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Downside risk and portfolio diversification in the euro-zone equity markets with special consideration of the crisis period

Tengdong Liu [a], Shawkat Hammoudeh [b,*], Paulo Araújo Santos [c]

[a] Tengdong Liu, School of Securities and Futures, SWUFE, Chengdu, China
[b] Lebow College of Business, Drexel University, Philadelphia, PA, United States
[c] School of Management and Technology of Santarém, Center of Statistics and Applications, University of Lisbon, Santarém, Portugal

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ABSTRACT

This study examines the Value-at-Risk for ten euro-zone equity markets individually and also divided into two groups: PIIGS (Portugal, Italy, Ireland, Greece and Spain) and the Core (Austria, Finland, France, Germany and the Netherlands), employing four VaR estimation and evaluation methods considered over the full period and the pre- and post-global crisis subperiods 1 and 2. The backtesting results are also evaluated according to the Basel capital requirements. The results demonstrate that the CEVT methods meet all the statistical criteria the best for most individual equity indices over the full period, but these results over the two subperiods for those two methods are mixed, compared to those the DPOT methods. Moreover, the two optimal group portfolios of the PIIGS and the Core as well as the grand portfolio that combines the ten indices do not show much diversification benefits. The PIIGS portfolio selects Spain's *IBEX* only, while that of the Core opts for Austria's *ATX* only in the full period and subperiod 1. However, Germany's *DAX* overwhelmingly dominates both the Core and the Grand portfolios in subperiod 2.

* Corresponding author.
E-mail addresses: tl342@drexel.edu (T. Liu), hammousm@drexel.edu (S. Hammoudeh), paulo.santos@esg.ipsantarem.pt (P.A. Santos).

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1. Introduction

The recent financial turmoil in the euro-zone countries has brought into focus the importance of financial risk management in those countries. The euro-zone debt crisis has affected their stock markets which are highly correlated because of increasing integration and harmonization in this area over time. The mounting risk and uncertainty resulting from the crisis have confounded investors, portfolio managers and policy-makers in the euro-zone as well as in other countries. In such an environment, it will be valuable and useful to examine the downside risk for these equity markets and figure out ways to diversify away risks under different time periods. It will also be particularly important to estimate risks during periods of extreme events like the 2007/2008 financial crisis that affected essentially all asset markets. We will examine the equity risk for the pre- and post-global financial crisis subperiods as well as the full period under consideration. Under such crisis circumstances, significant and extreme drops in prices and returns of these assets have become highly probable, with potentially damaging consequences on portfolios of individuals and institutions. These circumstances have also made risk management strategies for highly volatile markets become more challenging, particularly when the percentages of violations of confidence targets have compounded.

The quantification of the size of potential losses and the assessment of risk levels for individual markets and their portfolios are fundamental in designing prudent risk management and portfolio strategies. Value-at-Risk (VaR) models have become an important instrument within the financial markets for quantifying and assessing downside market risks associated with asset price fluctuations. They determine the maximum expected loss an asset or a portfolio can generate over a certain holding period, with a pre-determined probability value. Therefore, a VaR model can be used to evaluate the performance of individual asset and portfolio managers by providing downside risk quantification. It can also help investors and portfolio managers determine the most effective risk management strategy for a given situation. Moreover, quantification of the extreme losses in those asset markets is important in the current market environment. Extreme Value Theory (EVT) provides a comprehensive theoretical forum through which statistical models describing extreme scenarios can be developed.

There is a cost for inaccurate estimation of the VaR in equity markets, which affects the efficiency and accuracy of risk assessments. Surprisingly, despite the increasing importance and rising correlation and risk and the need for more portfolio diversification in the euro-zone markets, there are only few studies that analyze the VaRs, the VaR-based optimal portfolio constructions and their efficient VaR frontiers for these markets. The studies that examine European portfolio diversification emphasize diversification through industries instead of countries. In our paper, we assess the significance of diversification of the equity markets for portfolio combinations of two groups of ten countries in the euro-zone as well as for all ten countries combined as a grand group. The two groups are: the PIIGS which includes Portugal, Ireland, Italy, Greece and Spain, and the Core which consists of Germany, France, Austria, Finland and the Netherlands. The risk in these countries of the two groups will be investigated for the full period and before and after the crisis for comparison purposes. It will be the subject of our future research to examine expanded portfolios of these euro-zone countries by diversifying the equity portfolios with other asset classes such as commodities.

Our current study expands the spectrum of equity diversifications in the euro-zone and deals with events that are more extreme than the regular behavior dynamics of the stock indices over different periods. Therefore, it constructs VaR-based optimal portfolios, examines their characteristics and performances for this zone, and ranks those optimal portfolios using VaR-based risk performance measures.[1]

The objective of this paper is to fill this gap in the financial risk management for the euro-zone equity markets and construct optimal portfolio strategies by using more up-to-date techniques that take into account the volatility asymmetry and clustering in the pre- and post-global financial crisis periods. This topic has not been researched adequately for the seemingly harmonious euro-zone, despite the global crisis and its implications for diversification within broad investment portfolios

[1] We have constructed the efficient VaR frontiers for the portfolios. However, the frontiers constructed don't have the proper shape to allow a tangency point. The graphs are available upon request.

and hedging capability. To achieve these objectives, this paper computes VaRs for the ten individual euro-zone market indices and their grouping into PIIGS and Core, using four estimation methods including RiskMetrics, Duration-based Peak-Over-Threshold (DPOT), and conditional Extreme Value Theory (CEVT) (for both the normal and skewed t-distributions) under different periods. Several portfolios have been constructed from the markets in the two groups as well as the ten index euro-zone group.

Based on the four backtesting evaluation criteria, the results show that the two CEVT methods stand out as the best models for satisfying the backtesting properties for the ten euro-zone equity indices for the full period but compete with DPOT in the subperiods. The RiskMetrics method performs the worst under the full period but performs better under the crisis period (subperiod 2) than the other two periods. The DPOT yields better performance for the Core countries than for the countries in the PIIGS group. The results also show that the VaR-based risk adjusted return ratio for the optimal portfolios for the PIIGS, the Core and the 10 index euro-zone groups varies over the three periods. This ratio is the greatest for subperiod 1 relative to the full period and subperiod 2 for the portfolios of the three groups. Additionally, the optimal diversification results suggest that each of the two well-integrated euro-zone groups should have mainly one euro-zone equity index in its optimal portfolio. Spain's *IBEX* index strongly dominates the PIIGS portfolio for the three periods, while Austria's *ATX* index overwhelms the Core and the expanded ten index equity (euro-zone) portfolios under the full period and subperiod 1. However, in subperiod 2 Germany's *DAX* dominates the Core and the euro-zone portfolios probably because Germany has acquired a great deal of firepower in this crisis period. There is also slightly more room for portfolio diversification for the three groups' portfolios under subperiod 1 than under the full period, reflecting less harmonization and integration over the shorter period. The results of the average daily capital charges for the two subperiods are different than for the full period. They are considerably lower in subperiod 1 than in the full period for both PIIGS and Core groups. On the other hand, those capital requirements are higher for some countries in subperiod 2 than the full period and subperiod 1.

The paper is organized as follows. After this introduction, Section 2 presents a review of the VaR literature on euro-zone and Europe. Section 3 provides the VaR estimation methods and the construction of the optimal portfolios for the euro-zone. Section 4 discusses the data and the empirical results for the three periods under consideration. Section 5 constructs the optimal portfolios for the three groups under the three periods. Section 6 concludes.

2. Literature review

The review of the literature does not produce many studies that apply the various VaR estimation methods to the euro-zone and European stock markets, whether as individual assets and/or portfolios.

Cotter (2004) applies the Extreme Value Theory, among others, to measure the downside risk for five European equity indices: the *ISEQ* (Ireland), *FTSE* 100 (UK), *CAC* 40 (France), *DAX* 100 (Germany) and *IBEX* 35 (Spain) from the beginning of 1998 to the end of April 1999, which obviously does not cover the recent financial crisis. Cotter's results show that the EVT-VaR dominates alternative approaches, such as the variance/covariance and Monte Carlo methods, in the tail estimation for those equity indices. Moreover, his results also suggest that there is a significant difference across those equity indices in terms of the downside risk during the sample period. Allen (2005) assesses five models which estimate the VaR thresholds for an equally-weighted portfolio comprising three European equity indices, *CAC* 40 (France), *FTSE* 100 (UK) and *SWI* (Switzerland), as well as the *S&P* 500 index. Allen finds the Portfolio-Spillover GARCH model (PS-GARCH) (see McAleer and Veiga, 2008a for more information) provides the best result in terms of meeting the requirement of the Basel Accord among the five models considered.

Billio and Pelizzon (2000) use a multivariate regime-switching (RS) model to estimate the VaRs for 10 individual Italian stocks and also for a portfolio based on these stocks. They find the RS approach outperforms the RiskMetrics and GARCH(1,1) models both in the single asset VaR forecasts and the portfolio VaR estimation.

In the context of optimal portfolio selection, many studies generally focus on using the VaR as an alternative risk measure to the traditional measures of risk that rely on the standard deviation (or variance). The literature includes: Jansen et al. (2000); Basak and Shapiro (2001); Gaivoronski and

Pflug (2005); Krokhmal et al. (2002); and Campbell et al. (2001). Campbell et al. (2001) solve for the optimal portfolios based on a Sharpe-like portfolio performance index, using the VaR from the historical distribution as the risk measure. The optimal portfolio they find is the one which maximizes the expected return subject to the specified levels of VaR constraints. They conclude that their method outperforms the traditional mean-variance framework because the latter is rooted in the assumption of normality which usually underestimates the downside risk. However, this method has not been used within the context of the global financial crisis. Gaivoronski and Pflug (2005) provide a method to calculate the mean-VaR efficient frontier using a smoothed VaR estimation. Their experimental results show that the mean-VaR efficient portfolios differ substantially from the mean-variance efficient portfolios. Particularly, for the portfolios which consist of 16 market indices: eight Morgan Stanley Equity Price Indices for USA, UK, Italy, Japan, Russia, Argentina, Brazil and Mexico, and eight Morgan Stanley Bond Indices for the same countries, the VaR optimal portfolios constitute a substantial improvement over the variance optimal portfolios in term of the magnitude of the estimated portfolio VaRs. In 50% of their experiments, the improvement is over 10%.

