	288	0.128	0.512	-1.018	8.583	0.442	-0.877	0.111
Chile	288	0.132	0.250	-0.669	5.579	0.204	-0.348	0.111
Indonesia	288	0.080	0.474	0.159	7.635	0.669	-0.523	0.174
Korea	288	0.031	0.374	0.170	5.589	0.530	-0.377	0.139
Malaysia	288	0.048	0.300	-0.215	6.965	0.402	-0.364	0.088
Mexico	288	0.149	0.324	-1.039	6.337	0.255	-0.407	0.109
Philippines	288	0.032	0.318	-0.161	4.660	0.357	-0.350	0.126
Taiwan	288	0.018	0.365	-0.112	4.399	0.382	-0.415	0.136
Thailand	288	0.041	0.389	-0.552	5.223	0.356	-0.420	0.101
Turkey	288	0.056	0.561	-0.022	4.096	0.573	-0.536	0.167

Table 1: Descriptive statistics presented here are based upon monthly excess returns denoted in US dollars ($) provided by MSCI. The sample contains in most cases returns between 5/1/1985 and 12/31/2011, except for emerging and a few additional markets where data is only available from 2/1/1988. Columns are denoted as follows: Number of observation (obs), mean indices (mean), standard deviation of indices (std dev), skewness of time series (skew), kurtosis of time series (kurt), highest monthly change (max), lowest monthly change (min) and correlation coefficient of the Baltic Dry Index with the respective index.

Predictive Quality of BDI

Table 2 summarizes findings related to in- and out-of-sample estimation statistics. Additionally, figures in appendices provide graphical insight on the cross-sectional and time-series characteristics of BDI. Coefficients of BDI growth rates are fairly stable across markets, especially when looking at the sample excluding emerging markets. As expected, g has a throughout positive loading and varies around 0.02 - 0.03. In case of emerging markets the pattern turns out to be more volatile – indices for e.g. Indonesia or Turkey are well above the average, while Argentina or Mexico are significantly below. Generally small coefficients are related to the high standard deviation of the shipping index, thus a 1% change in g corresponds to a few basis point changes in future returns. These results are in line with Bakshi et al. (2011). When investigating the progress of average loadings throughout the timeframe we find a steady increase during the first quarter of the observation window. A subsequent jump is due to the sudden availability of additional – mostly emerging markets – data, which generally come along with significantly higher β coefficients. We also find a massive outlier around 2008, which can be explained by the enormous decrease of the BDI index during the financial crisis.

Investigating out-of-sample R^2 across indices we repeatedly see a changing mean when comparing emerged and emerging markets. These results are in-line with our findings for in-sample R^2 in terms of decreasing explanatory and forecasting power compared to previous literature. Average measures deviate in the positive range around 1.5-2 for developed

	obs	$\bar{\beta}^{OLS}$	$1-\bar{p}^{OLS}$	$\bar{R}^2$	R^2_{OS}	RMSE	HR	θ^{logit}	Chow F
Regional markets									
World	137	0.0222	0.82	1.3	3.9	0.5778	0.52	14.1870	2.1571
EAFE	137	0.0200	0.73	0.8	2.0	0.6330	0.51	40.8220	1.3502
Europe	137	0.0225	0.77	1.2	2.7	0.7002	0.47	28.4811	2.4655*
Emerging	105	0.0437	0.95	1.9	2.2	0.7462	0.58	27.0902	0.7199
G7	137	0.0217	0.82	1.3	4.2	0.5600	0.51	12.0287	2.2304
G7 markets									
Canada	137	0.0298	0.83	1.4	3.1	0.7716	0.50	103.3490	0.6529
France	137	0.0191	0.69	0.5	0.7	0.8028	0.51	8.8319	2.4837*
Germany	137	0.0269	0.74	0.8	1.6	0.9225	0.47	7.3956	3.3133**
Italy	137	0.0186	0.66	0.4	0.8	0.8441	0.49	12.8306	1.4278
Japan	137	0.0128	0.52	-0.2	-0.9	0.5942	0.49	164.6738	0.5802
UK	137	0.0167	0.66	0.8	4.5	0.5989	0.48	130.6183	1.4603
USA	137	0.0229	0.84	1.3	5.1	0.5553	0.55	10.8781	3.2268*
Emerged markets									
Australia	137	0.0126	0.55	0.4	1.6	0.7930	0.55	47.7716	0.1721
Austria	137	0.0305	0.70	1.4	3.2	1.0341	0.53	15.4970	2.2551
Belgium	137	0.0223	0.72	0.7	-0.9	0.9114	0.46	27.4830	2.7203*
Denmark	137	0.0414	0.98	2.2	1.6	0.7643	0.50	54.9900	0.8324
Greece	105	0.0308	0.69	0.5	0.4	1.1311	0.58	53.4409	5.5192***
Hong Kong	137	0.0145	0.53	-0.1	-0.2	0.7678	0.53	396.8115	0.4118
Ireland	105	0.0370	0.91	2.4	6.0	0.7969	0.52	97.5956	5.3123***
Netherlands	137	0.0246	0.79	1.0	0.4	0.8184	0.47	25.8675	3.0076*
Norway	137	0.0393	0.83	1.1	0.4	1.0470	0.48	96.3826	0.1829
Portugal	105	0.0324	0.87	1.0	-2.5	0.6957	0.52	125.7992	1.8340
Singapore	137	0.0384	0.88	0.7	1.3	0.8231	0.50	31.5491	0.0612
Spain	137	0.0148	0.63	0.2	-1.4	0.8885	0.55	51.2313	1.4854
Sweden	137	0.0427	0.91	1.2	1.5	1.0064	0.49	105.9418	1.2457
Switzerland	137	0.0193	0.74	0.7	2.5	0.5853	0.52	-59.5243	3.0126*
Emerging markets									
Argentina	105	0.0019	0.43	-0.3	-2.0	1.1355	0.57	593.6874	0.3595
Brazil	105	0.0717	0.96	0.7	-2.1	1.0227	0.49	270.2059	0.0878
Chile	105	0.0456	0.94	1.5	-6.3	0.7269	0.55	112.1156	0.2606
Indonesia	105	0.1063	0.99	2.4	-1.5	1.0586	0.59	5.5817	0.1566
Korea	105	0.0633	0.97	1.3	-0.2	0.9134	0.56	-27.7139	0.2482
Malaysia	105	0.0415	0.92	0.5	-5.5	0.5645	0.58	20.2893	0.4581
Mexico	105	0.0069	0.43	0.1	-0.2	0.7488	0.56	-28.2501	1.8180
Philippines	105	0.0472	0.92	1.0	1.9	0.7539	0.59	-27.8998	0.0061
Taiwan	105	0.0567	0.94	1.0	3.4	0.7391	0.52	15.6168	0.2642
Thailand	105	0.0668	0.96	1.1	-6.3	0.9151	0.56	23.5253	0.5401
Turkey	105	0.1387	0.99	2.8	-4.3	1.2880	0.57	7.6802	0.0014

Table 2: Estimation statistics are calculated for every index based upon all available observations with a recursive procedure. Description of columns is to be interpreted as follows: Number of out-of-sample observations (obs), average coefficient of BDI predictor ($\bar{\beta}$), average certainty of BDI predictor ($1-p^{OLS}$), average adjusted R^2, out-of-sample R^2 (R^2_{OS}), root mean squared error (RMSE), hit ratio (HR), coefficient of logit regression (θ) and F-statistics of Chow's structural break test (Chow F). Negative values of logit coefficients are denoted in parentheses and significant F results are marked (*$< 0,1$; **< 0.05, ***< 0.01).

countries, while for emerging markets out-of-sample statistics of BDI tend to be significantly lower. Moreover, a geographical and economical break is noticeable. Broad market indices – such as the MSCI World index, G7 index or the United States stock market – achieve highest results and can be acceptably forecasted by the BDI. Additionally, North American and European countries with leading macroeconomic and consumption indicators reach a significantly higher average value. Out-of-sample sample results against time provide the clearest picture about BDI as a predictor for excess returns. Hereby we differentiate between two groups based on the number of available observations and contrary to previous depictions we do not combine series with various length. This is inevitable as indices with less observations, which are mostly related to emerging markets, show a greatly different pattern and a combined figure would be strongly biased by the outliers. We also provide root mean squared errors (RMSE), whereby we measure squared deviation of our forecasts from realized values: $RMSE = \sqrt{\frac{1}{T}\sum_{t=0}^{T-1}(r_{t+1}-\hat{r}_{t+1})^2}$. As this measure partially corresponds to the nominator of the out-of-sample R^2, we only calculate it to solely observe estimation errors. Furthermore, due to its common application it enables us to make our predictor comparable with other studies. We can identify a pattern where residuals are on average smaller for diversified regional indices and G7 markets.

With the help of logistic regressions we finally investigate the influence of adjusted R^2 results on the direction of upcoming market movements, hence the relationship between in in-sample and out-of-sample statistics. This is crucial for this analysis as negative results would undermine a linkage between certainty levels and goodness-of-fit of the quantitative predictor. We find positive effects on odds ratios. 33 out of 37 of the tested indices exhibit a strong positive effect on hits when R^2 statistics are increasing. This supports our proposed procedure and allows us to construct our enhanced optimal portfolio based on BDI derived predictions.

Portfolio Characteristics

This section discusses the results of applying the BDI as the quantitative predictor to the mixed estimation introduced in this study to derive an *enhanced* optimal portfolio in a global equity portfolio setting. The performance of the EOP relative to the prior is presented in Figure 3. The first depiction of Figure 3 shows the cumulative return of both portfolios across an investment period of 200 months. As we can see the EOP can generate a small degree of excess return over the observation period, where outperformance steadily increases over time and can be retained.