The literature on equity portfolio diversification in Europe and euro-zone focuses on comparing diversification over countries with diversification over industries. In 1990 and before the creation of the euro-zone, some studies find that diversification over countries yields more efficient portfolios than diversification over industries (see Heston and Rouwenhorst, 1995). This result has been attributed to the unification process and the harmonization of economic policies in the euro-zone. In the 2000s, the literature finds evidence of increasing consequences for the industry factors in driving asset returns in European financial markets but the dominance remained for the country factors (see Rouwenhorst, 1999; Carrieri et al., 2004; Ge'rard et al., 2002; Adjaoute' and Danthine, 2001, 2002, 2004). This result has been aided by the information technology/internet "bubble" (known as IT-hype). Adjaoute' and Danthine (2001) find that diversification opportunities within the then 15 member euro-zone have been reduced. The authors find the culprit to be the convergence of economic structures and homogenization of economic shocks rather than the disappearance of risk.

More recently, employing the mean–variance approach and using recent data, Moerman (2008) finds strong evidence that diversification over industries yields more efficient portfolios than diversification over countries even when the IT-hype is accounted for. Therefore, the evolution of the literature on euro-zone equity market diversification increasingly supports diversification within industries instead of across national markets.

We explore in this study diversification using different combinations of euro-zone stock markets and over different risk-ridden time periods.

3. VaR forecasts models

in this section, we explicitly explain the empirical models that we use to estimate the VaRs for the ten individual equity index returns and the returns for the optimal portfolio based on VaRs for the three periods.

3.1. RiskMetrics

The first method we apply in this paper to estimate the VaRs is the RiskMetrics approach, which is widely used by financial institutions, regulatory departments and portfolio investors. This method is developed by Morgan (1996) and the conditional volatility in this method is estimated based on the exponentially weighted moving average (IGARCH) method:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2$$

where σ_t^2 is the forecast of conditional volatility, $\lambda = 0.94$ is the decay parameter,[2] and ε_{t-1} is the last period residual which follows the standard normal distribution. The VaR is calculated as follows:

[2] λ is set at 0.94 for our daily data as suggested in RiskMetrics.

$$VaR_t = Z_p \sigma_t$$

where Z_p is the standard normal quantile for $p = 0.01$.

The RiskMetrics model is relatively easier to implement than other methods. However, this model is subject to criticism because it ignores the asymmetric effect, the violation of the normality and the risk in the tails of the distribution as often observed in the equity return data. As a remedy, we apply the Extreme Value Theory in the following two promising methods CEVT and DPOT to get a better proxy of the tail distribution.

3.2. Conditional Extreme Value Theory (CEVT)

This approach is a hybrid of a time-varying volatility model and the Peaks-Over-Threshold (POT) method suggested by the Extreme Value Theory (Appendix A provides more details about the POT method). As proposed by Diebold et al. (2000) and McNeil and Frey (2000), we take a two-step process to forecast the VaRs. We first fit an AR(1)-GARCH(1,1) framework with the index return data, estimate $\widehat{\mu}_{t+1|t}$ and $\widehat{\sigma}_{t+1|t}$ and calculate the implied residuals. In the second step, we obtain the p-quantile value for the residual distribution by applying the POT method based on the Extreme Value Theory. Although the filter with normal innovations can remove the majority of clustering, it may still generate a misspecified model. In order to address the misspecification, we also use the filter with the skewed student-t distribution.

The one-day-ahead VaR forecast of CEVT method is calculated with the following equation:

$$\widehat{\text{VaR}}^{CEVT}_{t+1|t}(P) = \widehat{\mu}_{t+1|t} + \widehat{\sigma}_{t+1|t}\widehat{z}_p$$

where $\widehat{\mu}_{t+1|t}$ is the estimated conditional mean, $\widehat{\sigma}_{t+1|t}$ is the estimated conditional standard deviation, which are obtained from the AR(1)-GARCH(1,1) process. Moreover, the quantile $\widehat{z}_p$ for the significance level p is obtained through a Peak-Over-Threshold procedure.[3]

3.3. Duration-based peaks over threshold (DPOT)

The benefit for using the duration-based estimation methodology to forecast the VaRs is to eliminate the tendency of clustering which could be generated through the POT method. The DPOT model focuses on excesses and the durations between excesses instead of the extreme values themselves. This class of models was recently proposed by Araújo-Santos and Fraga-Alves (2012).

Let $x_1, x_2, \ldots x_n$ be the excess returns above the threshold u. t_0 is the start of time, t_i is the day of excess i, and duration $d_i = t_i - t_{i-1}$, and $d_{i,v} = d_i + \ldots + d_{i-v+1} = t_i - t_{i-v}$. At day t, after the excess n, we define the duration since the excess n as d^t. Therefore the information set up to time t is the last durations $d_n, d_{n-1}, d_{n-2}, \ldots d_1$ and d^t.

We also define at day t, after the excess n, $d_{t,1} = d^t$, $d_{t,2} = d^t + d_n$ and for $v = 3,4,\ldots$, $d_{t,v} = d^t + d_n + \ldots + d_{n-v+2}$, which represents the duration until day t since the proceeding v excesses. We assume a Generalized Pareto Distribution for the excess Y_i, which is above the threshold u, such that

$$Y_t \sim GPD\left(\gamma, \sigma_t = g\left(\alpha_1, \ldots, \alpha_k, d^t, d_n + \ldots + d_{n-v+2}\right)\right)$$

where $\gamma, \alpha_1, \ldots, \alpha_k$ are parameters to be estimated.

The one-day-ahead VaR forecast by the POT method is calculated with the following equation as derived in Appendix A:

$$\widehat{VaR}^{POT}_{t+1|t}(P) = \mu + \frac{\widehat{\sigma}_t}{\widehat{\gamma}}\left(\left(\frac{n}{n_x p}\right)^{\widehat{\gamma}} - 1\right)$$

where $\widehat{\sigma}_t = g(\alpha_1, \ldots, \alpha_k, d^t, d_n + \ldots + d_{n-v+2})$

[3] The detail of the POT method is discussed in the Appendix A.

Both the conditional expected value and the conditional variance for the excesses depend on d^t and the last v durations between excesses, respectively, as follows:

$$E[Y_t|\Omega_t] = \frac{\sigma_t}{1-\gamma}, \ (\gamma < 1); \ \text{ and } \ V[Y_t|\Omega_t] = \frac{(\sigma_t)^2}{(1-2\gamma)}, \ \left(\gamma < \frac{1}{2}\right)$$

where Ω_t is the information set which is available until t.

The empirical study of the equity indices suggests an inverse relationship between the expected value and variance of excesses, and the durations between excesses. This relationship is captured by the duration-based term $1/(d_{i,v})^c$, $c > 0$, which is incorporated in the parameter σ_t:

$$\sigma_t = \alpha \frac{1}{(d_{i,v})^c}$$

Therefore, the *DPOT* VaR estimator turns to be

$$\widehat{\text{VaR}}_{t+1|t}^{\text{DPOT}(v,c)}(p) = \mu + \frac{\widehat{\alpha}}{\widehat{\gamma}\,(d_{i,v})^c}\left(\left(\frac{n}{n_x p}\right)^{\widehat{\gamma}} - 1\right)$$

To estimate the parameters $\widehat{\gamma}$ and $\widehat{\alpha}$, we set $v = 3$, and $c = 0.75$ and apply the Nelder and Mead algorithm to maximize the following log likelihood function:

$$\begin{aligned}\log L(\gamma,\alpha) &= \log \prod_{i=v}^{n} fY_i(y_i) = \log \prod_{i=v}^{n} \left(\frac{\alpha}{(d_{i,v})^c}\right)^{-1} \left(1 + \frac{\gamma}{\alpha} y_i (d_{i,v})^c\right)^{-\left(\frac{1}{\gamma}+1\right)} \\ &= -\sum_{i=v}^{n} \log\left(\frac{\alpha}{(d_{i,v})^c}\right) - \left(\frac{1}{\gamma}+1\right) \sum_{i=v}^{n} \log\left(1 + \frac{\gamma}{\alpha} y_i (d_{i,v})^c\right)\end{aligned}$$

4. Data description and empirical results

4.1. Data description

In this study, we explore the ten euro-zone equity markets individually and in the two selected groups, in addition to the portfolio that combines all ten markets. As indicated earlier, the first group includes the five PIIGS countries: Portugal, Italy, Ireland, Greece and Spain; and the second group, the Core, consists of: Austria, Finland, France, Germany and the Netherlands. The countries in the second group are chosen to match the countries in the first group but have less issues of sovereign debt. We use daily percentage log returns based on the closing spot index values for those ten European equity market indices. To be consistent with the entry dates of all countries' memberships in the euro-zone, we select the daily full sample period which starts on January 2, 2001 and ends on March 8, 2013, thereby yielding a total of 3179 observations of percentage log returns, $r_t = 100 \times (lnp_t - lnp_{t-1})$. This sample period is also divided into pre- and post-global financial crisis subperiods.

The data is obtained from DataStream. Table 1 summarizes the notation and sources for those ten country equity indices included in this paper. The descriptive statistics for the full period are given in Table 2. Over this sample period, the Austrian Traded Index (*ATX*) has the highest average return among the equity indices, while the Greek ATHEX Composite Share Price Index (*ATHEX*) yields the lowest. It's interesting to note that the positive average return goes across some countries of both groups which are Spain, Austria and Germany. This across group positive performance is not strongly affected over the full period by the recent sovereign debt crisis. However, the overall un-weighted average returns over the sample period for the PIIGS group is still negative standing at −0.01, while the average for the Core group is positive and equal to 0.006. Over the first subperiod, which is from January 2, 2001 to November 30, 2007, the average return is all positive for all the markets in both groups, with Austria having the highest average return while the Netherlands yielding the lowest. Not surprisingly, the

Table 1
List of stock market indices.

Name	Symbol	Description	Country
Amsterdam Exchange Index	*AEX*	This market capitalization weighted index is composed of a maximum of 25 of the most actively traded[a] securities on the exchange.	The Netherlands
ATHEX Composite Share Price Index	*ATHEX*	This market capitalization weighted index is composed of the 60 largest[b] companies that traded in the Big Cap category of the Athens stock exchange.	Greece
Austrian Traded Index in EUR	*ATX*	This market capitalization weighted index comprises the 20 with the highest liquidity and market value.	Austria
CAC 40	*CAC*	This market capitalization weighted index composes the 40 largest equities measured by free-float market capitalization and liquidity companies listed on Euronext Paris equity market.	France
Deutscher Aktien Index	*DAX 30*	This market capitalization weighted index composes the 40 largest equities measured by free-float market capitalization and liquidity companies listed on Frankfurt Stock Exchange.	Germany
FTSE MIB (Milano Italia Borsa)	*FTSE*	This index consists of the 40 most-traded stock classes on the exchange.	Italy
IBEX 35(Iberia Index)	*IBEX*	This index is composed of the 35 most liquid securities traded on the Spanish Market	Spain
IBEQ overall index	*ISEQ*	This index is composed of the 20 companies with the highest trading volume and market capitalization liquid securities traded on the Irish Stock Exchange.	Ireland
	OMXH	OMX HELSINKI (OMXH) – FINLAND	Finland
	PSI	PORTUGAL *PSI* GENERAL	Portugal

Notes: All data are obtained from DataStream.
[a] The selection is made on an annual review date in March. It is based on the share turnover over the previous year.
[b] The companies are ranked on the basis of their trading value excluding blocks.

average return is negative for all the ten markets for the crisis subperiod which ranges from December 1, 2007 to March 8, 2013.