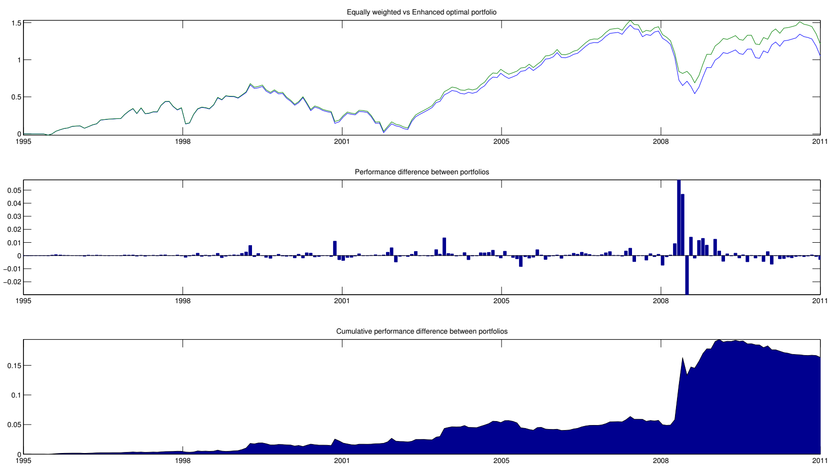

Figure 3: This figure provides a graphical presentation of the results in form of a composite of three separate plots: (1) cumulated portfolio performance, (2) Performance difference between optimal and enhanced portfolio and (3) cumulative performance difference.

Additional illustrations present monthly return differences, as well as the cumulative of the respective. Whilst the monthly outperformance is relatively stable until 2008, a significant shock is observable during the market downturn caused by the financial crisis. This is even more striking when looking at the cumulative return differences, where a significant increase is observable. Predictive power of BDI – in terms of adjusted R^2 – shows the same pattern as the cumulative return depiction. The previous analysis of the BDI as a global predictor has already shown that its goodness-of-fit varies significantly over time and cross-sectionally. Given the overall low level of goodness-of-fit, scaling factor τ was increased to 0.5 in order to provide a clear basis for evaluation. Overall results show a positive trend and accumulation of excess return, which can be retained at a high level. These results are in-line with the simulation analysis and confirm the validity of this model on empirical grounds. Furthermore, looking at downside risk in form of raw and mean lower partial moments even reflecting a decrease of 0.02 for raw and central LPM's. This again confirms that the methodology at hand can generate excess return relative to the prior, whilst avoiding additional downside risk on the *enhanced* optimal portfolio.

5 Conclusion

Given the accordance of empirical and simulated results we can prove that – given any arbitrary quantitative predictor – optimal portfolios can be *enhanced* in terms of excess return without the burden of additional downside risk, measured as second order lower partial moments. This is achieved by employing a Bayesian framework and allowing any existing portfolio to be *enhanced* by means of quantitative predictions. We show that the certainty regarding the predictions can be directly derived from a linear regressions in form of adjusted R^2 and, thereby, providing a implicit dependency between quantitative forecasts and their weight on the mixed estimation. Consequently, the model self-adjusts rapidly to changing market conditions by tilting weights towards the underlying portfolio to protect investors from experiencing additional downside deviation relative to their prior.

In an empirical study we analyze the Baltic Dry Index in terms of predictive power and – given its practicability – apply it as a universal predictor in a global equity index portfolio. Consequently, we are able to show that historically generated excess return on the basis of quantitative predictions can be preserved over the long-run whilst not exposing the investor to additional downside risk. Results prove to be robust and can be confirmed in a simulation setting.

Application of our method includes, but is not limited to, the field of enhanced indexation. The advantage in this approach is the possibility to specify any index as the basis for deriving the prior and enhancing the respective by means of quantitative predictions to generate excess return. At the same time investors are not exposed to additional downside risk, but only benefit from forecasts where the models predictive power is strong. Given the models closed form, the implementation overheads are low and the investors acceptable degree of deviation from the underlying index can be specified via the weighting factor τ.

References

Amenc, N., El Bied, S., & Martellini, L. (2003, September). Predictability in Hedge Fund Returns. *Financial Analysts Journal*, *59*(5), 32–46.

Bakshi, G., Panayotov, G., & Skoulakis, G. (2011). *The Baltic Dry Index as a Predictor of Global Stock Returns, Commodity Returns, and Global Economic Activity* (Working Paper). McDonough School of Business, Georgetown University.

Basak, S., & Chabakauri, G. (2010). Dynamic Mean-Variance Asset Allocation. *The Review of Financial Studies*, *23*(8), 2970-3016.

Bawa, V., Brown, S., & Klein, R. (1979). Estimation Risk and Optimal Portfolio Choice. *North-Holland Publ. Co., New York.*, 190.

Best, M. J., & Grauer, R. R. (1991). Sensitivity Analysis for Mean-Variance Portfolio Problems. *Management Science*, *37*(8), 980-989.

Bevan, A., & Winkelmann, K. (1998). *Using the Black-Litterman Global Asset Allocation Model: Three Years of Practical Experience* (Tech. Rep.). Fixed Income Research, Goldman, Sachs & Company.

Beverelli, C. (2010). *Oil Prices and Maritime Freight Rates: An Empirical Investigation* (Tech. Rep.). United Nations Conference on Trade and Development.

Black, F., & Litterman, R. (1991). Asset Allocation. *The Journal of Fixed Income*, *1*(2), 7-18.

Black, F., & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*, *48*(5), 28-43.

Braga, M., & Natale, F. (2007). TEV Sensitivity to Views in Black-Litterman Model. In *20th Australasian Finance & Banking Conference 2007 Collection.*

Carhart, M. M. (1997, March). On Persistence in Mutual Fund Performance. *The Journal of Finance*, *52*(1), 57-82.

Chen, S., Meersman, H., & Voorde, E. Van de. (2010). Dynamic Interrelationships in Returns and Volatilities between Capesize and Panamax Markets. *Maritime Economics & Logistics*, *12*(1), 65-90.

Chopra, V., & Ziemba, W. (1993). The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice. *The Journal of Portfolio Management*, *19*(2), 6-11.

Christodoulakis, G. A. (2002). *Bayesian Optimal Portfolio Selection: The Black-Litterman Approach* (Tech. Rep.). Faculty of Finance, Sir John Cass Business School City University, London.

Chung, K., & Ha, Y. (2010a). The Empirical Analysis for the Effect of Global Finance-Crisis to the Baltic Dry Index. *Journal of Shipping and Logistics*, *64*, 1-16.

Chung, K., & Ha, Y. (2010b). The Structural Change in the BDI Function due to the Global Financial Crisis: Using the Kalman Filter. *Journal of Shipping and Logistics*, *65*, 217-231.

Connor, G. (1997). Sensible return forecasting for portfolio management. *Fianncial Analsyt Journal*, *54(5)*, 44-51.

Da Silva, A., Lee, W., & Pornrojnangkool, B. (2009). The Black-Litterman Model for Active Portfolio Management. *The Journal of Portfolio Management*, *35*(2), 61-70.

Drobetz, W. (2001). How to Avoid The Pitfalls in Portfolio Optimization? Putting the Black-Litterman Approach at Work. *Financial Markets and Portfolio Management*, *15*(1), 59-75.

Fabozzi, F. (1999). *Investment Management.* New Jersey: Prentice Hall.

Fabozzi, F., Focardi, S., & Kolm, P. (2006). Incorporating Trading Strategies in the Black-Litterman Framework. *The Journal of Trading*, *1*(2), 28-37.

Garlappi, L., Uppal, R., & Wang, T. (2007). Portfolio Selection with Parameter and Model Uncertainty: A Multi-Prior Approach. *The Review of Financial Studies*, *20*(1), 41-81.

Giacometti, R., Bertocchi, M., Rachev, S. T., & Fabozzi, F. J. (2007). Stable Distributions in the Black-Litterman Approach to Asset Allocation. *Quantitative Finance*, *7*(4), 423-433.

Green, R. C., & Hollifield, B. (1992, December). When Will Mean-Variance Efficient Portfolios be Well Diversified? *The Journal of Finance*, *47*(5), 1785-1809.

He, G., & Litterman, R. (1999). *The Intuition Behind Black-Litterman Model Portfolios* (Tech. Rep.). Goldman, Sachs & Co. (Investment Management Research).

Horst, J., de Roon, F., & Werker, B. (2006). An Alternative Approach to Estimation Risk. In *Advances in Corporate Finance and Asset Pricing.* Elsevier Amsterdam, Luc Renneboog, eds.

Hyup Roh, T. (2007). Forecasting the Volatility of Stock Price Index. *Expert Systems with Applications*, *33*(4), 916-922.

Idzorek, T. M. (2002). A step-by-step Guide to the Black-Litterman Model: Incorporating user-specified Confidence Levels. In *Forecasting Expected Returns in the Financial Markets* (p. 17-38). Elsevier.

Jensen, M. C. (1968, May). The Performance of Mutual Funds in the Period 1945-1964. *The Journal of Finance*, *23*(2), 389-416.

Jones, R., Lim, T., & Zangari, P. (2007). The Black-Litterman Model for Structured Equity Portfolios. *The Journal of Portfolio Management*, *33*(2), 24-33.

Kan, R., & Zhou, G. (2007, September). Optimal Portfolio Choice with Parameter Uncertainty. *Journal of Financial and Quantitative Analysis*, *42*(03), 621-656.

Ko, B. W. (2010). A Study on the Change of the Dynamics of Dry Bulk Market before and after the 2008 Global Financial Crisis. *Ocean Policy Research*, *25*(2).

Ko, B. W. (2011, June). An Application of Dynamic Factor Model to Dry Bulk Market. *International Journal of Maritime Affairs and Fisheries*, *3*(1), 69-82.

Kosowski, R., Timmermann, A., Wermers, R., & White, H. (2006, December). Can Mutual Fund "Stars" Really Pick Stocks? New Evidence from a Bootstrap Analysis. *The Journal of Finance*, *61*(6), 2551-2595.

Leung, M. T., Daouk, H., & Chen, A.-S. (2000). Forecasting Stock Indices: A Comparison of Classification and Level Estimation Models. *International Journal of Forecasting*, *16*(2), 173-190.

Mankert, C. (2006). *The Black-Litterman Model* (Tech. Rep.). School of Industrial Engineering and Management, Royal Institute of Technology.

Martellini, L., & Ziemann, V. (2007). Extending Black-Litterman Analysis Beyond the Mean-Variance Framework. *The Journal of Portfolio Management*, *33*(4), 33-44.

Merton, R. C. (1980, December). On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics*, *8*(4), 323-361.

Michaud, R. (1989). The Markowitz Optimization Enigma: Is 'Optimized' Optimal? *Financial Analysts Journal*, 31-42.

Mishra, A., Pisipati, S., & Vyas, I. (2011). An Equilibrium Approach for Tactical Asset Allocation: Assessing Black-Litterman Model to Indian Stock Market. *Journal of Economics and International Finance*, *3*(10), 553-563.