In terms of volatility as defined by the standard deviation, the Greek *ATHEX* has the highest volatility, while the Portuguese *PSI* interestingly has the lowest over the full period. Higher volatility also

Table 2
Descriptive statistics for index returns (Full period).

Group 1	*ATHEX*	*FTSE*	*IBEX*	*ISEQ*	*PSI*
Mean	−0.0297	−0.0209	0.0084	−0.0024	−0.0067
Median	0.0225	0.0379	0.0241	0.0673	0.0361
Maximum	14.6372	12.3809	14.9681	9.9495	11.6804
Minimum	−11.3662	−10.8636	−10.6569	−15.1513	−12.6447
Std. Dev.	1.9338	1.7958	1.7761	1.6601	1.4214
Skewness	0.0104	−0.0847	0.0605	−0.6330	−0.1900
Kurtosis	7.6699	8.4052	8.7985	10.0645	10.6151
Jarque-Bera	2888.743	3873.771	4455.648	6822.989	7700.365
Group 2	*AEX*	*ATX*	*CAC*	*DAX*	*OMXH*
Mean	−0.0084	0.0373	−0.0034	0.0169	−0.0125
Median	0.0519	0.0718	0.0367	0.0678	0.0050
Maximum	12.3159	12.6114	12.1434	12.3696	9.9142
Minimum	−11.8564	−12.5361	−11.7369	−9.6010	−16.3144
Std. Dev.	1.7309	1.7884	1.7500	1.7740	1.9276
Skewness	−0.0936	−0.2886	0.0262	−0.0521	−0.2365
Kurtosis	9.19862	9.6776	8.5546	7.3448	7.3602
Jarque-Bera	5094.092	5950.555	4087.315	2501.937	2547.906
Observations	3179	3179	3179	3179	3179

Notes: The full sample covers the period 1/2/2001–3/8/2013.The indices and their associated markets are as follows: *ATHEX* (Greece), *FTSE* (Italy), *IBEX* (Spain), *ISEQ* (Ireland), *PSI* (Portugal), *AEX* (the Netherlands), *ATX* (Austria), *CAC* (France), *DAX* (Germany), and *OMXH* (Finland). The descriptive statistics for the two subperiods are discussed but are not tabulated. They are however available upon request.

goes across both euro-zone groups. The un-weighted average volatility for the PIIGS is 1.718, while the average for the Core is 1.795 over the full period. In subperiod 1 the volatility is much lower for both groups than for the full sample. However, the volatility over subperiod 2 is almost 50% higher than in subperiod 1 and is also considerably higher than over the full period due to both the Great Recession in the United States and the debt crisis in the euro-zone.

The skewness results over the full period are also mixed across the two groups. Seven indices (*AEX* for the Netherlands, *ATX* for Austria, *DAX* for Germany, *FTSE* for Italy, *ISEQ* for Ireland, *OMXH* for Finland and *PSI* for Portugal) have negative skewness statistics, which means the mass of the distribution of returns is concentrated on the right part. However, the returns for the other three mixed countries (*ATHEX* for Greece, *CAC* for France and *IBEX* for Spain) are positively skewed, which implies a higher chance of getting lower return in the equity markets of these countries. All the series have a Kurtosis value higher than 3, which means their distributions are more peaked than the normal distribution. Moreover, The Jarque-Bera statistic suggests a rejection of the normality hypothesis for the distributions of all the series. Interestingly, skewness is negative for all markets in both groups under subperiod 1, while they are mixed across groups over the crisis subperiod. The kurtosis is mixed among the two groups over the two periods but is still higher than the normal distribution. However, it is generally higher over the full period than the two subperiods.

4.2. Structural break tests and choice of two subperiods

We use a dataset which covers a subperiod that overarches the financial crisis and its aftermath as well as the more tranquil period that precedes it. The consideration of changes in these two subperiods should reflect changes in expectations about risk and investors' behavior. Therefore, abrupt changes in time-series dynamics are expected to lead us to investigate whether there is a structural break in the time series. In the second subperiod, the markets are more volatile than in the first subperiod of the sample, thereby we apply a structural break test to the unconditional variance. We apply the IT test (Inclán and Tiao, 1994), which does not impose any functional form on the conditional mean or the volatility process. Assuming that $\{r_t\}_{t=1}^{T}$ is a stochastic process where T is the number of random variables, the IT test statistic is given as

$$IT = \sqrt{\frac{T}{2}}\max_{1\le k\le T}\left|D_T(k)\right|, \text{with } D_T(k) = \left[\left(\sum_{t=1}^{k} r_t^2 \Big/ \sum_{t=1}^{T} r_t^2\right) - k/T\right],$$

where $D_T(0) = D_T(T) = 0$, $k = 1,\ldots, T$. The estimator of the break-date, denoted by $\widehat{k}$, is obtained by solving $\text{argmax}_k|D_T(k)|$. In our case r_t denotes the return at day t. The significance of the estimator $\widehat{k}$ is addressed through the IT test statistic whose asymptotic distribution is the supremum of a standard Brownian Bridge process (SSBB) under certain restrictions. The results of the tests and the break dates estimates are given in Table 3 for the (percentage log) returns of the ten euro-zone time series. In all cases, the null hypothesis of no structural breaks in the unconditional variance is clearly rejected, with the observed values of the IT test statistic being much greater than the 99th percentile from the SSBB that is equal to 1.628. As expected, the evidence that we obtain on structural breaks in the unconditional variance is very strong for all of the time series. In eight of the ten cases, the break date estimate occurs during 2008 and in one case it happens during 2007, while the only exception to a date occurring between 2007 and 2008 is the *OMXH* time series which has its break date taking place during 2002. The *OMXH* result is due to an exceptional volatility in the *OMXH* index occurring in the beginning of the sample. The great majority of the break dates is occurring in the period 2007–2008, and this supports a choice of a first period until the end of 2007 and a second period since the final of 2007. We choose the first subperiod from January 2, 2001 to November 30, 2007, and the second subperiod, the crisis period, from December 1, 2007 to March 8, 2013. This time separator is a well-known date and coincides officially with the beginning of the Great Depression in the United States and foreshadows the euro-zone debt crisis. Moreover, choosing a break date before the actual occurrence of the euro-zone debt crisis in 2009 accommodates our estimation over a rolling window of 1000 observations.

Table 3
IT structural break in variance test and break date estimation.

Group 1	*ATHEX*	*FTSE*	*IBEX*	*ISEQ*	*PSI*
Break date	9-2-2008	7-3-2008	9-2-2008	9-3-2008	12-3-2002
$\widehat{k}$	2001	1958	2001	2002	501
IT test statistic	7.5994*	14.0466*	9.7601*	6.9829*	6.3377*
Group 2	*AEX*	*ATX*	*CAC*	*DAX*	*OMXH*
Break date	9-2-2008	7-3-2008	9-2-2008	9-3-2008	12-3-2002
$\widehat{k}$	2001	1958	2001	2002	501
IT test statistic	7.5994*	14.0466*	9.7601*	6.9829*	6.3377*

Notes: The full sample covers the period 1/2/2001–3/8/2013.The indices and their associated markets are as follows: *ATHEX* (Greece), *FTSE* (Italy), *IBEX* (Spain), *ISEQ* (Ireland), *PSI* (Portugal), *AEX* (the Netherlands), *ATX* (Austria), *CAC* (France), *DAX* (Germany), and *OMXH* (Finland). The estimated number $\widehat{k}$ refers to the number of observations where the break takes place. * All the observed IT test statistics are much greater than the critical point, which is 1.628, at the 1% significance level.

4.3. Backtesting results

In this section, we assess the accuracy and the performance of the VaR estimation models used in this paper over the three periods under consideration. Following the approach proposed by Campbell et al. (2001), we obtain one-day-ahead VaR forecasts for each model. For every equity index, the VaR forecast is calculated with a rolling window of 1000 days. Therefore, we obtain 2179 one-day-ahead 99% VaR forecasts for each index per method over the full sample period. The programs that are used to obtain the one-day-ahead VaR forecasts and to apply the accuracy tests are written in the *R* language (R Development Core Team, 2008). The primary tool for assessing the accuracy of the interval forecasts is to monitor the binary sequence generated by observing whether the return r_t on day t is in the tail region specified by the VaR at time $t-1$. This sequence is referred to as the hit sequence:

$$h_t = \begin{cases} 1, if \;\; r_t < \mathrm{VaR}_{t|t-1(p)} \\ 0, if \;\; r_t \geq \mathrm{VaR}_{t|t-1(p)} \end{cases}$$

Christoffersen (1998) shows that evaluating interval forecasts can be reduced to examining whether the hit sequence satisfies the unconditional coverage (UC) and independence (IND) properties. When both properties are validated, we say that the hit sequence satisfies the conditional coverage (CC) property. In order to test the UC hypothesis, we apply the Kupiec test (Kupiec, 1995), while to test the CC hypothesis we apply the conditional coverage test developed by Christoffersen (1998). To test the IND hypothesis alone, we apply the independence test that was recently introduced in the literature by Araújo-Santos and Fraga-Alves (2012). This test is based on durations between consecutive violations and until the first violation. We refer to this test as the Maximum-Median (MM) independence test.

4.3.1. Percentage of violations

The RiskMetrics method gives the highest percentage of violations among all the methods for all of the equity indices across two groups under the whole period. The PIIGS indices have generally similar percentage of violations compared to the Core group according to the RiskMetrics model. In the PIIGS group, Italy has the highest percentage of violations, while in the Core group the Netherlands has the highest violations. The DPOT generally gives more violation percentages to the PIIGS than to the Core. The CEVT methods give lower percentage of violations for the Core than the PIIGS with the exception of Austria and Finland. These methods give lower violations than DPOT for the countries in the PIIGS group but this result is mixed for the Core countries for those methods.