Nantell, T., & Price, K. (1982). Mean-lower partial moment asset pricing model: Some empirical evidence. *Journal of Financial and Quantitative Analysis*, *17(5)*, 763-782.

Newey, W., & West, K. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 703-708.

Palomba, G. (2008, January). Multivariate GARCH Models and the Black-Litterman Approach for Tracking Error Constrained Portfolios: An Empirical Analysis. *Global Business and Economics Review*, *10*(4), 379-413.

Satchell, S., & Scowcroft, A. (2000). A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Portfolio Construction. *Journal of Asset Management*, *1*(2), 138-150.

Schöttle, K., Werner, R., & Zagst, R. (2010). Comparison and Robustification of Bayes and Black-Litterman Models. *Mathematical Methods of Operations Research*, *71*, 453-475.

Stopford, M. (2009). *Maritime Economics* (3rd ed.). New York: Allen and Unwin.

Tsolakis, S. (2005). *Econometric Analysis of Bulk Shipping Markets Implications for Investment Strategies and Financial Decision-Making.* Unpublished doctoral dissertation, Erasmus University Rotterdam.

Verbeek, M. (2004). *A Guide to Modern Econometrics.* New York: John Wiley & Sons.

Xu, J., Yip, T., & Marlow, P. (2011). The Dynamics Between Freight Volatility and Fleet Size Growth in Dry Bulk Shipping Markets. *Transportation Research Part E: Logistics and Transportation Review*, *47*(6), 983-991.

A Appendix

A.1 Specification of certainty Matrix Ω

By choosing a classical ordinary least squares (OLS) estimator it is easy to show the relationship between high certainty levels and reliable estimations. For simplicity we use assume regression equations with the same number of independent variables k across indices and also consider time series with the equal lengths.

$$\Omega_t = \begin{pmatrix} 1-R^2_{1,t} & 0 & \cdots & 0 \\ 0 & 1-R^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-R^2_{i,t} \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} \frac{1/(t-k)\sum_{j=1}^{t}\varepsilon^2_{1,j}}{1/(t-1)\sum_{j=1}^{t}(r_{1,j}-\bar{r}_1)^2} & 0 & \cdots & 0 \\ 0 & 1-R^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-R^2_{i,t} \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} \frac{t-1}{t-k}[\sum_{j=1}^{t}(r_{1,j}-\bar{r}_1)^2]^{-1}\sum_{j=1}^{t}\varepsilon^2_{1,j} & 0 & \cdots & 0 \\ 0 & 1-R^2_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1-R^2_{i,t} \end{pmatrix} \tag{3}$$

Once we substitute the formula for adjusted R^2 into the equation we can back out a matrix that represents diagonal elements of the covariance matrix of the returns, thus its variances.

$$= \frac{1}{t-k}\begin{pmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_i^2 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{j=1}^{t}\varepsilon^2_{1,j} & 0 & \cdots & 0 \\ 0 & \sum_{j=1}^{t}\varepsilon^2_{2,j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{j=1}^{t}\varepsilon^2_{i,j} \end{pmatrix} \tag{4}$$

$$= \frac{1}{t-k}\, diag(\Sigma)^{-1} \begin{pmatrix} \sum_{j=1}^{t}\varepsilon^2_{1,j} & 0 & \cdots & 0 \\ 0 & \sum_{j=1}^{t}\varepsilon^2_{2,j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{j=1}^{t}\varepsilon^2_{i,j} \end{pmatrix} \tag{5}$$

Last term of (5) is a matrix with the very elements we minimize during the OLS procedure when predicting equity returns. As the variance and the scalar are predetermined by the sample it is easy to see that our linear predictor singularly maximizes certainty by minimizing Ω.

A.2 Restructuring the Black-Litterman equation

To isolate τ and Ω in the initial Black-Litterman equation we restructure the formula following closely Mankert (2006):

$$\mathbf{z}^* = \Big[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big]^{-1} \Big[(\tau\Sigma)^{-1}\mathbf{z} + P'\Omega^{-1}\hat{\mathbf{r}}\Big] \tag{6}$$

First we multiple (6) with $\tau\Sigma$ and its inverse as an identity matrix:

$$= \Big[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big]^{-1} (\tau\Sigma)^{-1}(\tau\Sigma) \Big[(\tau\Sigma)^{-1}\hat{\mathbf{r}} + P'\Omega^{-1}\mathbf{z}\Big] \tag{7}$$

$$= \Big[I + \tau\Sigma P'\Omega^{-1}P\Big]^{-1} \Big[\hat{\mathbf{r}} + \tau\Sigma P'\Omega^{-1}\mathbf{z}\Big] \tag{8}$$

Now we extend the second term by $\tau\Sigma P'\Omega^{-1}P\hat{\mathbf{r}}$ and its additive inverse:

$$= \Big[I + \tau\Sigma P'\Omega^{-1}P\Big]^{-1} \Big[\hat{\mathbf{r}} + \tau\Sigma P'\Omega^{-1}\mathbf{z} + \tau\Sigma P'\Omega^{-1}P\hat{\mathbf{r}} - \tau\Sigma P'\Omega^{-1}P\hat{\mathbf{r}}\Big] \tag{9}$$

$$= \Big[I + \tau\Sigma P'\Omega^{-1}P\Big]^{-1} \Big[\big(I + \tau\Sigma P'\Omega^{-1}P\big)\,\hat{\mathbf{r}} + \tau\Sigma P'\Omega^{-1}\,(\mathbf{z} - P\hat{\mathbf{r}})\Big] \tag{10}$$

Once again we multiple with an identity matrix, this time by $\Omega + P'\tau\Sigma P$ and its inverse:

$$= \Big[I + \tau\Sigma P'\Omega^{-1}P\Big]^{-1} \cdots \tag{11}$$

$$\Big[\big(I + \tau\Sigma P'\Omega^{-1}P\big)\,\hat{\mathbf{r}} + \tau\Sigma P'\Omega^{-1}(\Omega + P'\tau\Sigma P)(\Omega + P'\tau\Sigma P)^{-1}\,(\mathbf{z} - P\hat{\mathbf{r}})\Big] \tag{12}$$

$$= \Big[I + \tau\Sigma P'\Omega^{-1}P\Big]^{-1} \cdots \tag{13}$$

$$\Big[\big(I + \tau\Sigma P'\Omega^{-1}P\big)\,\hat{\mathbf{r}} + \big(I + \tau\Sigma P'\Omega^{-1}P\big)\,\tau\Sigma P(\Omega + P'\tau\Sigma P)^{-1}\,(\mathbf{z} - P\hat{\mathbf{r}})\Big] \tag{14}$$

After some simple algebra we obtain a modified form of the equation with unified τ and Ω variables:

$$= \mathbf{z} + \tau\Sigma P(\Omega + P'\tau\Sigma P)^{-1}\,(\mathbf{z} - P\hat{\mathbf{r}}) \tag{15}$$

$$\mathbf{z}^* = \mathbf{z} + \Sigma P\Big[\frac{\Omega}{\tau} + P'\tau\Sigma P\Big]^{-1}\,(\mathbf{z} - P\hat{\mathbf{r}}) \tag{16}$$

Given that we hold *absolute* views on all assets at all times we can further simplify by dropping out P, which is the identity matrix assigning views to the respective asset where one does not hold a prediction on each asset or states *relative* views:

$$\mathbf{z}^* = \mathbf{z} + \Sigma\Big[\frac{\Omega}{\tau} + \tau\Sigma\Big]^{-1}\,(\mathbf{z} - \hat{\mathbf{r}}) \tag{17}$$

A.3 In- and Out-of- sample statistics of BDI

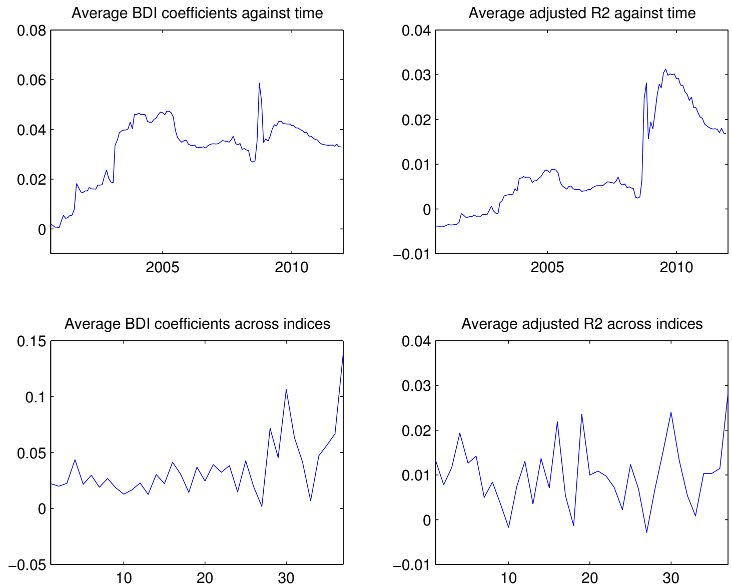

Figure 4: In-sample statistics are depicted as time series averages across indices as well as cross sectional averages against time. BDI coefficients and adjusted R^2 measures against time are composed of 22 indices for the first 32 observations and all 37 indices for the subsequent 105 observations. This causes a remote structural brake in the upper row figures. Numbering of indices is according to the listing in tables in appendices **??** and 2.

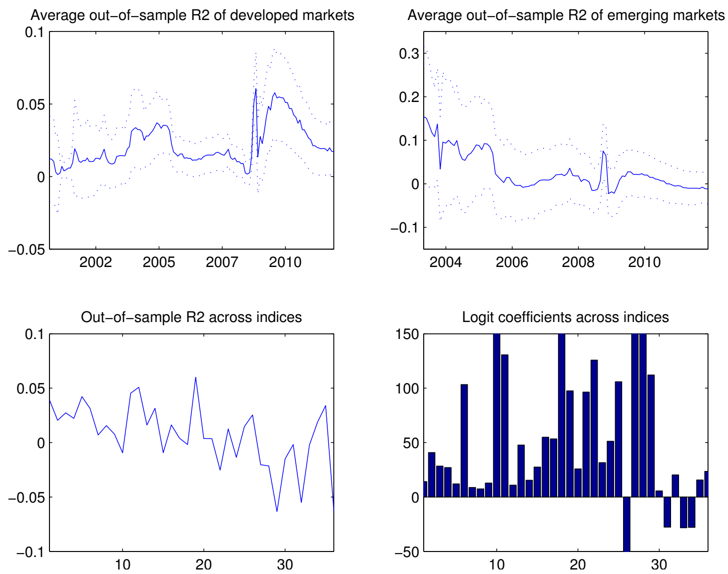

Figure 5: Out-of-sample estimation results are are depicted as time series averages across indices as well as cross sectional averages against time. Out-of-sample R^2 statistics are sorted into two groups according to available number of observations. Dotted lines represent one sigma band deviations across indices. Note the different scaling of the y-axis for figure in the upper row. Numbering of indices is according to the listing in tables in appendices **??** and 2.