The results of percentage of the violations for the subperiod 1 are lower for both groups than for the full sample period, still with the RiskMetrics having the highest percentages. Comparing the methods for subperiod 1, the CEVTs generally give lower numbers than all the other methods for the PIIGS, with the exceptions are Italy and Spain. However, in the Core the DPOT gives lower violation percentages than the two CEVT methods. In the volatile subperiod 2, the percentages are much higher than for subperiod 1 but are closer to the numbers for the full period. Overall, the violation results for the three periods imply that the accuracy of the percentage of violations in the VaR estimation models is country- and period- specific. Thus, decision makers cannot assume in advance that one size fits all the countries in those groups of the euro-zone for this evaluation property.

4.3.2. Unconditional coverage test (UC test)

The results of the UC test are given in Tables 4A–D for the full and the two subperiods. Again, the RiskMetrics performs very poorly over the full period, leading to rejections at 1% significance level of the UC hypothesis that all the hit sequences are equal to zero for the ten equity indices. This suggests that the percentages of violations are higher than 1% in all cases. This result signifies the evolving nature of the volatilities in equity markets according to this method. Nonetheless, the DPOT method improves the UC results significantly over the RiskMetrics method for the Core group, giving rise to only two (*CAC* and *AEX*) rejections out of the five indices at the 5% level. However, in the PIIGS group, the DPOT leads to an improvement for only one country (Ireland) for this property over the RiskMetrics. In contrast, both CEVT models provide much more reliable results in terms of no rejection of the UC property for all the equity indices in both groups, compared to the RiskMetrics and DPOT methods. This implies that the application of the Extreme Value Theory in approximating the tail distributions of returns can help improve the accuracy of the VaR forecasts significantly for both euro-zone groups.

The poor performance of the violation property by the RiskMetrics persists differently over the two subperiods. In subperiod 1, this method gives rise to four rejections (*ISEQ*, *FTSE*, *ATHEX* and *IBEX*) in the PIIGS group, and three (*ATX*, *OMXH* and *AEX*) in the Core group. Strangely, its performance is better in the crisis period, subperiod 2, for the two groups, leading to only two rejections in the PIIGS group and no rejection in the Core group.

The above UC test only focuses on the frequency of the violations of VaR forecasts, but does not consider the case of the clustering zeros and ones in the hit consequence. As a remedy, we conduct the following conditional coverage (CC) test as provided in Christoffersen (2009), accounting for the dynamics of the exceptions by jointly testing for the unconditional coverage and the serial independence of the hit sequence.

4.3.3. Conditional coverage test (CC test)

The results for the conditional coverage (CC) test are also presented in Tables 4A–D for the three periods. In terms of this property for the full period, the RiskMetrics still performs very poorly as expected, with rejections of the null hypothesis for all the ten equity indices, as has been the case with the previous two properties. With the DPOT approach, the CC results have not improved much over the RiskMetrics, with nine rejections for the countries in the both groups and the only exception is Ireland. For the CEVT models, all the VaR forecasts for all ten indices pass the CC test as has also been the case with the previous properties.

Under the subperiods, the estimation models perform better compared to the full period, probably because of taking into account the structural breaks. Under subperiod 1, the RiskMetrics performs better in this subperiod than the full period, giving rise to four rejections (Ireland, Italy, Greece and Spain) in the PIIGS group and only three (Austria, Finland, France and the Netherlands) in the Core group. The performance of the DPOT improves significantly over this subperiod, having one rejection which is Greece in the PIIGS group. The CEVT methods have no rejection for both groups. As we have seen before, the RiskMetrics performs well in the crisis period (subperiod 2), having no rejections whatsoever, and so does the DPOT method compared to the full period and subperiod 1. The moving window starts out with a high volatility which carries on throughout this crisis period. The same logic applies to the two CEVT methods.

4.3.4. Maximum-Median independence (MM) test

In order to provide more insights into the independence property of the equity index returns, we apply the MM independence test. This more recent test has more power than the CC test in testing the independence hypothesis because it considers all types of clustering, while the CC test is only sensitive to the violations following the Markov-Chain process.[4] The results of the MM test are included in Tables

[4] The CC test is based on the assumption that the probability of a violation is only affected by the most recent period.

Table 4-A
Backtesting results for individual equity indices (Full period – One day ahead forecasts).

	RiskMetrics	DPOT	CEVT - n	CEVT - sstd
PIIGS countries				
Portugal (*PSI*)				
% of viol.	0.0197	0.0146	0.0119	0.0119
Kupiec uc	16.2476 (0.00)	4.222 (0.03)	0.7737 (0.37)	0.7737 (0.37)
MM ind	0.2506 (0.61)	0.5718 (0.49)	1.9862 (0.22)	2.1955 (0.19)
Christ. cc	16.2939 (0.00)	5.1864 (0.07)	1.7759 (0.41)	1.7759 (0.41)
Ireland (*ISEQ*)				
% of viol.	0.0220	0.0110	0.0087	0.0087
Kupiec uc	23.7157 (0.00)	0.2191 (0.63)	0.3771 (0.53)	0.3771 (0.53)
MM ind	1.8957 (0.20)	3.2615 (0.10)	0.8967 (0.50)	0.8967 (0.50)
Christ. cc	24.4420 (0.00)	1.4566 (0.48)	2.3759 (0.30)	2.3759 (0.30)
Italy (*FTSE*)				
% of viol.	0.0234	0.0146	0.0123	0.0128
Kupiec uc	28.7155 (0.00)	4.2226 (0.03)	1.1694 (0.27)	1.6400 (0.20)
MM ind	3.0297 (0.10)	3.8682 (0.06)	2.5265 (0.19)	2.0840 (0.20)
Christ. cc	31.1887 (0.00)	5.1864 (0.07)	1.8521 (0.39)	2.3751 (0.30)
Greece (*ATHEX*)				
% of viol.	0.0192	0.0151	0.0123	0.0119
Kupiec uc	14.8922 (0.00)	5.0320 (0.02)	1.1694 (0.27)	0.7737 (0.37)
MM ind	−0.2074 (0.72)	0.0682 (0.68)	0.1583 (0.67)	0.1394 (0.62)
Christ. cc	16.2169 (0.00)	5.4443 (0.06)	1.8521 (0.39)	1.4059 (0.49)
Spain (*IBEX*)				
% of viol.	0.0206	0.0146	0.0119	0.0119
Kupiec uc	19.0996 (0.00)	4.2226 (0.03)	0.7737 (0.37)	0.7737 (0.37)
MM ind	1.4516 (0.31)	1.3251 (0.31)	4.3491 (0.05)	4.3491 (0.05)
Christ. cc	19.1266 (0.00)	4.6998 (0.09)	1.4059 (0.49)	1.4059 (0.49)
CORE countries (Full period – One day ahead forecasts)				
Austria (*ATX*) mm				
% of viol.	0.0206	0.0100	0.0123	0.0123
Kupiec uc	19.0996 (0.00)	0.0020 (0.96)	1.1694 (0.27)	1.1694 (0.27)
MM ind	0.4192 (0.56)	0.8367 (0.44)	−1.1124 (0.96)	−1.1124 (0.96)
Christ. cc	20.0976 (0.00)	1.5113 (0.46)	2.0663 (0.35)	2.0663 (0.35)
Finland (*OMXH*)				
% of viol.	0.0188	0.0091	0.0105	0.0105
Kupiec uc	13.5854 (0.00)	0.1527 (0.69)	0.0666 (0.79)	0.0666 (0.79)
MM ind	5.1946 (0.0)	1.3479 (0.33)	1.6749 (0.32)	1.6749 (0.32)
Christ. cc	13.6676 (0.00)	0.5217 (0.77)	0.5587 (0.75)	0.5587 (0.75)
France (*CAC*)				
% of viol.	0.01881	0.0142	0.0091	0.0096
Kupiec uc	13.5854 (0.00)	3.4766 (0.06)	0.1527 (0.69)	0.0292 (0.86)
MM ind	1.8604 (0.25)	12.0077 (0.00)	0.0480 (0.66)	−0.0007 (0.73)
Christ. cc	13.6676 (0.00)	4.0247 (0.13)	0.5217 (0.77)	0.4374 (0.80)
Germany (*DAX*)				
% of viol.	0.0201	0.0137	0.0091	0.0091
Kupiec uc	17.6504 (0.00)	2.7960 (0.09)	0.1527 (0.69)	0.1527 (0.69)
MM ind	2.0924 (0.18)	8.9104 (0.00)	1.1743 (0.37)	1.1743 (0.37)
Christ. cc	18.7510 (0.00)	3.6417 (0.16)	0.5217 (0.77)	0.5217 (0.77)
The Netherlands (*AEX*)				
% of viol.	0.0224	0.0151	0.0123	0.0119
Kupiec uc	25.3408 (0.00)	5.0320 (0.02)	1.1694 (0.27)	0.7737 (0.37)
MM ind	4.6858 (0.03)	5.4026 (0.02)	−0.4891 (0.84)	0.3846 (0.55)
Christ. cc	27.7319 (0.00)	5.4443 (0.06)	1.8521 (0.39)	1.4059 (0.49)

Notes: The numbers in parenthesis are the *p*-values. The full period is 1/2/2001–3/8/2013.

Table 4-B
Backtesting results for individual equity indices (Subperiod 1 – One day ahead forecasts).