A.4 Statistical Properties of Estimations

Furthermore, we also make use of a common measure from the field of asset management. We calculate hit ratios to observe whether forecast match the direction of future market movements. This basic measure reveals a lot. Most investment managers reduce their exposure radically when expecting negative real returns, on the other hand the increase in asset value is less relevant. Although different definitions circulate in literature, we determine hits and hit ratios as follows (see Amenc, El Bied, and Martellini (2003) or Hyup Roh (2007)):

$$HR = \frac{1}{T}\sum_{t=1}^{T} h_t, \text{ where } h_t = \begin{cases} 1 & \text{if } \operatorname{sgn}(r_t) = \operatorname{sgn}(\hat{r}_t) \\ 0 & \text{if } \operatorname{sgn}(r_t) \neq \operatorname{sgn}(\hat{r}_t) \end{cases} \tag{18}$$

Consequently, we compare signs of real and estimated returns at every time t. Whenever the signs match the function takes the value 1, else 0. To obtain the final hit ratio we calculate an arithmetic average across all hits h_t. Simple 'hits' gain further relevance when verifying the relationship between in-sample estimations and out-of-sample forecasts.

To underline the statistical relationship between in-sample estimation measures and out-of-sample predictability we use a logistic (logit) econometric model, similarly to Leung, Daouk, and Chen (2000). A binary choice model is necessary due to the selected dependent variable for this estimation. Hereby we investigate the relationship from a classical asset management perspective as we are solely interested in whether higher R^2 values consequently result in higher hit ratios, regardless of total deviation. In case of single factor and other non-sophisticated forecast models this is especially meaningful as squared deviations from real returns are commonly large. Therefore we calculate 'hits' for every month and regress changes of R^2 on it. Changes are necessary as consecutive values of R^2 exhibit a clear trend and heteroscedasticity, hence it violates the assumption of stationarity. By generating ΔR^2 we overcome this issue, although we devote an initial observation.

$$\ln\left(\frac{p_{i,t}^{logit}}{1-p_{i,t}^{logit}}\right) = \gamma_i + \theta_i \cdot \Delta R_{i,t-1}^2 + \nu_{i,t} \tag{19}$$

Left-hand side of Equation 19 is commonly referred to as log odds ratio, where the probability of observing outcome 1 is $p_{i,t} = P\{h_{i,t} = 1|\Delta R_{i,t-1}^2\}$. Thereupon, coefficients of the logit model can be interpreted as effects on the log odds ratio. As such, we look for positive outcomes for θ underlining that a positive change in our adjusted R^2 estimations increases the odds of a 'hit' when predicting expected returns (Verbeek, 2004). Hence, a higher R^2 measure promises a better quality of our predictor. Note that we make use of lagged changes of R^2 in order to realize the correct dependency of future returns on in-sample estimations. We compute this measure across all indices i and verify legitimate influence of the related confidence. Positive coefficients of logit regressions consequently indicate that it is proper to set the Ω matrix of the BL model according to the goodness of fit of in-sample estimations.

A.5 Overview on Baltic Dry Index

BDI prices are based on weighted averages of twenty global routes and not, as the name might suggest, only on routes around the Baltic states. BDI originated from the Baltic Freight Index (BFI), which was set up in May 1985 to provide a generally accepted base index for freight derivatives. In November 1999 the BFI was replaced by the BDI, which is a constitute representing the average price for the different vessel sizes: Baltic Exchange Cap-Size Index (BCI), Baltic Exchange Panamax Index (BPI) and Baltic Exchange Handymax Index (BHMI). Based on the change in construction of the BDI we test the time series for structural breaks. With the help of the Chow test we separate the sample into two parts – prior and after November 1999 – and regress index returns on lagged BDI growth rates with an OLS procedure. We calculate F results by comparing the sum of squares (SSR) of the restricted and unrestricted models: $F = \frac{(SSR_r - SSR_{ur})/k}{SSR_{ur}/(n-2k)}$, where k denotes number of restricted regressors including the intercept and n the number of observation. As shown in Table 2 we find little evidence for structural breaks and only in case of a small number of indices are the F results significant.

The requirement by market participants for derivatives is obvious when considering the volatility of this index, where prices are determined based on supply and demand. Supply in this case is defined by the available vessels in the market, in other words the total amount of aggregated tones transportable by available ships. On the other hand, demand is based on the industry's need for raw materials. As the production of a new vessel takes between 2-3 years, the supply side is rather inelastic and therefore the index is driven by changes in demand (Stopford, 2009). This of course can also create a bias within the index. Sudden increases in supply, due to a large order of vessels, will decrease the index value although the demand has not actually reduced. Respectively, economic outlook is also not negative as might be implied by the BDI. Given these shortcoming, the study by Bakshi et al. (2011) analyses whether the index really is a reliable indicator for future economic activity, as well as for commodity and asset prices.