	RiskMetrics	DPOT	CEVT - n	CEVT - sstd
PIIGS countries				
Portugal (*PSI*)				
% of viol.	0.0111	0.0099	0.0074	0.0062
Kupiec uc	0.1114 (0.73)	0.0002 (0.98)	0.5731 (0.44)	1.3416 (0.24)
MM ind	5.5904 (0.11)	1.8021 (0.32)	15.5802 (0.01)	39.6823 (0.01)
Christ. cc	0.3179 (0.85)	0.1611 (0.92)	0.6584 (0.71)	1.3967 (0.49)
Ireland (*ISEQ*)				
% of viol.	0.0174	0.0099	0.0087	0.0087
Kupiec uc	3.6543 (0.05)	0.0002 (0.98)	0.1420 (0.70)	0.1420 (0.70)
MM ind	2.6011 (0.18)	3.0391 (0.18)	1.9310 (0.44)	1.9310 (0.44)
Christ. cc	9.0437 (0.01)	0.1611 (0.92)	4.1221 (0.12)	4.1221 (0.12)
Italy (*FTSE*)				
% of viol.	0.0199	0.0087	0.0099	0.0124
Kupiec uc	6.1809 (0.01)	0.1420 (0.70)	0.0002 (0.98)	0.4479 (0.50)
MM ind	−0.1859 (0.74)	0.8266 (0.64)	0.5171 (0.57)	−0.0351 (0.72)
Christ. cc	6.8517 (0.03)	0.2626 (0.87)	0.1611 (0.92)	0.7051 (0.70)
Greece (*ATHEX*)				
% of viol.	0.0211	0.0174	0.0124	0.0111
Kupiec uc	7.6399 (0.00)	3.6543 (0.05)	0.4479 (0.50)	0.1114 (0.73)
MM ind	1.5969 (0.37)	2.9955 (0.15)	1.2338 (0.39)	1.3392 (0.50)
Christ. cc	8.4766 (0.01)	5.0634 (0.07)	0.7051 (0.70)	0.3179 (0.85)
Spain (*IBEX*)				
% of viol.	0.0223	0.0124	0.0149	0.0149
Kupiec uc	9.2190 (0.00)	0.4479 (0.50)	1.7111 (0.19)	1.7111 (0.19)
MM ind	0.0017 (0.68)	0.6432 (0.53)	0.9230 (0.45)	0.9230 (0.45)
Christ. cc	10.0698 (0.00)	0.7051 (0.70)	2.0853 (0.35)	2.0853 (0.35)
CORE countries (Subperiod 1 – one day ahead forecasts)				
Austria (ATX)				
% of viol.	0.0199	0.0136	0.0111	0.0111
Kupiec uc	6.1809 (0.01)	0.9872 (0.32)	0.1114 (0.73)	0.1114 (0.73)
MM ind	0.0749 (0.66)	1.4799 (0.45)	0.4763 (0.69)	0.4763 (0.69)
Christ. cc	6.8517 (0.03)	3.2187 (0.20)	0.3179 (0.85)	0.3179 (0.85)
Finland (*OMXH*)				
% of viol.	0.0186	0.0087	0.0136	0.0136
Kupiec uc	4.8496 (0.02)	0.1420 (0.70)	0.9872 (0.32)	0.9872 (0.32)
MM ind	0.6458 (0.59)	0.6464 (0.68)	0.4683 (0.67)	0.4683 (0.67)
Christ. cc	5.4383 (0.06)	0.2626 (0.87)	1.3003 (0.52)	1.3003 (0.52)
France (*CAC*)				
% of viol.	0.0136	0.0049	0.0062	0.0074
Kupiec uc	0.9872 (0.32)	2.5153 (0.11)	1.3416 (0.24)	0.5731 (0.44)
MM ind	1.5367 (0.44)	1.2358 (0.46)	13.3922 (0.06)	0.3513 (0.64)
Christ. cc	1.3003 (0.52)	2.5453 (0.28)	1.3967 (0.49)	0.6584 (0.71)
Germany (*DAX*)				
% of viol.	0.0136	0.0062	0.0062	0.0062
Kupiec uc	0.9872 (0.32)	1.3416 (0.24)	1.3416 (0.24)	1.3416 (0.24)
MM ind	2.2629 (0.32)	5.3782 (0.22)	7.6242 (0.15)	7.6242 (0.15)
Christ. cc	1.3003 (0.52)	1.3967 (0.49)	1.3967 (0.49)	1.3967 (0.49)
The Netherlands (*AEX*)				
% of viol.	0.0223	0.0087	0.0124	0.0124
Kupiec uc	9.2190 (0.00)	0.1420 (0.70)	0.4479 (0.50)	0.4479 (0.50)
MM ind	3.7330 (0.09)	5.3138 (0.16)	9.9265 (0.01)	8.3978 (0.02)
Christ. cc	12.7565 (0.00)	0.2626 (0.87)	0.7051 (0.70)	0.7051 (0.70)

Notes: The numbers in parenthesis are the *p*-values. Subperiod 1 is 1/2/2001–11/30/2007.

Table 4-C
Backtesting results for individual equity indices (Subperiod 2 – One day ahead forecasts).

	RiskMetrics	DPOT	CEVT - n	CEVT - sstd
PIIGS countries				
Portugal (*PSI*)				
% of viol.	0.0213	0.008	0.008	0.008
Kupiec uc	3.6718 (0.05)	0.1626 (0.68)	0.1626 (0.68)	0.1626 (0.68)
MM ind	0.6683 (0.53)	6.2424 (0.33)	6.5920 (0.32)	6.5920 (0.32)
Christ. cc	4.0446 (0.13)	0.2071 (0.90)	0.2071 (0.90)	0.2071 (0.90)
Ireland (*ISEQ*)				
% of viol.	0.0133	0.0026	0.0053	0.0026
Kupiec uc	0.3810 (0.53)	2.8768 (0.08)	0.9938 (0.31)	2.8768 (0.08)
MM ind	4.9439 (0.24)	Na	2.9706 (0.31)	NA
Christ. cc	0.5233 (0.76)	2.8674 (0.23)	1.0059 (0.60)	2.8674 (0.23)
Italy (*FTSE*)				
% of viol.	0.0213	0.008	0.0106	0.0106
Kupiec uc	3.6718 (0.05)	0.1626 (0.68)	0.0164 (0.89)	0.0164 (0.89)
MM ind	3.9278 (0.13)	16.3900 (0.16)	4.5193 (0.16)	4.5193 (0.16)
Christ. cc	4.0446 (0.13)	0.2071 (0.90)	0.1043 (0.94)	0.1043 (0.94)
Greece (*ATHEX*)				
% of viol.	0.016	0.016	0.016	0.016
Kupiec uc	1.1537 (0.28)	1.1537 (0.28)	1.1537 (0.28)	1.1537 (0.28)
MM ind	−0.8444 (0.98)	−0.8444 (0.98)	−0.8444 (0.98)	−0.8444 (0.98)
Christ. cc	1.3615 (0.50)	1.3615 (0.50)	1.3615 (0.50)	1.3615 (0.50)
Spain (*IBEX*)				
% of viol.	0.016	0.0106	0.008	0.008
Kupiec uc	1.1537 (0.28)	0.0164 (0.89)	0.1626 (0.68)	0.1626 (0.68)
MM ind	2.8292 (0.22)	6.0177 (0.11)	−0.1084 (0.97)	−0.1084 (0.97)
Christ. cc	1.3615 (0.50)	0.1043 (0.94)	0.2071 (0.90)	0.2071 (0.90)
Core countries (Subperiod 2 – One day ahead forecasts)				
Austria (*ATX*) m				
% of viol.	0.0133	0.0053	0.0106	0.0106
Kupiec uc	0.3810 (0.53)	0.9938 (0.31)	0.0164 (0.89)	0.0164 (0.89)
MM ind	4.6288 (0.26)	0.1205 (0.91)	3.2924 (0.22)	3.2924 (0.22)
Christ. cc	0.5233 (0.76)	1.0059 (0.60)	0.1043 (0.94)	0.1043 (0.94)
Finland (*OMXH*)				
% of viol.	0.016	0.0106	0.0106	0.0106
Kupiec uc	1.1537 (0.28)	0.0164 (0.89)	0.0164 (0.89)	0.0164 (0.89)
MM ind	4.6446 (0.12)	6.3043 (0.10)	3.6885 (0.20)	3.6885 (0.20)
Christ. cc	1.3615 (0.50)	0.1043 (0.94)	0.1043 (0.94)	0.1043 (0.94)
France (*CAC*)				
% of viol.	0.016	0.016	0.008	0.008
Kupiec uc	1.1537 (0.28)	1.1537 (0.28)	0.1626 (0.68)	0.1626 (0.68)
MM ind	4.2155 (0.14)	5.3707 (0.09)	6.0639 (0.34)	6.0639 (0.35)
Christ. cc	1.3615 (0.50)	1.3615 (0.50)	0.2071 (0.90)	0.2071 (0.90)
Germany (*DAX*)				
% of viol.	0.016	0.0106	0.008	0.008
Kupiec uc	1.1537 (0.28)	0.0164 (0.89)	0.1626 (0.68)	0.1626 (0.68)
MM ind	4.2155 (0.13)	5.7762 (0.12)	6.063 (0.34)	6.063 (0.34)
Christ. cc	1.3615 (0.50)	0.1043 (0.94)	0.2071 (0.90)	0.2071 (0.90)
The Netherlands (*AEX*)				
% of viol.	0.0186	0.008	0.0106	0.0106
Kupiec uc	2.2666 (0.13)	0.1626 (0.68)	0.0164 (0.89)	0.0164 (0.89)
MM ind	1.7178 (0.47)	6.1479 (0.34)	5.7762 (0.12)	5.7762 (0.12)
Christ. cc	2.5513 (0.27)	0.2071 (0.90)	0.1043 (0.94)	0.1043 (0.94)

Notes: The numbers in parenthesis are the *p*-values. Subperiod 2 is 12/3/2007–3/8/2013.

Table 4-D
Backtesting results for individual equity indices (Full Period – One week ahead forecasts).

	RiskMetrics	DPOT	CEVT - n	CEVT-sstd
PIIGS countries				
Portugal (*PSI*)				
% of viol.	0.0275	0.0160	0.0160	0.0183
Kupiec uc	9.1902 (0.0024)	1.3766 (0.2406)	1.3766 (0.2406)	2.4792 (0.1153)
MM ind	−0.6549 (0.8894)	5.8767 (0.1432)	0.0211 (0.8329)	−0.1124 (0.7600)
Christ. cc	10.1753 (0.0061)	4.2181 (0.1213)	4.2181 (0.1213)	4.8135 (0.0901)
Ireland (*ISEQ*)				
% of viol.	0.0229	0.0160	0.0183	0.0206
Kupiec uc	5.4226 (0.0198)	1.3766 (0.2406)	2.4792 (0.1153)	3.8372 (0.0501)
MM ind	1.0000 (0.4442)	−0.4684 (0.9379)	1.9804 (0.2952)	1.8626 (0.4131)
Christ. cc	5.9208 (0.0517)	1.6185 (0.4451)	2.7968 (0.2469)	4.2402 (0.1200)
Italy (*FTSE*)				
% of viol.	0.0229	0.0137	0.0183	0.0183
Kupiec uc	5.4226 (0.0198)	0.5653 (0.4521)	2.4792 (0.1153)	2.4792 (0.1153)
MM ind	−0.0567 (0.7288)	0.6765 (0.5553)	1.8813 (0.3097)	1.8813 (0.3093)
Christ. cc	6.9716 (0.0306)	4.0201 (0.1339)	4.8135 (0.0901)	4.8135 (0.0901)
Greece (*ATHEX*)				
% of viol.	0.0252	0.0160	0.0206	0.0206
Kupiec uc	7.2130 (0.0072)	1.3766 (0.2406)	3.8372 (0.0501)	3.8372 (0.0501)
MM ind	0.2096 (0.7353)	−0.1583 (0.8737)	1.5968 (0.4590)	1.5968 (0.4590)
Christ. cc	8.4568 (0.0145)	1.6185 (0.4451)	5.7460 (0.0565)	5.7460 (0.0565)
Spain (*IBEX*)				
% of viol.	0.0229	0.0137	0.0183	0.0137
Kupiec uc	5.4226 (0.0198)	0.5653 (0.4521)	2.4792 (0.1153)	0.5653 (0.4521)
MM ind	−0.6432 (0.9005)	−0.2777 ((0.8418)	−0.2079 ((0.7879)	−0.4223 ((0.8871)
Christ. cc	5.9208 (0.0517)	0.7412 (0.6902)	2.7968 (0.2469)	0.7412 (0.6902)
CORE countries (Full period – One week ahead forecasts):				
Austria (*ATX*) mm				
% of viol.	0.0229	0.0160	0.0206	0.0183
Kupiec uc	5.4226 (0.0198)	1.3766 (0.2406)	3.8372 (0.0501)	2.4792 (0.1153)
MM ind	8.9263 (0.0193)	−1.0177 (0.9981)	0.4763 (0.6911)	−1.1114 (0.9946)
Christ. cc	6.9716 (0.0306)	1.6185 (0.4451)	5.7460 (0.0565)	4.813 (0.0901)
Finland (*OMXH*)				
% of viol.	0.0206	0.0114	0.0114	0.0091
Kupiec uc	3.8372 (0.0501)	0.0936 (0.7596)	0.0936 (0.7596)	0.0292 (0.8642)
MM ind	−0.8278 (0.9738)	−0.6923 (0.9942)	−0.3404 (0.9509)	0.7358 (0.5745)
Christ. cc	5.7460 (0.0565)	0.2132 (0.8988)	0.2132 (0.8988)	0.1020 (0.9502)
France (*CAC*)				
% of viol.	0.0252	0.0137	0.0229	0.0206
Kupiec uc	7.213 (0.0072)	0.5653 (0.4521)	5.4226 (0.0198)	3.8372 (0.0501)
MM ind	1.6124 (0.4284)	−0.9279 (0.9947)	1.7077 (0.3188)	1.8131 (0.4214)
Christ. cc	7.8164 (0.0200)	0.7412 (0.6902)	5.9208 (0.0517)	4.2402 (0.1200)
Germany (*DAX*)				
% of viol.	0.0275	0.0160	0.0229	0.0229
Kupiec uc	9.1902 (0.0024)	1.3766 (0.2406)	5.4226 (0.0198)	5.4226 (0.0198)
MM ind	1.5254 (0.3341)	−0.8662 (0.9901)	−0.0567 (0.7272)	−0.0567 (0.7275)
Christ. cc	13.5835 (0.0011)	1.6185 (0.4451)	6.9716 (0.0306)	11.2452 (0.0036)
The Netherlands (*AEX*)				
% of viol.	0.0206	0.0137	0.0137	0.0137
Kupiec uc	3.8372 (0.0501)	0.5653 (0.4521)	0.5653 (0.4521)	0.5653 (0.4521)
MM ind	0.0485 (0.7978)	0.8349 (0.5167)	1.5353 (0.3821)	1.5353 (0.3811)
Christ. cc	5.7460 (0.0565)	0.7412 (0.6902)	0.7412 (0.6902)	0.7412 (0.6902)

Notes: The numbers in parenthesis are the *p*-values. The full period is 1/2/2001–3/8/2013.
Notes: The numbers in parenthesis are the *p*-values. Full period is 1/2/2001–3/8/2013. We use a rolling window of 200 weeks to make VaR estimations.

4A–D for all the periods. Over the full period, RiskMetrics fails the MM test for one index (*FTSE*) in the PIIGS group and two (*OMXH* and *AEX)* in the Core group. The DPOT model passes the MM test for all indices except Ireland and Italy in the PIIGS group at the 10% level, and France, Germany and the Netherlands in the Core group. However, the CEVT models outperform the RiskMetrics and DPOT models since they pass the MM test for all the countries, with the exception of Spain in the PIIGS group.

Under the subperiods, the performance of the models with respect to the MM test is different than under the full period. For subperiod 1, the RiskMetrics signals only one rejection for all the countries which is for the Netherlands in the Core group, while the DPOT leads to no rejections to any country in both groups. Surprisingly, the CEVT methods fail to pass this test for Portugal in the PIIGS group, while the CEVT-n rejects this test for France and the Netherlands in the Core group. Under subperiod 2, the RiskMetrics does better than in subperiod 1, leading to no rejections in any group. The DPOT leads to marginal rejection to Finland and France in the Core group and no rejections in the PIIGS group under this crisis period. The CEVT methods perform better under this period, with no rejections for any country.

Based on the four evaluation criteria, the CEVT methods stand out as the best models for backtesting properties for the ten euro-zone equity indices for the full period but the results are not conclusive for the subperiods. Moreover, it seems that the RiskMetrics method performs better under the crisis period (subperiod 2) than the other two periods. The DPOT yields better performance for the Core countries than for the countries in the PIIGS group.

We carried out the one week ahead forecasts. We provide the new backtesting results in Table 4-D. The new results suggest that DPOT works better than the CEVT methods for the weekly data as supported by the lower violation percentages. This exercise suggests that changes of the frequency of data change the performance of the estimation methods.

4.3.5. Basel capital requirement

In 1996 the Basel Committee on Banking Supervision (BCBS) issued an Amendment to the Basel I Capital Accord, in which the financial institutions are required to calculate their market risk Minimum Capital Requirement (MCR) based on their own VaR models by using the following formula:

$$\mathrm{MCR}_{t+1} = \max\left(\frac{m_c}{60}\sum_{i=1}^{60}\mathrm{VaR}_{t-i+1};\mathrm{VaR}_t\right)$$

where $m_c = 3 + k$ and $k \in [0,1]$. The MCR is the maximum between the previous day's VaR and the average of the last 60 daily VaRs increased by the multiplier m_c. The multiplier m_c is determined by the backtesing results for the internal VaR models. Essentially, the greater the number of the violations when the actual loss exceeds the daily VaR forecast during the last 250 trading days, the higher the value of the multiplier m_c. A high daily capital charge is undesirable as it reduces profitability while large violations may lead to bank failures, as the capital requirements implied by the VaR threshold forecasts may be insufficient to cover the realized losses. The details of the Basel Accord's three-zone approach are included in Table 5-A.

We present in Table 5-B results of the number of days in the red zone and the daily capital requirements for the ten individual equity indices for the full period, using the four VaR estimation methods. However, the results for the two subperiods are available upon request. Under the full period, the two CEVT methods are more reliable and accurate than the RiskMetrics and DPOT methods as they give zero number of entries in the red zone. It is interesting to note that the CEVT-sstd has generally the lowest number of violations among all the methods, while the RiskMetrics gives the highest. DPOT generally comes in between with the exception of Austria and Finland for which it gives lower violations than the CEVT methods. These results for the number of violations are also not reported but can be available upon request.

The RiskMetrics gives nonzero number of days in the red zone for all countries in the PIIGS group, ranging from 30 days for Italy to 364 days for Ireland. On the other hand, the DPOT gives very spotty results, producing high red day numbers for Italy, Portugal and Spain while giving zero days for Ireland and Greece. We must add that RiskMetrics gives better results for the Core group by producing zero days for two countries, namely Finland and France. DPOT still gives similar spotty performance for the

Table 5-A
Basel accord penalty zones.

Zone	Number of violations	*k*
Green	0–4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: The number of violations is accumulated for the last 250 trading days.

Core group, producing very high red days for Germany, France and the Netherlands while giving zeros for the remaining countries in this group. Having said all that, we should not be surprised that the two CEVT methods produce zero days in the red zone for both groups.

It is also worth noting that under the full period, the CEVT methods do not give the lowest average daily capital charges for all the indices, despite producing zero entries in the red zone. Moreover, the CEVT-normal method generates lower capital charges than the CEVT-sstd for three indices in the GIIPS group and four in the Core groups. The exceptions are PSI and *IBEX* in the PIIGS group and Finland in the Core group. Interestingly, the RiskMetrics gives the lowest capital charges for seven out of the ten countries, which are the lowest capital charges among the four VaR estimation methods. The results of the average daily capital charges for the two subperiods are different than for the full period. They are considerably lower in subperiod 1 than in the full period for both euro-zone groups. However, they are somewhat below the average of the full period for some and higher for other countries. The interesting thing about those subperiods is that they give zero entry days in the red zone for all VaR methods and for all countries of both groups in both subperiods. We are concerned about these subperiods' results because we use rolling windows of 1000 observations to do the backtesting and additional 250 observations to calculate the daily capital requirements for each sub period. Thus, it is possible that the zero entry days in the red zone may not be indicative of increasing accuracy of those methods in the second subperiod but because we only test the observations 1250 days after the onset of the crisis period. The impact of the financial crisis on VaR estimations could have been absorbed in the foregone 1250 observations at the beginning of the estimation of the second subperiod. Therefore, while we report this finding we do not include the tables of those results in this paper.

5. VaR-based optimal portfolios

We construct three optimal portfolios for the PIIGS, CORE and the whole ten euro-zone equity markets for three periods. We apply the procedure proposed by Campbell et al. (2001), which constrains the maximum expected loss by the estimated VaR for a chosen investment horizon at the 99% confidence level. This is a portfolio construction model developed following the line of Arzac and Bawa (1977). We use the similar notation presented in Campbell et al. (2001). The amount invested is denoted by $W(0)$ and r_f is the interested rate that the investor can borrow and lend for the period T. With n assets, $\gamma(i)$ denotes the fraction invested in asset i, $q(p,P)$ denotes the p-quantile of the return distribution for portfolio P and VaR(p,P) is the Value-at-Risk for portfolio P. The following measure for risk is proposed:

$$\varphi(p,P) = W(0)r_f - \mathrm{VaR}(p,P).$$

The mathematical problem resumes to find the optimal portfolio P' by choosing the fractions $\gamma(i)$ that maximize the risk return ratio $S(P)$:

$$\max_P S(P) = \left(r(P) - r_f\right)\Big/(\varphi(p,P))$$

Investors will choose fractions $\gamma(i)$ to maximize $S(P)$.

Table 5-B
Daily capital charges for individual indices (Full period).