The economic relevance of the BDI is based on the fact that a positive outlook on behalf of the companies increases their demand for raw materials, which subsequently increases the prices for shipping dry bulk. Therefore, the shipping of raw materials today is related to future industrial output. The factors affecting shipping prices can be summarized as follows: (1) Commodity/raw material demand, (2) Fleet supply, (3) Seasonal Pressures, (4) Bunker prices, (5) Choke points, (6) Market sentiment, (7) Port congestion, (8) Labour relations, (9) Piracy and (10) New arctic shipping routes (Tsolakis, 2005; Stopford, 2009). As we can see the number of factors impacting the index is significant and drives volatility of the index. Besides, this index is free of speculation, as one does not book freights unless one has actually got something to ship.

A.6 Literature review

References for this paper are gathered from two major fields. First we take a look at the Black-Litterman model and secondly we introduce contributions related to the Baltic Dry Index.

The BL approach was developed by Black and Litterman (1991) and attempts to improve the applicability of quantitative portfolio optimization in practice. Main constraints of Markowitz are agreed to be the extreme portfolio allocation (Black and Litterman (1992), Green and Hollifield (1992) and Basak and Chabakauri (2010)), sensitivity of portfolio weights (Best & Grauer, 1991), information aggregation (Merton, 1980) and estimation errors of input variables (Michaud (1989) and Chopra and Ziemba (1993)). Especially the reduction of estimation errors has played a significant role and many sophisticated models have been developed to reduce these misspecifications (Bawa, Brown, and Klein (1979), Horst, de Roon, and Werker (2006), Kan and Zhou (2007) and Garlappi, Uppal, and Wang (2007)). The BL approach aims to overcome the majority of these short-comings and in addition allows the user to incorporate personal views on the basis of Bayesian inference. Meanwhile numerous studies have evaluated the BL model, including Bevan and Winkelmann (1998), He and Litterman (1999), Satchell and Scowcroft (2000), Drobetz (2001), Christodoulakis (2002), Idzorek (2002) and most recently Schöttle, Werner, and Zagst (2010). Additional studies have been concerned with a clear specification of the required input parameters. Giacometti, Bertocchi, Rachev, and Fabozzi (2007) improve the classical BL model by considering different distributions (normal-, student- and stable distributions) to describe asset returns and evaluate the results by means of more sophisticated risk measures. Furthermore, Palomba (2008) implements a multivariate GARCH model into the BL approach in order to reflect the changes in volatility of asset returns and can thereby significantly improve the model's asset allocation. Further studies concerned with the application of the BL model include those by Braga and Natale (2007), Jones, Lim, and Zangari (2007), Martellini and Ziemann (2007), Da Silva et al. (2009), Mishra, Pisipati, and Vyas (2011).

Literature on the Baltic Dry Index is limited. Some studies have been published during the last two years, where one strand is focusing on the determination mechanisms and the second strand is analyzing the index's dynamic properties (Ko, 2011). The study by Chung and Ha (2010b) analyses the impact of the financial crisis on the BDI by applying a Kalman filter to identify possible structural changes in the index during the period April 2007 to August 2008. The results show that there have been three structural breaks related to the beginning of the subprime mortgage crisis, the collapse of Lehman Brothers and the period of lowest equity prices in the US. A subsequent study by Chung and Ha (2010a) identifies a co-integration relationship between explanatory variables such as US equity returns, Eurodollar interest rate and the iron import of China. Their results have further been confirmed by Beverelli (2010) confirming the dependency of freight rates on the oil price and iron ore. Ko (2010) evaluates the relationship between demand, supply and freight rates by applying a recursive VAR model and could thereby identify a statistically significant negative correlation between ship capacity and BDI, as well as a positive relationship between transport demand and BDI. Furthermore, Chen, Meersman, and Voorde (2010) analyze the relationship of daily returns and volatilities of BDI constitutes. Their findings are inline with Xu, Yip, and Marlow (2011) and imply that the impact on the BDI is not constant over time but can be driven by different underlying indices at different times.

Based on the high volatility observable for the Baltic Dry Index and the partial dependency on macroeconomic variables, Bakshi et al. (2011) have been the first to conduct an in-depth analysis on the predictability of international equity indices by means of the BDI growth rate. The fact that freight rates are determined before industrial output gives rise to the idea that the BDI acts as an indicator for economic growth. Furthermore, the fact that equity markets are also an indicator for the real economy and therefore predetermined, motivated Bakshi et al. (2011) to see if the BDI has also got predictive power with regard to equity markets. Their investigation yields significant results for the predictive power of the BDI regarding international equity index returns, commodity index returns and global economic activity. Finally, they have applied the BDI as an indicator for the equity market within a simple Markowitz portfolio strategy and stated Sharpe ratios above a benchmark. Based on their findings this study incorporates the 3-month BDI growth rate into the Black-Litterman model, while also introducing a procedure to determine an optimal confidence matrix for estimating portfolio weights.