	Number of days	Daily capital charges		
	In the red zone	Mean	Maximum	Minimum
PIIGS countries:				
Portugal (*PSI*)				
RiskMetrics	35	11.6004	33.1698	3.5135
DPOT	217	12.3813	24.3401	3.5094
CEVT - sstd	0	12.0973	37.8660	4.3410
CEVT - sn	0	12.1900	39.7377	4.2341
Ireland (*ISEQ*)				
RiskMetrics	364	14.2645	43.2118	5.2024
DPOT	0	15.3002	32.8284	5.7656
CEVT - sstd	0	14.5641	45.2622	6.3504
CEVT - sn	0	14.3882	40.3708	6.2358
Italy (*FTSE*)				
RiskMetrics	30	14.7416	40.6310	4.1421
DPOT	227	15.0634	33.8649	4.9261
CEVT - sstd	0	14.6386	42.4715	5.0902
CEVT - sn	0	14.7380	44.3543	5.0295
Greece (*ATHEX*)				
RiskMetrics	135	15.6059	39.6072	5.3262
DPOT	0	17.3810	30.9675	6.0816
CEVT - sstd	0	15.8273	38.7486	6.5970
CEVT - sn	0	16.0504	39.73.3	6.5493
Spain (*IBEX*)				
RiskMetrics	5	13.9895	40.1655	5.9757
DPOT	56	15.2360	29.8339	5.3908
CEVT - sstd	0	14.5866	43.2858	5.6790
CEVT - sn	0	14.4169	42.0580	5.7405
Core countries (Full period):				
Austria (*ATX*)				
RiskMetrics	84	15.1615	51.9808	6.0915
DPOT	0	16.1930	34.0801	5.9128
CEVT - sstd	0	15.2896	50.9050	6.9044
CEVT - sn	0	15.3544	51.8988	6.6779
Finland (*OMXH*)				
RiskMetrics	0	13.0033	34.8865	5.5072
DPOT	0	14.7372	31.9103	6.2303
CEVT - sstd	0	13.4321	32.4414	6.6855
CEVT - sn	0	13.3558	32.8589	6.4298
France (*CAC*)				
RiskMetrics	0	13.0848	39.0218	4.7202
DPOT	248	14.7477	29.7777	5.5496
CEVT - sstd	0	13.1469	37.3101	5.6133
CEVT - sn	0	12.9324	38.2218	5.5892
Germany (*DAX*)				
RiskMetrics	78	13.0136	41.8529	4.6088
DPOT	248	13.9568	29.2133	5.2953
CEVT - sstd	0	12.9175	41.4741	5.3827
CEVT - sn	0	12.9564	41.1182	5.4133
The Netherland (*AEX*)				
RiskMetrics	76	12.6622	42.6374	4.5256
DPOT	244	13.5340	32.8022	4.7890
CEVT - sstd	0	12.8701	45.8070	4.8979
CEVT - sn	0	12.8887	46.2707	4.8657

Notes: Full period is 11/2/2004–3/8/2013.

We set r_f equal to 2.06% which is the 10-year U.S. Treasury rate available on the last day of the sample period. $W(0)$ is equal to $1000 as the money invested in the portfolio. The VaR is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR.

The VaR-based risk adjusted returns of the optimal portfolios for the PIIGS, Core and the 10 index euro-zone groups vary over the three periods as displayed in Tables 5-A and 5-B. As expected, the risk-return ratio is the greatest for subperiod 1 for the optimal portfolios of the three groups relative to the full period and subperiod 2. This is not surprising because subperiod 1 is the most prosperous of all three periods under consideration. Surprisingly over this subperiod, the PIIGS portfolio yields the greatest risk-return ratio followed by the performance of the most diversified portfolio that includes the ten markets in the euro-zone under consideration. The portfolio for the Core group which is more economically stable and balanced than the PIIGS group comes last over this subperiod. Obviously, the PIIGS countries were living beyond their means in subperiod 1. The full period yields better risk-return ratios than the crisis period (subperiod 2) for the Core and the ten index euro-zone groups but not for the PIIGS group. Strangely, the performance of the PIIGS group is better in subperiod 2 than over the full period, probably because of the foregone 1250 observations in this crisis period.

In terms of optimal portfolio diversification, the optimal portfolio of each group over the three periods is overwhelming dominated by one market index, with marginal weights for the other markets in the respective group. For example, under the full period the PIIGS portfolio is dominated by Spain's *IBEX* index (99.97%), with a 0.03% weight for Greece's *ATHEX* index and zero weights for the other members of this group. In the Core group under this full period, the portfolio dominance is for Austria's *ATX* index (99.09), with 0.9% and 0.03% weights for Germany' *DAX* and Finland's *OMXH* indices, respectively.

There is also a slightly more portfolio diversification for the three groups' portfolios under subperiod 1 than under the full period. This finding may be the result of greater integration and harmonization in the euro-zone over the full period than subperiod 1. Spain's *IBEX* still dominates the PIIGS group in this subperiod and full period as does Austria's *ATX* still dominate the Core and the euro-zone portfolios. These indices give relatively high performance during the historical period. However, in subperiod 2, while *IBEX* remains dominating the PIIGS's portfolio, Germany's *DAX* dominates the Core and the euro-zone portfolios. Germany's has acquired more power in the crisis period than in the full period and Subperiod 1.

As the countries in the PIIGS group drifted apart over the crisis period (subperiod 2), the results show that there is a slight increase in the room for diversification in the optimal portfolio of this group, compared the portfolios of subperiod 1 and the full period. Here Spain's *IBEX* optimal weight drops to 97.7%, while each of the other four members of this group has a positive optimal weight, although small, under this subperiod. This result underlines the importance of diversification with other asset classes like commodities and equity indices in other regions like the *S&P* 500 index in constructing optimal portfolios for the equity markets of the euro-zone countries.

In terms of ranking the portfolios over the three periods as shown in Tables 6A–D, the most diversified equity (grand) portfolio that combines the ten indices is ranked # 1 followed by that of the Core over the full period. Over the most prosperous subperiod (subperiod 1), the PIIGS portfolio ranks first, followed by that of the most diversified group. Finally, the Core portfolio followed by the grand portfolio is ranked over the PIIGS portfolio, reflecting the center of the euro-zone debt crisis. This is also clear evidence that in the subperiod 1 there is economic convergence between the euro-zone countries but in the subperiod 2 there is economic divergence between these countries. With economic divergence, the European integration as a means of attempting to heal the breaches caused by old wars and to solidify the ties between the various countries of Europe for the benefit of all of their citizens is violated. This implication is relevant because it shows a violation of the basic foundation of EMU construction but this topic is beyond the scope of this study.

6. Conclusions

Using the recent daily data from 2001 to 2013, we explore the downside risks for ten individual equity indices in the euro-zone countries. These countries are divided into two groups: the PIIGS countries (Portugal, Italy, Ireland, Greece and Spain) and the Core countries (Austria, Finland, France,

Table 6-A
Estimated VaR-optimal portfolios (Full period).

ATHEX (%)	IBEX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 1: The PIIGS portfolio				
0.03	99.97%	47.03111962	0.0000770	−1.14818E-07

ATX (%)	OMXH (%)	DAX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 2: The Core portfolio					
99.09	0.01	0.9	56.16501634	0.0003100	4.05234E-06

ATX (%)	DAX (%)	IBEX	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 3: The 10 index euro-zone portfolio					
97.21%	0.04%	2.73%	53.34847917	0.000366	5.31519E-06

Notes: Portfolio 1 includes the five equity indices of the PIIGS countries. Daily returns are used in order to find the optimal portfolio at the point where the return-risk ratio $S(P)$ is maximized. The risk-return ratio equation is given by $P'\text{:}\max_P S(P) = (r(P) - r_f)/(\phi(p,P))$, where P is the optimal portfolio, $\phi(p,P) = W(0)r_f - VaR(p,P)$ is the performance measure for risk, $W(0)$ is the amount invested, r_f is the 10 year Treasury rate available on the last day of the sample period which is equal to 2.06%. The VaR for $1000 held in the portfolio is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR. The full period is 1/2/2001–3/8/2013. Notes: Portfolio 2 includes the five equity indices for the Core countries. Notes: Portfolio 3 includes all the ten equity indices in the euro-zone.

Table 6-B
Estimated VaR-optimal portfolios (Subperiod 1).

PSI (%)	ISEQ (%)	ETSE (%)	ATHEX (%)	IBEX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 4: The PIIGS portfolio							
0.01	1.05	0.29	0.15	98.5%	33.67059669	0.0054890	0.000160573

ATX (%)	CAC (%)	DAX (%)	AEX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 5: The Core portfolio						
95.11	4.81	0.03	0.05	31.91562487	0.0009988	2.87137E-05

ATX (%)	DAX (%)	IBEX	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 6: The 10 index euro-zone portfolio					
92.0	0.25	7.75	31.3882	0.001	2.92339E-05

Notes: Portfolio 1 includes the five equity indices for of the PIIGS countries. Daily returns are used in order to find the optimal portfolio at the point where the return-risk ratio $S(P)$ is maximized. The risk-return ratio equation is given by $P'\text{:}\max_P S(P) = (r(P) - r_f)/(\phi(p,P))$, where P is the optimal portfolio, $\phi(p,P) = W(0)r_f - VaR(p,P)$ is the performance measure for risk, W(0) is the amount invested, r_f is the risk-free return is the 10 year Treasury rate available on the last day of the sample period which is equal to 2.06%. The VaR for $1000 held in the portfolio is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR. Subperiod 1 is 1/2/2001–11/30/2007. Notes: Portfolio 2 includes the five equity indices of the Core countries. Notes: Portfolio 3 includes all the ten equity indices in the euro-zone.

Table 6-C
Estimated VaR-optimal portfolios (Subperiod 2).

PSI (%)	ISEQ (%)	ETSE (%)	ATHEX (%)	IBEX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 7: The PIIGS portfolio							
1.63	0.42	0.01	0.01	97.7	63.0857	−0.00054	−9.9753E-06

CAC(%)	DAX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 8: The Core portfolio				
0.5	99.5	62.34635676	−0.000075	−2.5246E-06

DAX (%)	CAC (%)	AEX (%)	Portfolio VaR ($)	Portfolio return	Risk-return ratio
Portfolio 9: The 10 index euro-zone portfolio					
99.89	0.04	0.07	62.32549576	−0.0000790	2.58963E-06

Notes: Portfolio 1 includes the five equity indices of the PIIGS countries. Daily returns are used in order to find the optimal portfolio at the point where the return-risk ratio $S(P)$ is maximized. The risk-return ratio equation is given by $P'\text{:}\max_P S(P) = (r(P) - r_f)/(\phi(p,P))$, where P is the optimal portfolio, $\phi(p,P) = W(0)r_f - \text{VaR}(p,P)$ is the performance measure for risk, $W(0)$ is the amount invested, r_f is the 10 year Treasury rate available on the last day of the sample period which is equal to 2.06%. The VaR for $1000 held in the portfolio is given for a daily time horizon and a 99% confidence level, where the historical distribution is used to estimate the VaR. Subperiod 2 is 12/1/2007–3/8/2013. Notes: Portfolio 2 includes the five equity indices of the Core countries. Notes: Portfolio 3 includes all the ten equity indices in the euro-zone.

Table 6-D
Ranking of portfolios over the three periods.

Ranking	Full period	Subperiod 1	Subperiod 2
1	10 index portfolio	PIIGS portfolio	Core portfolio
2	Core portfolio	10 index portfolio	10 index portfolio
3	PIIGS portfolio	Core portfolio	PIIGS portfolio

Notes: The ranking of portfolios is ordered by the VaR based risk-return ratios reported above.

Germany and the Netherlands) and also all combined in one grand group. The backtesting is estimated for the individual countries using four VaR estimation methods: RiskMetrics, DPOT, CEVT-normal and CEVT-student *t* and evaluated utilizing four evaluation criteria (percentage of violations, unconditional coverage, conditional coverage and maximum median), in addition to computing the daily capital charges as specified in the Basel Accord. The sample period is classified into three periods: the full period, the pre-crisis period (subperiod 1) and the post-crisis period (subperiod 2).

We also explore the downside risk for optimal portfolios of the two groups as well as for the ten index (euro-zone) portfolio for the three periods. We test for the most appropriate Value-at-Risk (VaR) method for the individual market indices under the three periods.

Given the evidence we collected for the individual equity index VaR forecasts, the basktesting evaluation criteria imply that the CEVT methods are the best performer among all the estimation methods for the full period. On the other hand, the RiskMetrics method performs the worst under the full period but performs better under the crisis period (subperiod 2) than the other two periods. The DPOT yields better performance for the Core countries than for the countries in the PIIGS group and it competes with the CEVT methods in the subperiods.

If the minimum capital requirement is the only concern, the RiskMetrics method gives the lowest mean capital requirement for the individual indices, which rewards the financial institutions who apply this method the opportunity to earn higher profits than other institutions who utilize different advanced VaR estimation methods such the CEVT methods. However, the probability that this risk management strategy would succeed is low because this model has the worst performance in terms of the number of entries in the red zone (which happens in 8 out of the 10 cases for the full period). With employing the RiskMetrics model, the probability of entering in the red zone is high and the consequences of this entering can be severe and damaging.

We examine portfolio diversifications across the ten equity indices. By assessing the historical performance of the VaR-based equity portfolios for the PIIGS and Core groups, the results demonstrate that the optimal portfolio is overwhelmingly dominated by one index for each group for the three subperiods, which implies limited diversification benefits within the euro-zone. We find that the optimal PIIGS portfolio is comprised of over 99% and 98% of the Spanish *IBEX* index, respectively, over the full period and subperiod 1 which has less harmonization and integration. Similarly, the optimal Core portfolio consists of about 99% of the Austrian *ATX* index in full period and 95% in subperiod 1, respectively. However, in subperiod 2 the countries in the PIIGS group have slightly drifted apart from each other under the pressure of the debt crisis and have a tad of more room for diversification as the dominance of *IBEX* dropped to 97.7%. The interesting thing about subperiod 2 is that *DAX* has replaced *ATX* as the dominant index in the Core and the grand euro-zone portfolios over this subperiod. During this crisis period, Germany holds all the punches and persists to be the strongest and most stable economy in Europe.

Thus, any diversification within the euro-zone markets is not expected to produce great diversification gains. Consequently, any diversification with other asset classes such as commodities like oil and gold should give greater diversification benefits. This conclusion will be our next research project.

Appendix A. The POT method for VaR forecasts

We consider the following Generalized Pareto Distribution (GPD):

$$G_{\gamma,\sigma}(y) = \begin{cases} 1-(1+\gamma y/\sigma)^{-\frac{1}{\gamma}}, & \gamma \neq 0 \\ 1-\exp\left(-\frac{y}{\sigma}\right), & \gamma = 0 \end{cases} \tag{1}$$

where $\sigma > 0$, and the support is $y > 0$ when $\gamma \geq 0$; and $0 \leq y \leq -\sigma/\gamma$ when $\gamma < 0$.

The probability that a random variable X assumes a value that exceeds a threshold u by at most y, given that it does exceed that threshold, can be represented by the excess distribution:

$$F_u(y) = P[X - u \le y | X > u] = \frac{F(y+u) - F(u)}{1 - F(u)} \tag{2}$$

for $0 \le y \le x^F - u$, where x^F is the right endpoint of F. The Extreme Value Theory (EVT) suggests the GPD (i.e., Eq. (1)) as an approximation for the excess distribution (i.e., Eq. (2)), for a sufficiently high threshold u for a wide class of distributions.

Let $\overline{F(y)} = 1 - F(y)$, Eq. (2) can be transformed as:

$$\overline{F_u(y)} = 1 - P[X - u \le y | X > u] = \frac{\overline{F(y+u)}}{\overline{F(u)}} \tag{3}$$

$$\text{Let } x = y + u, \ \overline{F_u(x-u)} = \frac{\overline{F(x)}}{\overline{F(u)}} \tag{4}$$

Smith (1987) proposed a tail estimator based on the approximation of a GPD to the excess distribution. For a sample of size of n_x, let n be the number of observations that are above the threshold u. Then, n/n_x is an estimator of $\overline{F(u)}$. Plug in $\overline{F_u(x-u)}$, the term $(1 + \gamma y/\sigma)^{-\frac{1}{\gamma}}$ obtained from Eqs. (1)–(4), we get the tail estimator:

$$\widehat{\overline{F(x)}} = \frac{n}{n_x}\left(1 + \widehat{\gamma}\frac{x-u}{\widehat{\sigma}}\right)^{-1/\widehat{\gamma}}, \text{valid for } x > u. \tag{5}$$

When we forecast the VaRs, we need to know the quantile z_p responding to the specified significance level p. For $p = \overline{F(x)}$, we invert Eq. (5) and get the VaR POT estimator:

$$\widehat{VaR}^{POT}_{t+1|t}(P) = \mu + \frac{\widehat{\sigma}}{\widehat{\gamma}}\left(\left(\frac{n}{n_x p}\right)^{\widehat{\gamma}} - 1\right)$$

References

Adjaouté, K., Danthine, J.P., 2001. EMU and Portfolio Diversification Opportunities. CEPR Discussion Paper No. 2962, London.
Adjaouté, K., Danthine, J.P., 2002. European financial integration and equity returns: a theory-based assessment. In: Gaspar, V., Hartmann, P., Sleijpen, O. (Eds.), The Transformation of the European Financial System. European Central Bank, Frankfurt, pp. 185–245.
Adjaouté, K., Danthine, J.P., 2004. Portfolio diversification: alive and well in Euroland! Appl. Financ. Econ. 14, 1225–1231.
Allen, D.E., 2005. Modeling and forecasting dynamic VaR thresholds for risk management and Regulation. In: Siamese International Conference on Modeling and Simulation, SIMMOD, The Rose Garden, Bangkok, Thailand.
Araújo-Santos, P., Fraga-Alves, M.I., 2012. A new class of independence tests for interval forecasts evaluation. Comput. Stat. Data Anal. 56 (11), 3366–3380.
Araújo-Santos, P., Fraga-Alves, M.I., 2013. Forecasting value-at-risk with a duration-based POT method. Mathematics and Computers in Simulation 94, 295–309.
Basak, S., Shapiro, A., 2001. Value-at-risk based risk management: optimal policies and asset prices. Rev. Financial Stud. 14, 371–405.
Billio, M., Pelizzon, L., 2000. Value-at-Risk: a multivariate switching regime approach. J. Empir. Finance 7, 531–554.
Campbell, R., Huisman, R., Koedijk, K., 2001. Optimal portfolio selection in a Value-at-Risk framework. J. Bank. Finance 25, 1789–1804.
Carrieri, F., Errunza, V., Sarkissian, S., 2004. Industry risk and market integration. Manag. Sci. 50, 207–221.
Christoffersen, P., 1998. Evaluating intervals forecasts. Int. Econ. Rev. 39, 841–862.
Christoffersen, P., 2009. Value-at-risk models. In: Andersen, T., Davis, R., Kreiss, J.-P., Mikosch, T. (Eds.), Handbook of Financial Time Series. Springer Verlag.
Cotter, J., 2004. Downside risk for European equity markets. Appl. Finance Econ. 14 (10), 707–716.
Diebold, F.X., Schuermann, T., Stroughair, J.D., 2000. Pitfalls and opportunities in the use of extreme value theory in risk management. J. Risk Finance 1 (2), 30–35.
Gaivoronski, A.A., Pflug, G., 2005. Value-at-risk in portfolio optimization: properties and computational approach. J. Risk 7, 1–31.
Gérard, B., Hillion, P., De Roon, F.A., Eiling, E., 2002. The Structure of Global Equity Returns: Currency, Industry and Country Effects Revisited (SSRN Working Paper).

Heston, S.L., Rouwenhorst, K.G., 1995. Industry and country effects in international stock returns. J. Portfolio Manag. 21 (3), 53–58.
Inclán, C., Tiao, C.J., 1994. Use of cumulative sums of squares for retrospective detection of changes of variance. J. Am. Stat. Assoc. 89, 913–923.
Jansen, D.W., Koedijk, K.G., de Vries, C.G., 2000. Portfolio with limited downside risk. J. Empir. Finance 7, 247–269.
Krokhmal, P., Palmquist, J., Uryasev, S., 2002. Portfolio optimization with conditional value-at-risk objective and constraints. J. Risk 4, 43–68.
Kupiec, P., 1995. Techniques for verifying the accuracy of risk measurement models. J. Deriv. 3, 73–84.
McAleer, M., da Veiga, B., 2008. Forecasting value-at-risk with a parsimonious portfolio spillover GARCH (PS-GARCH) model. J. Forecast. 27 (1), 1–19.
McNeil, A., Frey, R., 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. J. Empir. Finance 7, 271–300.
Moerman, G.A., 2008. Diversification in euro area stock markets: country versus industry. J. Int. Money Finance 27, 1122–1134.
R Development Core Team: R, 2008. A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0. URL. http://www.R-project.org.
Rouwenhorst, K.G., 1999. European equity markets and the EMU. Finan. Anal. J. 55 (3), 57–64.
Smith, R., 1987. Estimating tails of probability distributions. Ann. Stat. 15, 1174–1207.